OPENING EYES TO MATHEMATICS

CONTACT & INSIGHT LESSONS / VOLUME 2

Debby Head, Libby Pollett and Michael J. Arcidiacano

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Debby Head
M.A., Elementary School Teacher

Libby Pollett
M.A., Elementary School Teacher

Michael J. Arcidiacono
D.Ed., The Math Learning Center

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Contents

Contact Lessons

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*Fraction Bars® is a registered trademark of Scott Resources, Inc.

Contact Lessons (continued)

Insight Lessons

Contact Lessons
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92 Measurement—Area

You Will Need

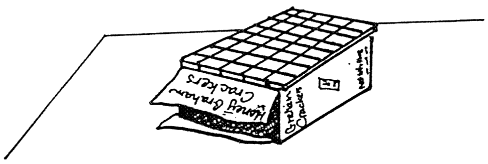
- Chapter 7, Measurement
- a box of graham crackers for each group of four children
- base ten counting pieces
- chart paper

Your Lesson

What is the area of the front of a graham cracker box?

Divide your class into groups of four. Have each group answer the above question using a child's hand as the unit of measure. Encourage them to make an estimate first. Conduct conversations with the groups as they work. Does it matter whose hand is used?

When the groups are ready, have them record their estimates and answers on chart paper. Discuss the results. There will likely be disagreement since hands are not all the same size and don't fit together easily. This can motivate a discussion of standard units of area. Remind the children that squares are customarily used for this purpose.



Ask the groups to estimate and measure the area of the front of their boxes once more, using a unit from their set of base ten pieces as a unit of area. Have them record their results on chart paper and conduct a show-and-tell discussion of their work.

Did any group tile the front with the help of mats and strips?

Was any group able to exactly cover the front with its pieces? If not, how might the total area be reported?

What would the front area be if it were rounded to the nearest ten? to the nearest hundred?

How many units would it take to cover the front of five graham cracker boxes?

Are the fronts of all graham cracker boxes the same size?

As time permits, discuss the use of other units of area. For example, would it be helpful to use an 8½ by 11 piece of paper as a unit for covering the front of the box? Or, how about a square centimeter? a square yard?

Emphasis

Standard units are fixed in size and shape and are used to communicate measurements easily and accurately.

An important part of the measurement process is to select a unit that is appropriate to the task—a square yard is not a convenient unit for measuring the area of the front of a graham cracker box.

If two boxes have fronts that are congruent (that is, the same size and shape), then the fronts will have the same area.

Note: The 100th Day Celebration is almost here and it's none too soon to begin preparing for it! We have children display collections of 100 items (comics, swimming ribbons, baseball cards, etc.) on that day; encourage them to begin thinking about what they might bring.

Additional Ideas

93 Measurement—Length

You Will Need

- Chapter 7, Measurement
- the graham cracker box patterns from Contact Lesson 91
- chart paper

for each child

- linear units
- rulers (inch and centimeter)
- paper

Your Lesson

Give each group of four one of the graham cracker box patterns formed in Contact Lesson 91. Ask the groups to examine the lengths of the rectangles present in their patterns and imagine ordering them from longest to shortest. How many different lengths are there? Which is longest?

Ask each child to trace the different lengths of the rectangles on paper and to number them from longest to shortest. Discuss questions such as these: How long is each line? About as long as a pencil? a popsicle stick? Children may have a difficult time discussing these lengths without some common point of reference. As with areas (Contact Lesson 92), standard units are used to measure and discuss lengths.

Have each child estimate and measure each length in terms of linear units. Post their answers on chart paper and discuss. Is there general agreement among the answers? Was each length an exact number of linear units long? If not, how did the children report their answers? Did they estimate fractional parts of a linear unit in any special way?

Ask the children to measure the lengths two more times using a centimeter and an inch ruler.

As time permits, discuss the use of other units of length. For example, could the lengths of the above rectangles be reported in terms of feet? or miles? Point out that one could measure the lengths with a unit such as miles, but it would not be appropriate to do so.

Teacher Tips

When measuring, children need to "begin at the beginning", placing the first linear unit (or their rulers) so that one end coincides with the beginning of the line being measured. You may wish to model this part of the measuring process.

Journal Writing

How did you feel about this activity and what did you learn from it?

94 Measurement—Perimeter

You Will Need

- Chapter 7, Measurement

for each group of four children

- a graham cracker box
- a set of linear units
- centimeter ruler

Your Lesson

Once again, your boxes appear. Give one to each group of four. Using a linear unit as the unit of length, ask the groups to:

Estimate the perimeter of the front of the box.

Determine the perimeter of the front and compare it with their estimates.

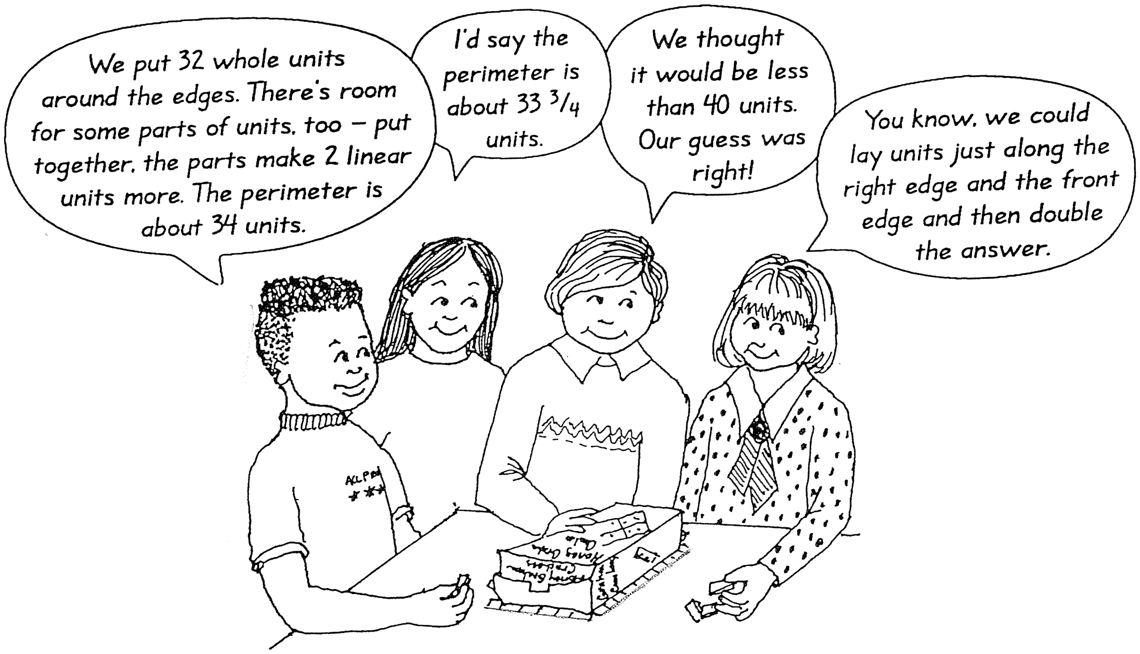
Write a brief paragraph to summarize their work, including a diagram that indicates their measurements.

Repeat the above activities once more, this time ask the groups to determine the perimeter of the front of their boxes using a centimeter ruler.

Discuss the results of these activities. What strategies were used to estimate and determine the perimeter? How does the perimeter of the front compare to each edge of the front?

If longer units were used to measure lengths, would it take more or less of them to determine the perimeter of the front of the box? if shorter units were used?

94 Measurement—Perimeter (continued)



As time permits, have the groups determine the perimeter of some of the other faces of their boxes.

Teacher Tips

To extend this lesson, compare the perimeters of the faces of a two-pound box of graham crackers with those of a one-pound box.

Additional Ideas

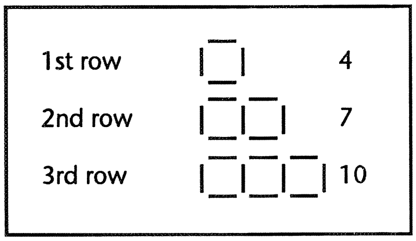
95 *Toothpick Patterns*

You Will Need

- Chapter 3, Patterns
- wall chart (as shown) which displays squares formed with toothpicks

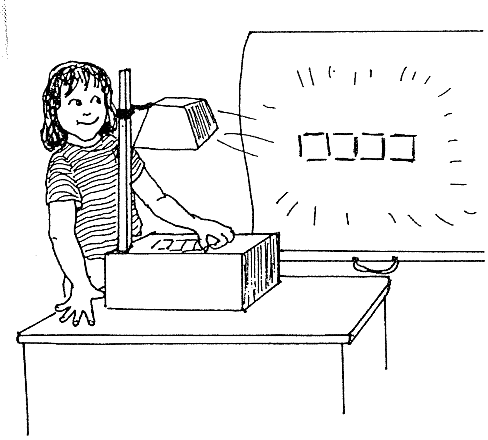
for each pair of children

- a handful of toothpicks
- calculator



Your Lesson

In this lesson, rows of squares are formed with toothpicks according to a specific pattern. Children will look at this pattern and then predict the number of toothpicks needed to build the squares in subsequent rows. The lesson is described in more detail in Chapter 3, pages 11–13, of the Teaching Reference Manual.



Display a wall chart of toothpick squares as shown above. Divide the class into pairs and, at their desks, have them build the rows pictured on the chart.

Tell the children you see a pattern in the rows and are picturing the 4th row in your mind. Ask a volunteer to build the 4th row at the overhead. (Also see Teacher Tips.)

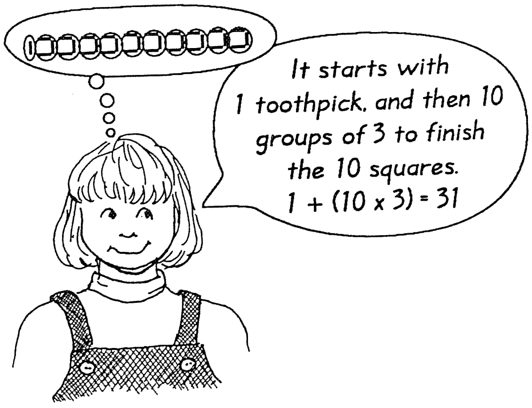
The children that are seated also build this row.

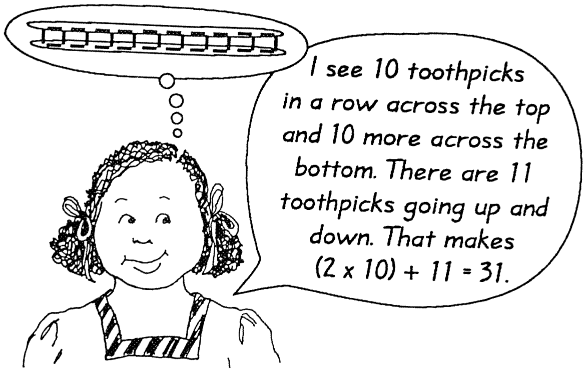
Conduct a show-and-tell discussion of the following questions (possible responses are indicated):

How many toothpicks are used in the 4th row?

95 Toothpick Patterns (continued)

Imagine the 10th row. What does it look like? How many toothpicks will be needed to make it?





Encourage children to describe their visualizations of the 10th row and to use this image to suggest a way for determining the required number of toothpicks. If necessary, use calculators to complete any cumbersome computations. Some children may also need to actually build this row.

Continue in the same way: What does the 50th row look like? the 100th row? etc. How many toothpicks will be needed in each case?

Invite the children to draw some general conclusions about the rows of squares. Some may even suggest a formula for the number of toothpicks needed to build each row.

Teacher Tips

The children may suggest other ways of building the 4th row. If one of these looks particularly interesting, extend and explore it too.

Our children enjoy imagining higher-numbered rows and using a calculator to determine the number of toothpicks in each case. In fact, some even came up with the following formula:

total number of toothpicks in a row = 1 + (n × 3)

Here n represents the number of squares in a row.

Additional Ideas

96 Mystery Box—Stamps; Toothpick Patterns

You Will Need

- Chapter 2, Sorting; Chapter 3, Patterns
- mystery box with some stamps inside
- chart paper, markers and 20 popsicle sticks *for each pair of children*
- a handful of toothpicks
- calculator

Your Lesson

Stamps can open windows for children by introducing them to other countries, stamp collecting, history and much more. The next unit is built around all kinds of stamps—metered, new, used, whatever!

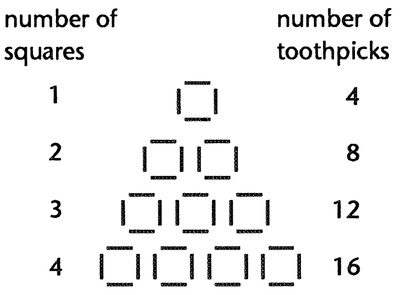
By using stamps as an emphasis, the mathematics should stick to the children just as a stamp sticks to a letter. The greater variety in your stamps, the more fun your lessons will be.

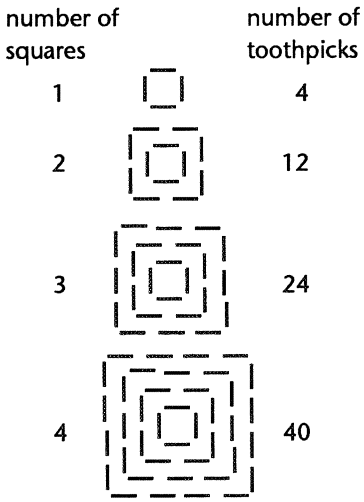
The theme of stamps begins in a few days (Lesson 101), so bring out the mystery box once more! You might carry it as if it weighed 20 pounds—when in fact it contains only a few postage stamps!

Follow the usual procedures, permitting no more than 20 questions each day. Make a design with 20 popsicle sticks and remove one stick for each question.

The second part of today's lesson is a variation of the toothpick activity that took place in Contact Lesson 95. It is described more fully in Chapter 3 of the Teaching Reference Manual (Lesson 2, pages 14–15).

Have each pair of children choose a pattern and then form 4 rows of toothpick squares. Have them put 1 square in the 1st row, 2 squares in the 2nd, 3 in the 3rd and 4 in the 4th. Tell them to select a pattern different from the one explored in Contact Lesson 95. Here are two possibilities:





When ready, ask the groups to imagine building additional rows of squares of their patterns and to answer the following questions:

What would the 5th row look like? How many toothpicks would be needed to build it?

Describe the 10th row. How many toothpicks are needed for it?

Conduct a show-and-tell discussion of each group's work.

Teacher Tips

This toothpick activity can be extended. Your children might enjoy describing other rows, such as the 50th, and predicting the number of toothpicks required to build them.

Additional Ideas

97 Measurement—Volume

You Will Need

- Chapter 7, Measurement
- three clear plastic jars of different shapes
- filler material such as rice or sand
- measuring cups or small paper cups
- masking tape and marker
- funnel (optional)

Your Lesson

In Contact Lessons 29 and 30, your children discussed strategies for comparing the volumes of various containers. Begin this lesson by conducting a similar discussion about the volumes of two of your plastic jars. How can the volume of each jar be measured? What is the difference in volume between the jars?

Move into a discussion of using measuring cups. How many full measuring cups can be poured into each jar? Keep a record of this process by calibrating each jar, using tape and a marker to indicate the level of each added cup.

The calibrated jars can now serve as a measuring tool for determining the capacity of other jars. Demonstrate this by conducting the following experiment:

Have a volunteer fill a third jar with rice and then pour the contents into one of the calibrated jars. How many cupfuls did the third jar hold?

Have a second volunteer do the same using the second calibrated jar.

How did the calibrations help determine the number of cupfuls? Were estimations (of partial cups) needed for the answers? Did both calibrated jars show the same volume for jar 3? How did the answers compare with each other? Were they the same? If not, what might explain the difference?

Teacher Tips

Theoretically, the answers obtained in the above experiment should be the same. Any difference might be explained by discrepancies normally associated with any measuring or estimating procedure. If the answers differ by a great amount, however, it may be due to more significant errors such as inaccurate calibration.

Homework

Ask your children to bring some clear containers from home for Contact Lesson 98.

98 Measurement—Volume

You Will Need

- Chapter 7, Measurement
- clear containers brought from home by children
- small paper cups for calibrating purposes*
- masking tape and markers
- filler material such as rice or sand
- a copy of Blackline 81 (Observation Record Sheet) for each group of four children
- funnels (optional)

Your Lesson

Continue the calibration activity begun in Contact Lesson 97 by asking each group of four to conduct this experiment:

Estimate the volume of each of their jars and predict how to order the jars according to increasing volume. Record the prediction on observation record sheets.

Use paper cups* and filler material to calibrate one of their jars.

Use their calibrated jar to determine the volume of each of their jars.

Have the groups describe their procedures and record their results on the observation record sheets. How did the actual order of volumes compare with the predicted order?

* The calibrated jars will be used in Contact Lesson 99, so each group must use the same-sized cup to make its calibration.

Homework

Ask each child to bring in another container from home (not necessarily clear) for Contact Lesson 99.

Additional Ideas

99 Measurement—Volume

You Will Need

- Chapter 7, Measurement
- calibrated jars created in Contact Lesson 98
- containers brought from home by the children
- filler material such as rice or sand
- a copy of Blackline 81 (Observation Record Sheet) for each child

Your Lesson

Place filler material and a calibrated jar from Contact Lesson 98 at various stations throughout the room. Identify each station by a letter.

Ask children to visit each station and use the materials there to measure the volume of the container they brought to school today. Have them record the letter of each station and the corresponding volume on their observation record sheets. When finished, have them examine their data and record their observations. Discuss the results.

Teacher Tips

In Contact Lesson 98, the same-sized cup was used to calibrate the jars used at each station. Theoretically, then, each child should obtain approximately the same volume for their container at each station.

Tomorrow is the 100th Day—get ready to celebrate! Are the collections of 100 items together? (See the note at the end of Contact Lesson 92.)

Additional Ideas

100 *The 100th Day Celebration!*

Your Lesson

Bring out the brass bands! When the 100th day of school arrives, we like to spend the entire day celebrating.

Start your day with special Calendar activities. Bundle the straws and fill the hundred's pocket of your Numberline Straw Pockets. What was different 100 school days ago? What were the children wearing that day? What was happening then? What was each child doing 100 months ago? Have groups of four record number statements that total 100 on chart paper. Share the equations that are created—we're sure they will be *so* incredible that you'll want to post them for the remainder of the day!

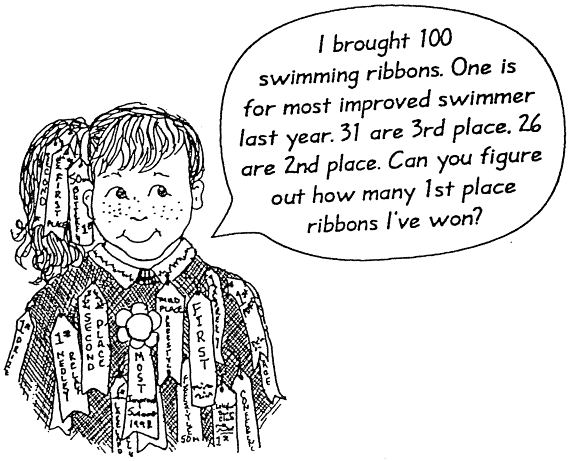
Have children with collections of 100 items arrange them in a way that demonstrates there are 100 of them. Provide time for children to examine the collections to see "100" displayed in several ways. How many 100s are there? How many items in all the collections? If there are edible items to be shared, how many should each person in the room receive?

One hundred can also be a theme of your other lessons during the day. For example, you might ask your children to write about life 100 years ago or 100 years from now. Or perhaps create a mosaic design using 100 small squares.

Listed below are several other appropriate activities. Have fun pursuing those of special interest to your children.

What is the 100th day of the calendar year?

Send a birthday card to someone who is 100 years old.



Determine the day, month and year of each child's 100th birthday.

How long would it take to do 100 jumping jacks? Check it out.

Perhaps the children would enjoy trying to follow 100 commands of Simon Says or bouncing among 100 balloons in the gym.

Have each child examine a copy of a national weather map for the day. Rounding to the nearest 10, what cities are expected to have temperatures equal to 100?

Give each group of four children 100 tile. Have them use these to determine n if 2n = 100? if 3n = 100? if 4n = 100? etc.

Find 100 words in a newspaper that have a prefix or suffix.

Predict the outcomes of 100 tosses of a fair coin. About how many heads will turn up? Conduct the experiment, devising ways to keep track of the results. Does anyone wish to revise their predictions after 10 tosses have been made? after 20 tosses? How did the overall results compare with the predicted ones?

Here is a special telephone number: 633-4951. Note that the last four digits form two numbers that total 100: 49 + 51 = 100. What other numbers can be found in a phone book that have this property?

What combinations of coins will total 100 cents?

Where could you go if you traveled 100 miles from school?

How long will it take each child's pulse to beat 100 times?

What items can be purchased in packages of 100?

How many days are there in 100 weeks? How many hours?

If you could leave school for 100 minutes, where would you go?

Five children are arranged in a circle and a ball is passed from person to person 100 times. How many times does each child receive the ball? How would this be answered if there were only 4 children in the circle?

Count out 100 pieces of popcorn. Eat and enjoy!

How many United States flags would it take to have 100 stripes?

Additional Ideas

101 *Stamps—Sorting and Fractions*

You Will Need

- Chapter 2, Sorting; Chapter 8, Fractions
- a collection of stamps for every two children
- individual chalk, chalkboards and erasers, paper

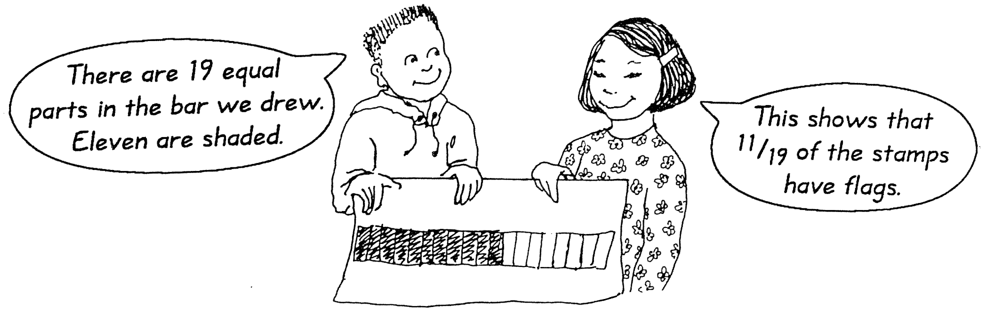
Your Lesson

By now, your children have probably identified the stamps in your mystery box. Begin the theme of stamps with the following activity which combines sorting and fractions.

Ask each team of children to sort a handful of stamps into two piles, giving each pile a title that describes how it was formed. In addition, ask them to record on their chalkboards the fraction of total stamps in each pile.

Have the teams write about their piles, including responses to the following: What sorting criteria was used to form the piles? What do they observe about them? For example, does a pile have more or less than one half of the stamps in it? What fraction of the stamps in each pile cost more than 15¢?

Sketch a fraction bar that depicts the portion of the total number of stamps contained in each pile and record observations about it.



Conclude the lesson by having some of the teams report about their work to the class.

Teacher Tips

Here are additional ideas to try:

What shapes are present on each team's stamps? Did some shapes appear more often than others? Were any particularly difficult to find? Did any shape appear in all of the stamps?

(Your children might also enjoy examining art or architectural prints for the occurrence of common shapes.)

Post a copy of current package mailing rates. Have volunteers weigh different objects in the room and then determine the appropriate postage for mailing each item. Record the information on a chart. Does it cost more to send items separately or to bundle them into one package?

102 Stamps—Area, Estimation and Place Value Counting

You Will Need

- Chapter 7, Measurement
- a page of stamps (or gummed perforated stickers)
- a double sheet of newspaper for each group of four children
- calculators

Your Lesson

Show the children a sheet of stamps and ask, "How many stamps are contained in this sheet?" Invite volunteers to describe ways to answer this question.

Give each group of four a double sheet of newspaper. Holding up one of these sheets and referring to the stamps above, ask the groups to investigate the following problem: If a stamp represents a unit of area, estimate the area of this sheet of newspaper.

Circulate among the groups as they work, noting the strategies they use to conduct the investigation. Some may use items found in the room (or built by the children) that have the same area as a stamp or block of stamps.

Ask the groups to summarize their work and report to the class. As part of this, have them demonstrate their estimation procedures. Did they use fractions in any way? How did their estimates compare? If there are differences among the answers, what might explain them?

Journal Writing

Write a brief paragraph describing how you feel about the procedure your group used to estimate the area of the newspaper.

103 *Stamps—Spatial Awareness*

You Will Need

- Chapter 11, Geometry

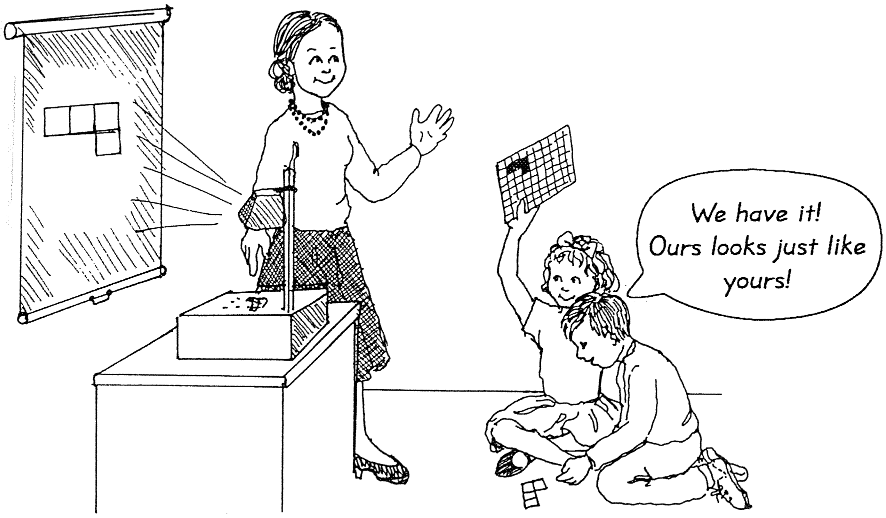
for each child

- several square stamps or tile
- a copy of Blackline 73 (2-cm Grid Paper)
- transparency of Blackline 73
- colored squares for the overhead

Your Lesson

It is common to find stamps that are square (or nearly square). In the activities of this two-day lesson, children use these stamps to form and combine different arrangements of 4 squares. These arrangements can also be formed from square tile.

Display the following arrangement of squares on the overhead and sketch a picture of it on a transparency of Blackline 73. Have children use their stamps (or tile) to build the same arrangement and record it on their squared paper.

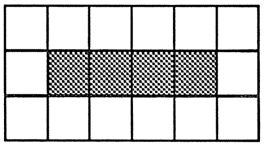


Note that neighboring squares share complete sides in the above arrangement.

How many other arrangements can be formed with 4 squares, where neighboring squares share complete sides?

Have the children work in groups of four to investigate this question. Ask each child to record the findings of their group on squared paper. Volunteers may then share results at the overhead. Here are some possibilities:

a)



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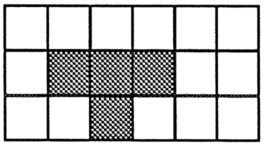
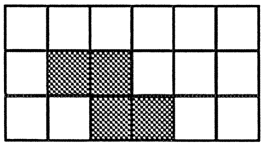
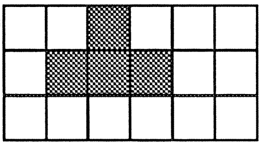


c)



d)



103 Stamps—Spatial Awareness (continued)

Discuss the arrangements. Are there any similarities or differences among them?

If there are arrangements like b) and d) in the above illustration, would the children consider them to be the same or different? Some may argue they are the same because arrangement b) can be turned to look like arrangement d). Others may insist they are different because of the way they are positioned.

Have all possible arrangements been found? How can one be sure?

What are the perimeters and areas of the arrangements?

What if 5 squares were used instead of 4?

Additional Ideas

104 *Stamps—Spatial Awareness*

You Will Need

- Chapter 11, Geometry
- two transparencies of Blackline 107 (cut out the patterns of 4 squares.)
- transparencies of Blacklines 108 and 109

for each child

- copies of Blackline 107 (Four Square Patterns) and Blackline 109 (Recording Sheet)
- scissors

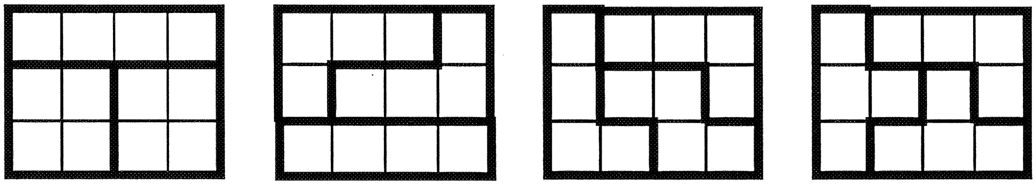
for each pair of children

- a copy of Blackline 108 (Rectangles for Four Square Patterns)

Your Lesson

In Contact Lesson 104, children built several patterns of 4 stamps or squares. Five of the possible patterns are shown on Blackline 107. Have each child cut out these five patterns.

Divide the class into pairs and distribute Blackline 108. Ask each pair to find 3 pieces from their combined sets of patterns that will exactly cover Rectangle A. Discuss their solutions in a show-and-tell manner. Here are some possibilities:



Distribute the recording sheet (Blackline 109). Ask the teams to now find 5 pieces from their combined sets of patterns that will exactly cover Rectangle B of Blackline 108. Have them search for several ways of doing this and to record each way on Blackline 109.

As time permits, have the teams share some of their solutions.

Additional Ideas

105 Stamps—Money and Story Problems

Your Lesson

There are many connections between money and stamps that can be discussed with your class. Here are a few possibilities:

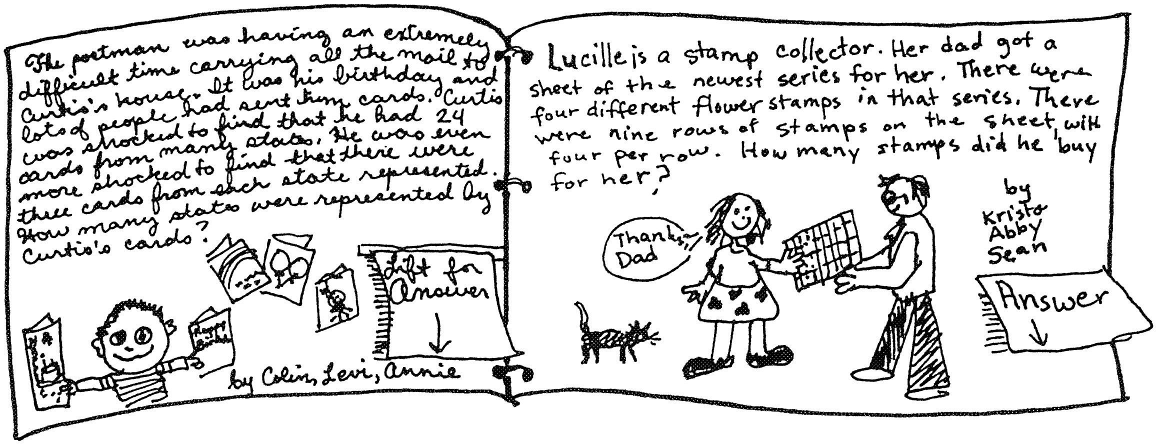
Invite your local postmaster to class to tell about postal operations. Some questions you might discuss: How are the prices for different stamps determined? How is the revenue obtained from the sale of stamps used? Why has the price of a first-class stamp risen so dramatically in recent years?

Have students compute the total value of various collections of stamps. (Use a calculator if needed.) Totals can then be represented with money from feely boxes.

Have children create a book of story problems involving stamps. They enjoy drawing pictures to go with each story and later having others read and solve the problems.

Some stories our children have written:

The postman was having an extremely difficult time carrying all the mail to Curtis' house. It was his birthday and lots of people had sent him cards. Curtis was shocked to find he had 24 cards from many states. He was even more shocked to find there were 3 cards from each state represented. How many states were represented by Curtis' cards?



Lucille is a stamp collector. Her dad got a sheet of the newest series for her. There were 4 different flower stamps in the series. There were 9 rows of stamps on the sheet with 4 per row. How many stamps did he buy for her?

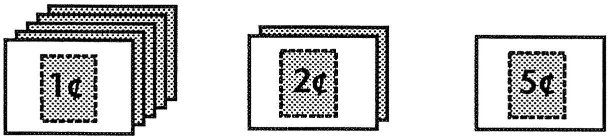
Jethro's mom wanted him to get her as many stamps as possible. Each stamp cost a quarter. She gave him $6.40. How many stamps could he get for her?

Additional Ideas

106 *Stamps—Probability*

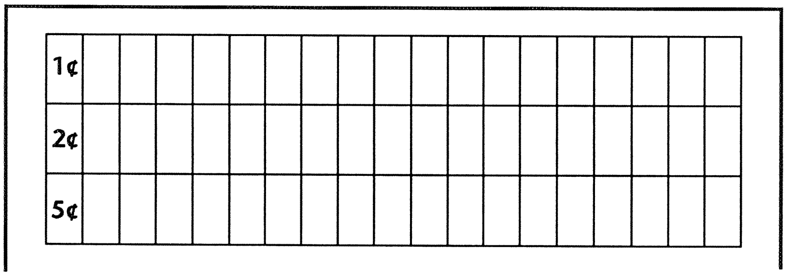
You Will Need

- Chapter 9, Probability
- a copy of Blackline 81 (Observation Record Sheet) for each child *for every two children*
- a deck of eight cards—the cards have stamps glued on them as shown:



5 cards 2 cards 1 card

- a copy of Blackline 41 (Three-Column Graph), labeled as shown:



Your Lesson

Distribute the materials to each pair. Have them examine their decks of eight cards and ask, "Suppose a card is randomly drawn from the deck, which stamp is most likely to appear?"

Have children record predictions about this question on their observation record sheets (Blackline 81). Discuss the predictions and then have the pairs conduct the following experiment:

Shuffle the cards and randomly draw one card from the deck.

Note the value of the stamp on the card and color a block in the corresponding column of the graph.

Replace the card in the deck.

Repeat these steps twenty times.

Have the teams summarize their results of this experiment on the record sheets.

Discuss the results. For each pair, which stamp appeared most often? If the results from all pairs are put together, which stamp appeared most often? How do the combined results compare with those of individual pairs? What might explain any similarities or differences? How do the results compare with the students' predictions? What might account for any differences?

Teacher Tips

Theoretically, the probability of drawing each type of stamp is shown in the following chart:

stamp	1 cent	2 cent	5 stamp
probability	5⁄8	2⁄8	1⁄8

This assumes that each draw is made in a random fashion. The results of the experiment may differ if this isn't the case or if the number of draws is not very large. Also, individual team results may differ from those of the combined group for the same reasons.

This is a great at-home activity. Children enjoy playing the game with an adult and then analyzing the data. Don't be surprised if the students play it many more times than you expect simply because they are hooked on the probability of it all.

Additional Ideas

107 Stamps—Problem Solving

You Will Need

- information from your local post office describing the kinds of stamps currently available for purchase
- calculators
- paper and pencils

Your Lesson

Gather information about kinds and prices of stamps that are currently available at the post office. Discuss this information in class and, if possible, bring in some samples.

Divide children into groups of four and have them work together on this problem:

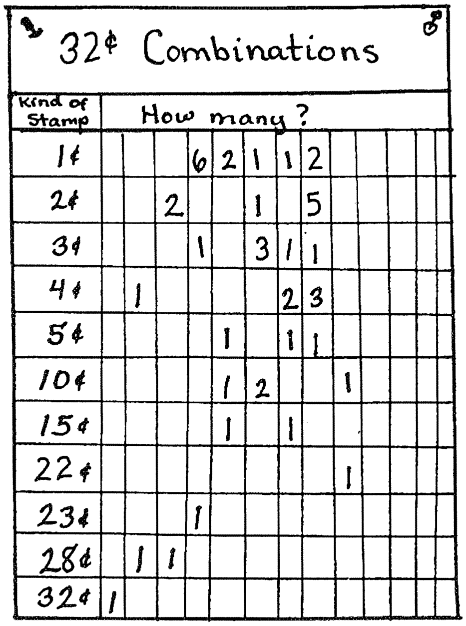
What combinations of stamps could be used to mail a letter that requires 32¢ postage?

Have the children record their solutions and model them with sketches. When ready, groups can share some of their solutions.

As time permits or as homework, children can search for combinations of stamps that could be used to mail letters which require an amount of postage other than 32¢.

Teacher Tips

Some children may organize their work on graph paper as shown here.



32¢ Combinations

Kind of Stamp	How many?												
1¢				6	2	1	1	2					
2¢			2			1		5					
3¢				1		3	1	1					
4¢		1					2	3					
5¢					1		1	1					
10¢					1	2			1				
15¢					1		1						
22¢									1				
23¢				1									
28¢		1	1										
32¢	1												

Additional Ideas

108 *Stamps—Extended Number Pattern*

You Will Need

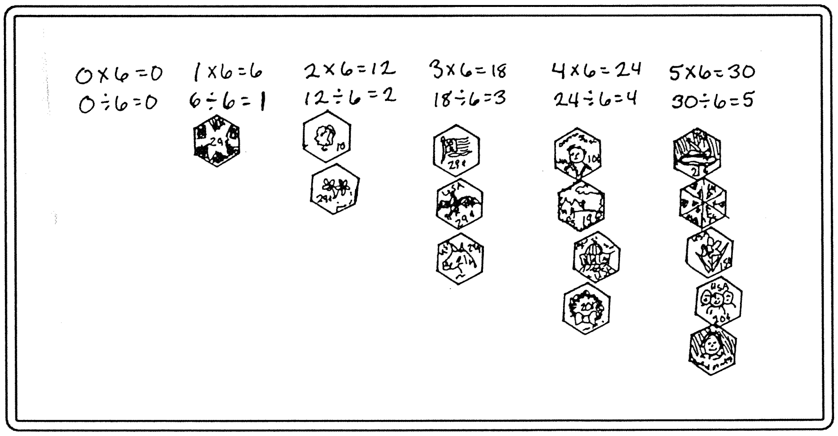
- Chapter 3, Patterns
- paper hexagon shapes (cut from Blackline 115) and crayons for each child
- bulletin board sized paper

Your Lesson

What if you received a letter from the Postmaster General inviting your class to design a new hexagonal stamp as part of a national contest? Would your class enjoy the challenge of creating a possible prize winner?

Have students make their designs on paper hexagons. The designs can then be displayed and discussed.

The hexagons can also be used to visualize multiples of six. This may be done by grouping the hexagons and noting the number of sides in each group as shown here:



In the above display, the hexagons within each group do not share sides. If larger groups were formed in the same way, how many sides would they show? Discuss questions such as:

How many sides would a group of 10 hexagons have?

How many sides would a group of 43 hexagons have?

How many hexagons would there be in a group that has 96 sides altogether?

Encourage the children to find answers for these questions in various ways and to share their thoughts. This is a good opportunity to combine visual thinking with mental arithmetic or with using calculators.

109–113 *Mystery Box—Paper; The 6 Number Pattern*

You Will Need

- mystery box with strips of colored paper inside

Your Lesson

It's time to bring out the mystery box once more. This time the mystery items are several strips of colored paper. Activities centered around paper begin with Contact Lesson 118.

Provide time for questioning over the next several days. Make a star with pattern blocks at the overhead and set each day's quota of questions by the number of pieces in the star. One question per piece—once the pieces are gone, the box is put away until the next day.

The stage has been set to investigate the counting progression 6, 12, 18, 24, 30, 36, etc. The activities of Contact Lesson 108 provided some initial experiences with this number sequence. Over the next five days, your children will strengthen their awareness of this progression. The goal is to help them think about these numbers in terms of pictures, patterns and songs.

Follow the general procedures used to explore previous counting patterns (see, for example, Contact Lessons 20–24) or refer to *Opening Eyes to Mathematics Through Musical Array-ngements.*

Teacher Tips

The first part of this exploration consists of modeling the 6 counting progression with rectangular arrays that have a dimension of 6. At this point in the year, it may suffice to build only the first few of these arrays and to then have the children cut arrays up to 6 by 10 from graph paper.

Many patterns can be observed when the multiples of 6 are shaded on a hundreds matrix, including the fact that multiples of 6 are also multiples of 3. You might wish to compare these two counting patterns.

Additional Ideas

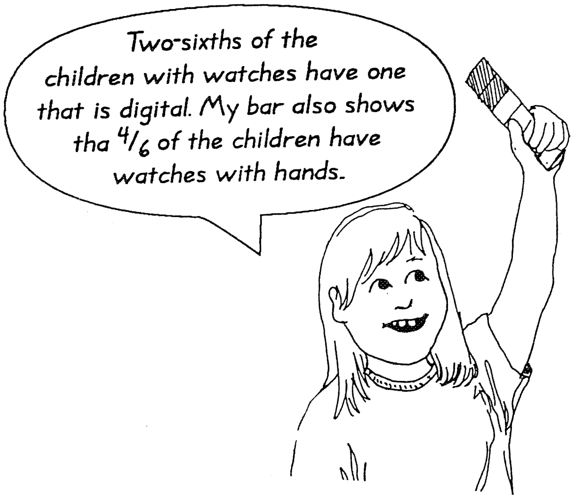
114 *Fractions—Say It, See It, Show It*

You Will Need

- Chapter 8, Fractions
- a set of Fraction Bars for each child
- individual chalkboards, chalk and erasers

Your Lesson

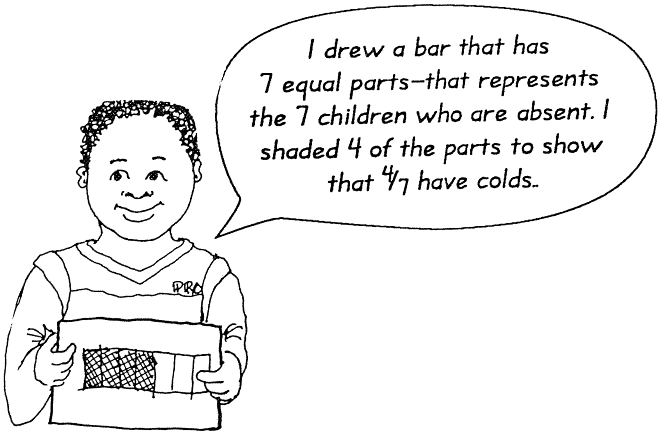
Today's lesson reviews modeling fractions with Fraction Bars. The main activity calls for children to show a bar that represents a fraction described to them in words. As an example, you might say to the class: "I see 6 of you are wearing a watch. Of the 6 watches, 2 are digital. What fraction of the children with watches have digital ones? Can you show a Fraction bar that models this fraction?"



Continue describing situations that call for a fraction to be modeled. Invite the children to describe situations, too.

Be sure to include situations that require the children to sketch bars not included in their sets. Here is an example:

Seven children are absent today. I understand that 4 of them have colds. What fraction of the absent children have colds? Sketch a bar that models this fraction.



Teacher Tips

Children might compare fractions as part of this review. Ask them to think of a pair of fractions, such as $\frac{1}{3}$ and $\frac{1}{4}$ (or $\frac{1}{2}$ and $\frac{6}{12}$). Have them find (or sketch) the appropriate Fraction Bars, write the fractions on their chalkboards, and indicate how they are related with either an =, < or > sign.

Additional Ideas

115–116 Fraction Bar Match

You Will Need

Version 1
- a set of Fraction Bars for every two children

Version 2
- a set of Fraction Bars for every group of four children

Version 3
- a set of Fraction Bars for every group of 2–4 children
- a set of fractional parts cut from Blacklines 83 and 84 for each group of 2–4 children—these need to be mounted on cardstock

Your Lesson

Continue the review of fractions that was begun in Contact Lesson 114 with the following versions of Fraction Bar Match.

Version 1

Divide the class into teams of two and give each team a set of Fraction Bars. Give teams a brief time to search for examples of bars that have the same amount of shading. Bars that match in this way represent fractions equivalent to each other.

When ready, ask each team to mix its bars together and deal them out between the two players. Each player examines their bars and identifies any that represent equivalent fractions. One point is awarded for each match. Players check their opponent's matches; if one should find a match missed by their opponent, then that player receives 5 extra points.

Note that there are some Fraction Bars for which no match exists.

Version 2

Divide your children into groups of four. Each group then forms teams of two and selects a number of Fraction Bars and scatters them face down on the floor. Each team of two, in turn, reveals two bars at a time. If the bars represent equivalent fractions, they are kept by that team; otherwise, they are placed back in position face down. Play continues within each group until both teams agree that no more matches are possible. (Of course, the teams will check the remaining bars every time to be sure!) The team with the most bars at the end of play is the winner.

Note that each group decides on the bars to be used in this game. Some choose sets in which each bar can be matched by some other bar in the set. Others like to include bars that cannot be matched. Still others may restrict the bars they use in some way, such as including only halves, fourths and sixths.

You can simplify play by using only yellow, blue and red bars until the children understand the game and are comfortable with it.

Version 3

Play this version of Fraction Bar Match in the following way:

Each group of children places their set of cards showing fractional parts face down. A set of Fraction Bars is then dealt to the group.

The first player in each group turns over a card and tries to match that fraction with a bar from their hand. If a match cannot be made, the card is placed face down at the bottom of the pile (or, if agreed to by the children, the other members of the group can try for a match). In some cases, a player may hold more than one bar that matches a fraction; the group must decide, prior to play, how many matching bars to count on any one play.

All players must agree when a match has been made.

The child with the most matches at the end of play is the winner.

Note that each card can be matched by at least one Fraction Bar.

Teacher Tips

We have found that children seem to enjoy playing this game, even at unusual times such as during play on a rainy day or while waiting for a late bus.

Additional Ideas

117 *Paper Graphing*

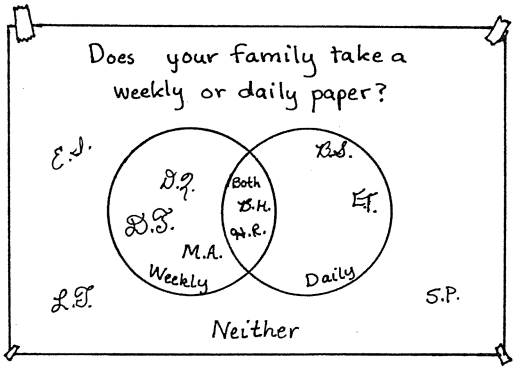
You Will Need

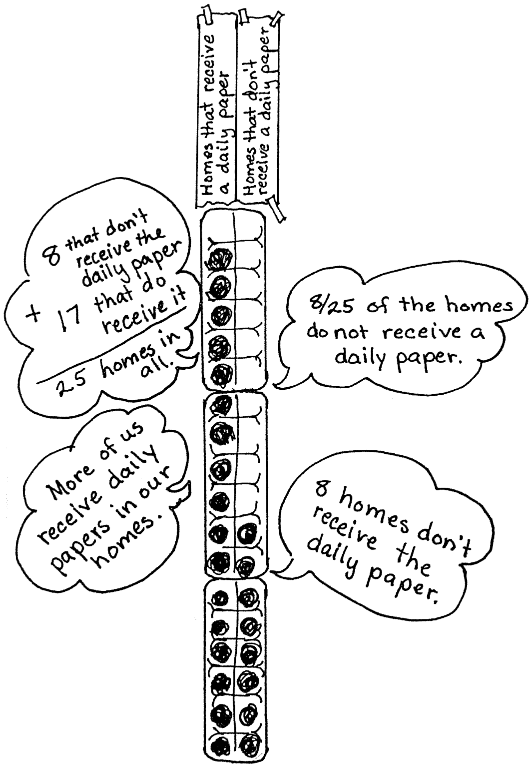
- Chapter 10, Data Analysis and Graphing
- materials for graphing, as needed

Your Lesson

What newspapers are sold in your town or city? Which of them are published locally? Which are dailies? Which are weeklies? What factors influence the availability of papers in your area? How many papers are delivered to the homes of your children?

Your class can research questions like these and summarize some of the information graphically. Some examples:





Additional Ideas

118 *Paper—Geometry*

You Will Need

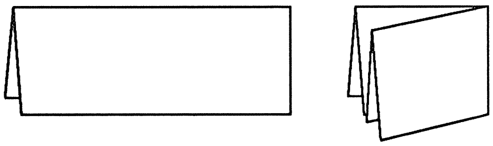
- Chapter 11, Geometry
- scratch paper, scissors and rulers for each child

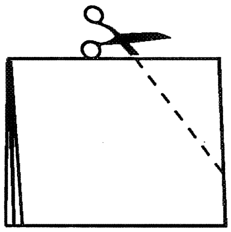
Your Lesson

We find that our children enjoy folding a piece of paper in various ways, cutting it and then examining the shapes. These activities motivate discussions of several geometric topics such as symmetry, congruence and other properties of shapes.

The paper-folding activity of today's lesson is summarized in Chapter 11 of the Teaching Reference Manual (Lesson 3, pages 90–91). This has been adapted from a lesson taught by David Fielker, formerly the director of The Abbey Wood Teachers' Centre, London, England.

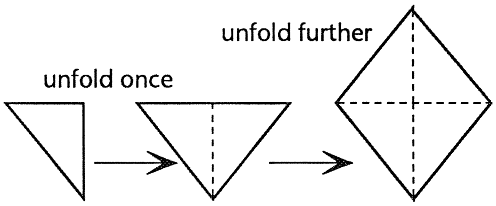
Begin by folding a piece of paper twice, as pictured here.





Draw a straight line from one fold to the other and start cutting along this line.

Before completing the cut, ask the children to predict what they will see when the part being cut off is unfolded. See the dialogue on pages 90–91 of the Teaching Reference Manual for examples of children's responses.



Complete the cut and unfold the shape.

Ask the children to duplicate the above activity and to share observations about the shape that has been unfolded. For example, they might observe the lines of symmetry created by the folds or that the folds are at right angles to each other.

It is possible to cut and unfold a square in this activity. Challenge the children to do this and to write directions for making the required cut.

As time permits, extend the lesson by folding the paper in other ways. In each case, make a single, straight cut across the folds. What will be seen when the cut-off part is unfolded? What observations can be made about the unfolded shape(s)?

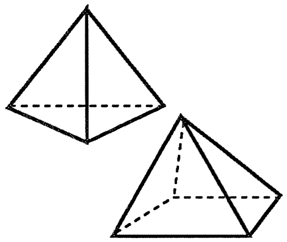
Teacher Tips

It is important for children to describe how they would make a cut so as to unfold a square. This will strengthen their awareness of the geometry present in the activity and provide an opportunity to review or enhance vocabulary.

If a computer is available, you might ask children to create a display of their unfolded shape with a language such as Logo.

119 *Paper—Spatial Relations*

You Will Need



- Chapter 11, Geometry
- a set of solid shapes (cube, rectangular prism, cylinder, sphere, triangular pyramid, square pyramid) placed in a feely bag
- for each group of four children: a model of a triangular pyramid (tetrahedron) and a square pyramid

for each child

- two copies of Blackline 76 (Folding Pyramids)
- scissors and tape

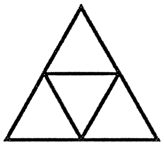
Your Lesson

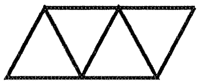
In Contact Lessons 70 and 91, your children created patterns that could be folded to form cubes and rectangular prisms, respectively. In this lesson, they will create patterns that will fold into pyramids.

Begin by asking the children to examine the solids in your feely bag. Ask a volunteer to feel one of the objects and to describe it to the class. Can the volunteer identify the object by feeling it? Can the others identify it from the description?

Remove the object and examine it. Were the guesses correct? How many faces does it have? How many edges? Does it have any symmetry? What do the faces look like?

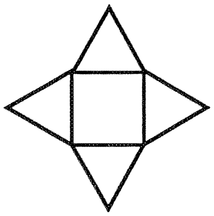
Repeat the above activity until all the objects in the bag have been identified.

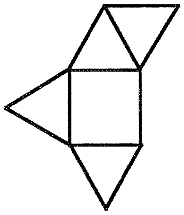




Review the activities of Lessons 70 and 91. Then, challenge each group of four to create patterns of four triangles that will fold up into a triangular pyramid. Give them a triangular pyramid to use as a model and have them cut their patterns from the triangles on Blackline 76. Two solutions are shown to the left.

Now have the groups create a pattern that will fold into a square pyramid. The triangles and squares on Blackline 76 can also be used for this purpose. Here are two solutions:





Conclude the lesson by asking the groups to share their patterns in a show-and-tell manner.

Note: You will need a variety of boxes to cover in Lesson 121. Have your children start bringing in shirt boxes, empty cereal boxes, shipping boxes, etc., for that lesson.

Journal Writing

How would you describe a triangular pyramid to a friend?

Additional Ideas

120 *Paper—Fractions*

You Will Need

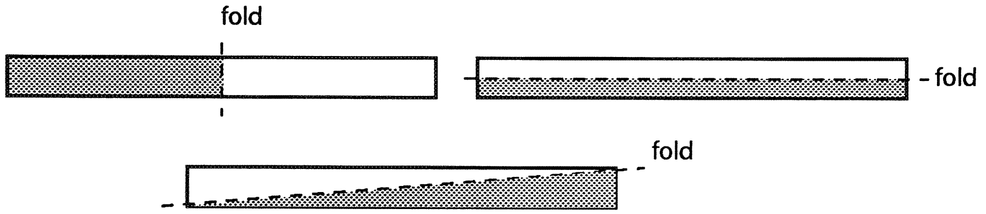
- Chapter 8, Fractions

  for each child
- a set of 1 × 11 inch paper strips, crayon, glue, art paper

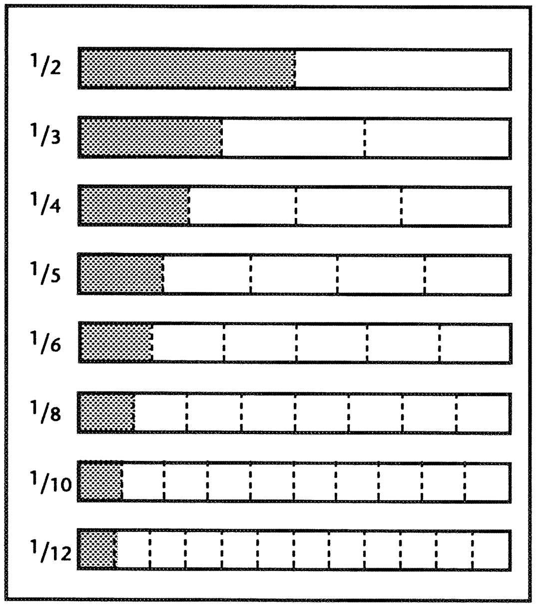
Your Lesson

Folding paper strips is an excellent tactile and visual way for students to learn about fractions.

Begin by asking students to fold one of their paper strips into two equal halves and to color in one of the halves. Students may do this in different ways; if so, have the children share these ways and what relationships they observe.



Extend this activity by challenging the students to fold other strips into fourths, eighths, thirds, sixths, fifths, tenths and twelfths. In each case, have them color in one of the sections and identify the fraction that section represents. When all strips have been folded, ask the children to order them according to the size of the shaded parts, glue them on art paper and label them.



Here are some questions to ask about these fractions:

What happens to the fraction's denominator as the shaded parts decrease in size? Why? Can the children picture a strip that has been divided into 24 equal parts? How would such a strip be related to the ones already made?

120 Paper—Fractions (continued)

Which fraction is twice as big as 1/12? Which is one-half as big as 1/4? How many one-sixths does it take to make 1/3?

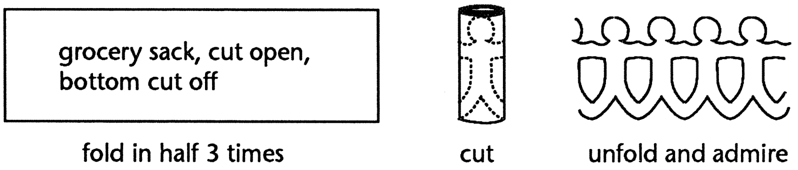
Show other strips of paper that have a shaded portion and ask the children to estimate how much is shaded and how much isn't.

Teacher Tip

Have your children imagine folding other objects (e.g., notebook paper or string) in half. Can they visualize half of the object? Then ask them to check their predictions by actually making the folds. Do the same for other fractional parts of the objects, such as 1/3, 1/4 or 2/3.

Homework

(Optional) Help revive the art of making paper dolls. Form a rectangle from a grocery sack by cutting off the bottom and cutting down the seam. Then follow the steps shown here.



What patterns can be observed when the dolls are unfolded? What symmetry is present? How many dolls were created from one bag? How many did the entire class make?

The children will enjoy coloring their dolls, perhaps with some color pattern in mind, and making them their friends.

Additional Ideas

121 *Paper—Geometry*

You Will Need

- Chapter 11, Geometry
- a variety of boxes brought by your children (cereal boxes, shirt boxes, etc.)
- paper to cover the boxes (wrapping paper, newspaper, newsprint, etc.)
- rulers

Your Lesson

Throughout the year, your children have created patterns that could be folded to exactly cover an object (see Contact Lessons 70 and 91). This lesson extends those experiences by asking the children to estimate an amount of paper needed to exactly cover a box.

Give a box and paper to each group of two to four children. Ask the groups to imagine your biggest department store is hiring people to wrap packages. Before hiring, they have a big Wrap-Off to determine which people to hire. Strut your stuff! Challenge your children to make a pattern that can be folded to cover the box without any overlap. Some may do this by tracing faces; others may find it helpful to measure the length and width of each face.

Have the groups make several patterns that will cover their boxes. Can each child within a group come up with a cover that is different from the others' in the group?

As time permits, ask the groups to show their patterns to the class.

Additional Ideas

122 Geometry—Spatial Relations

You Will Need

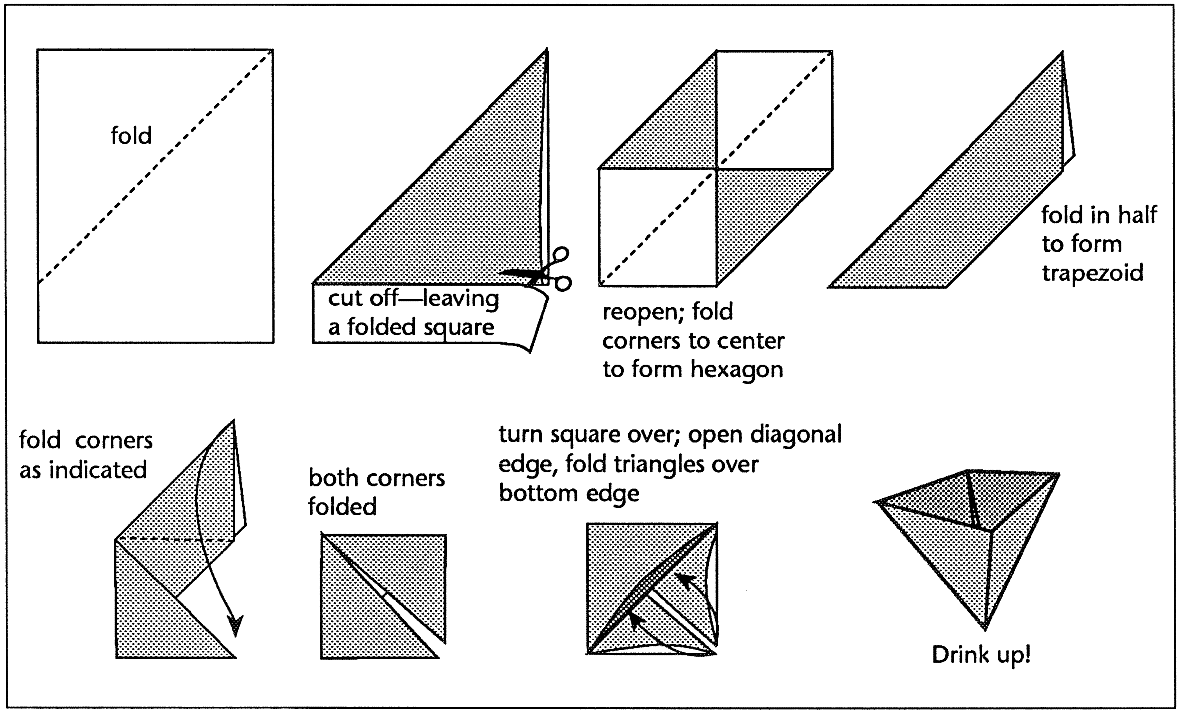
- Chapter 11, Geometry
- an empty tube from a roll of paper towels
- several sheets of paper for each child

Your Lesson

Here are some additional activities that focus on folding paper to create 3-dimensional shapes:

Show the children a paper towel tube. Ask them to notice its shape and to predict what it will look like if it is unrolled at the seam and flattened out. Check the predictions by unrolling the tube.

Have your children figure out ways to create a drinking cup by folding a sheet of paper without the aid of glue, tape, staples or scissors. You might later want to share how it's been done Kentucky style! Slick as a ribbon!



Ask the children to notice the shapes encountered during this process.

Teacher Tip

Check out Origami books from your library. Your children will love making other creations from paper.

123 *Paper—Paper Chain Problem Solving*

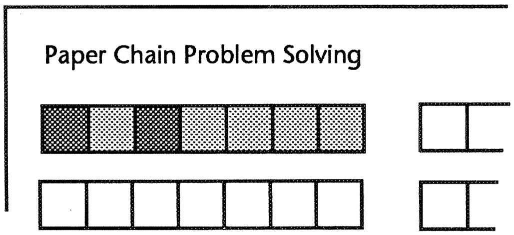
You Will Need

- Chapter 3, Patterns
- transparency of Blackline 82
- two 7-link paper chains, each formed from blue and green strips in the following way: blue, green, blue, green, green, green, green

for each pair of children

- 10–12 strips of blue paper and 25–30 strips of green paper
- a copy of Blackline 82 (Paper Chain Problem Solving)

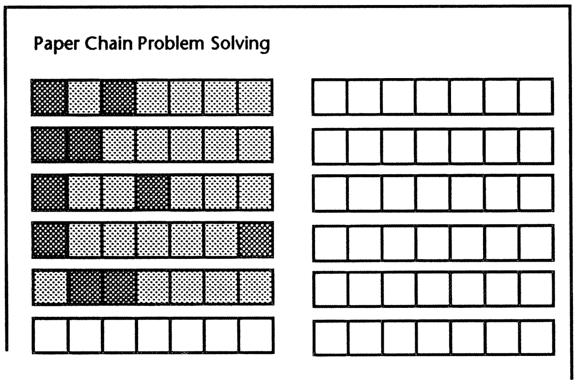
Your Lesson



Children seem to take great pleasure in creating patterns with paper: paper chains, paper dolls, paper snowflakes, etc. Show them the two 7-link chains described above. Tell the children that you have made these chains according to a 7-step pattern and record the pattern on the transparency of Blackline 82 as shown. Have the children also make a record of the pattern on their copies of the blackline.

Link the chains together so that two repetitions of the pattern are displayed.

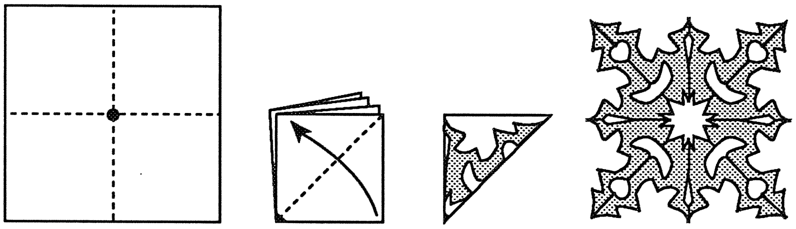
Working in pairs, have the children form a paper chain using a 7-step pattern of 2 blue and 5 green strips that is different from the one discussed above. Have them extend their chains so there are at least two repetitions of the pattern. Conduct a show-and-tell discussion of the children's work and decorate the room with their chains. Several patterns may be shared and each can be recorded on Blackline 82. Here are some possibilities:



As time permits, challenge the children to find other patterns of 2 blue and 5 green strips that could be added to the list on Blackline 82. How many different patterns are there? How might the search for them be organized?

Homework

(Optional) Another patterning activity children enjoy is creating snowflake designs. Discuss the nature of snowflakes and then help the children prepare papers that can be cut to create designs:



They can take these papers home and cut patterns on all edges. When the paper is unfolded, a snowflake appears. Display the flakes when they are returned and discuss the patterns of shapes and symmetry that are present.

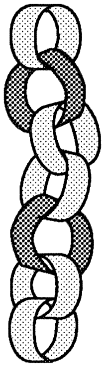
Additional Ideas

124 *Paper—Creating a 7-Step Pattern Chain*

You Will Need

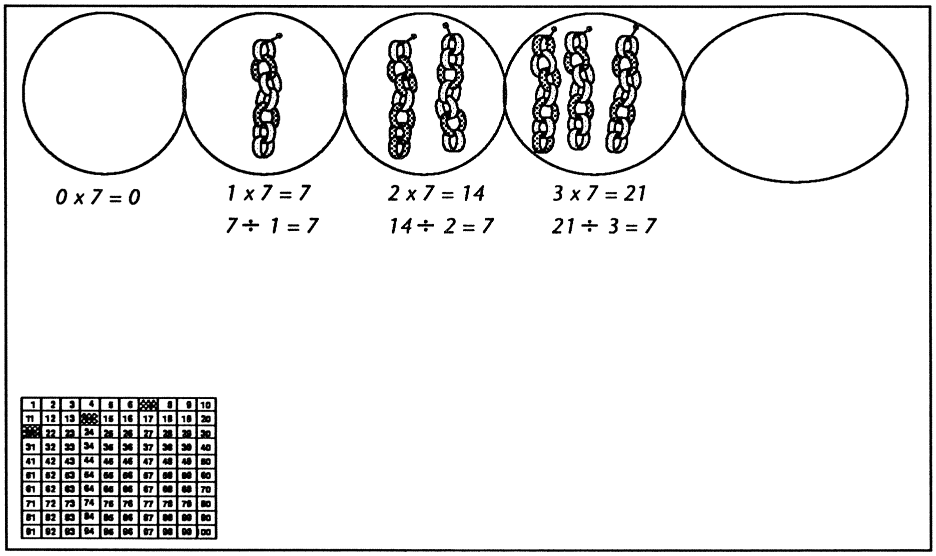
- several blue and green strips of paper
- glue for each child
- calculators as needed

Your Lesson



Ask each child to create a 7-link paper chain using blue and green strips of paper, selecting whatever combination of strips they prefer.

Prepare a display of these chains which models the 7 counting pattern, perhaps as shown here.



Discuss questions such as: If this extended number pattern were continued, how many links would there be in a group of 10 chains? How many in a group of 15 chains? How many 7-link chains would there be in a group that had 140 links altogether?

As time permits, have each child put together another 7-link chain and combine these into one long chain. Have them estimate and measure the length of this chain.

Additional Ideas

Similar displays can be created using paper flowers with 7 petals or paper kites with 7-piece tails.

125–128 *The 7 Counting Pattern; Mystery Box—Peanuts*

Your Lesson

Use the activities of Contact Lesson 124 to motivate an exploration of the counting progression 7, 14, 21, 28, 35, 42, 49, etc. This exploration will be conducted over the next four days with the help of pictures, patterns and song.

Follow the general procedures used in previous explorations of this sort or refer to the booklet, *Opening Eyes to Mathematics Through Musical Array-ngements.*

On the fourth day of this set of lessons, place some peanuts in your mystery box in honor of March being National Peanut Month. The peanut lessons will begin with Contact Lesson 135. Enjoy these lessons with your children. However, be warned! They may drive you nuts!

Sixteen questions per day are allowed for this month's mystery. For each question pour one cup of rice or sand into a gallon container. When the container is full, the box is put away until tomorrow when the process repeats.

You might kick off the peanut unit by reading George Washington Carver's biography to your children. Ours were fascinated by his imagination! His creative uses of the peanut inspired us to come up with some creative mathematical uses. We think he'd like to try the balancing act found in Contact Lesson 137.

Additional Ideas

129 Measurement—Area

You Will Need

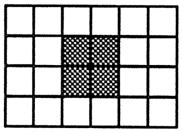
- Chapter 7, Measurement
- copies of Blackline 73 (2-cm Grid Paper) and a marker for each child
- overhead transparency of Blackline 73
- pens for the overhead

Your Lesson

During the next few days, your children will be investigating areas and perimeters of shapes that can be created on a geoboard. Today's lesson provides some readiness experiences for this investigation.

Ask each child to color one of the squares on Blackline 73, while you do the same at the overhead. Tell the children that the area of this square is 1 square unit and ask them to color a region that has an area of 4 square units. Volunteers may show regions such as the following at the overhead.

a)



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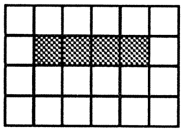

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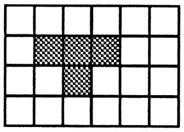

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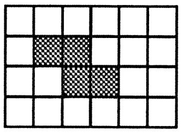


Discuss the regions that have been presented. How are they alike? What differences exist among them? What is the perimeter of each region?

Alike	Different
Each has an area = 4 sq. units. They all have some right angles. Each is made up of 4 squares. Etc.	The first one (a) has a perimeter of 8. The others have a perimeter of 10. The first 2 are rectangles; the last two aren't. Only one is a square. Etc.

Repeat this activity, only this time ask the children to color a region that has an area of 8 square units.

Conclude the lesson by having the children work the following problem: On a clean copy of Blackline 73, color in at least 4 regions that have an area of 12 square units. Write a paragraph that describes how your regions are alike and how they are different.

Additional Ideas

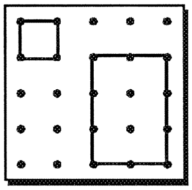
130 Measurement—Geoboard Areas

You Will Need

- Chapter 7, Measurement
- a geoboard and rubber bands for each child
- for the overhead: a transparent geoboard or a transparency of Blackline 68 (Geoboard Recording Paper)

Your Lesson

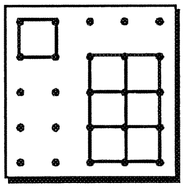
Children enjoy creating shapes on a geoboard and sharing their creations with others. Begin this lesson by providing a period of free exploration during which the children can make geoboard designs of their own. Discuss the designs in a show-and-tell manner.



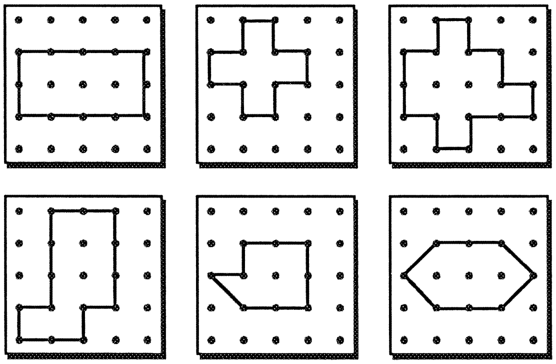
When ready, form the two geoboard shapes shown to the left on a transparent geoboard and show them to the class. Ask the children to form the same shapes on their geoboards.

How are these shapes alike? How are they different?

Tell the children that the small square has an area of 1 square unit. Ask them to use additional rubber bands to show that the area of the larger shape is 6 square units. Invite volunteers to describe how they see this area. Some may count the unit squares one at a time; others may see 3 groups of 2 square units; etc. Using the shape on your transparent geoboard, summarize the discussion by numbering the 6 squares or by coloring them.



Show the children other geoboard shapes, such as those shown here. In each case, ask them to form the shape on their geoboards and to determine its area. Have them demonstrate their thinking about each area.



Some children will likely use other rubber bands to show each of the unit squares contained in a shape; others may be able to visualize the area without having to do this. It helps to encourage the children to estimate each area prior to determining it.

Homework

If your children feel confident about the activities of this lesson, you might ask them to determine the area of a few shapes that have been drawn on geoboard recording paper (Blackline 68).

131 *Recording Geoboard Designs*

You Will Need

- transparency of Blackline 68 (Geoboard Recording Paper)
  for each child
- a geoboard and rubber bands
- Blackline 68 (Geoboard Recording Paper)

Your Lesson

Have each child make a geoboard shape using a single rubber band and sketch it on their recording paper.

Ask a volunteer to describe their shape and record it at the overhead on Blackline 68. The children at their seats also make a sketch of the same shape on their recording paper, identifying it by the initials of its creator. Discuss the shape further. What does it look like? What observations do the children have about it?

Discuss the shapes made by other children in the same way.

Teacher Tips

The task of recording a geoboard shape may be difficult for some children. Provide assistance as needed here and encourage the children to help one another.

Additional Ideas

132 Geoboard Areas

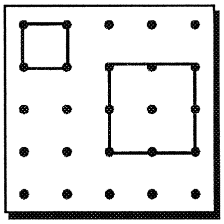
You Will Need

- Chapter 7, Measurement
- transparent geoboard for the overhead

for each child

- geoboard and rubber bands
- Blackline 68 (Geoboard Recording Paper)

Your Lesson

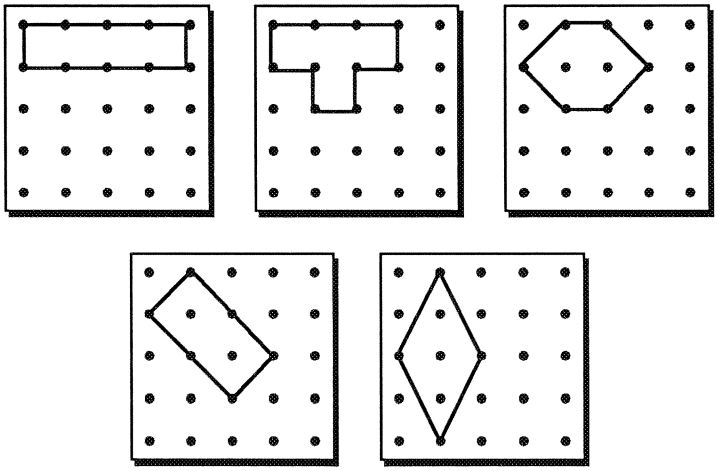


Form the two geoboard shapes shown to the left on a transparent geoboard at the overhead.

Tell the children that the small square has an area of 1 square unit. Invite a volunteer to show that the area of the larger square has an area of 4 square units.

Divide the class into teams of two. Ask the children to form a geoboard shape (different from the one above) that has an area of 4 square units. Have them record their shape on Blackline 68 and prove to their partner that its area is 4. Allow time for some of the children to show their shapes to the class at the overhead in a show-and-tell manner.

Several responses are possible (see following illustration). As time permits, repeat the above activity, encouraging the children to search for other shapes that have an area of 4.



Journal Writing

Make a geoboard shape that has an area of 5 square units. Sketch your shape on geoboard recording paper and describe it. Show how you know that its area is 5.

Additional Ideas

133–134 *Geoboard Perimeters*

You Will Need

- Chapter 7, Measurement
- for the overhead: a transparent geoboard or a transparency of Blackline 110

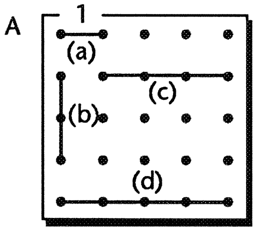
for each child

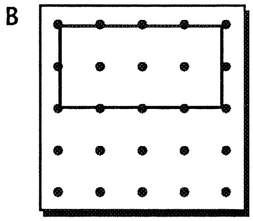
- a geoboard and rubber bands
- Blackline 110 (Geoboard Recording Paper, adjoining boards)

Your Lesson

Day 1

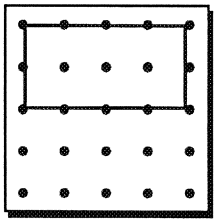
Place rubber bands as illustrated by A on an overhead geoboard and show them to the class. Tell the children the rubber band (a) represents one unit of length. Knowing this, discuss the lengths of the other bands.





Now form the rectangle shown on B at the overhead and ask the children to do the same at their seats.

Working with a partner, have the children determine the perimeter of this rectangle. Discuss their procedures. What is the area of this rectangle? What are its dimensions?



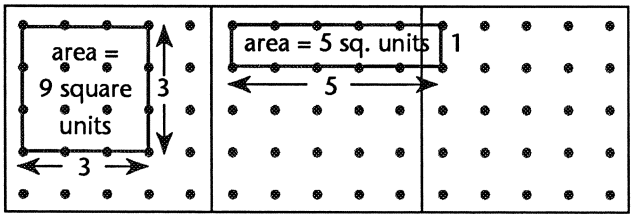
"We counted the units around the rectangle. The perimeter is 12."

"There are 4 units of the top and on the bottom. Each side has 2 units. That makes 4 + 4 + 2 + 2 = 12 units."

"The area is 8 square units. The dimensions are 2 and 4 line units."

The perimeter of the above rectangle is 12 units. Ask the teams to form other rectangles that have this perimeter, recording each on Blackline 110. (They may have to place their geoboards side by side in some cases.) Have them also record the dimensions and areas of their rectangles. Some examples are pictured here:

Other rectangles that have a perimeter of 12 units.

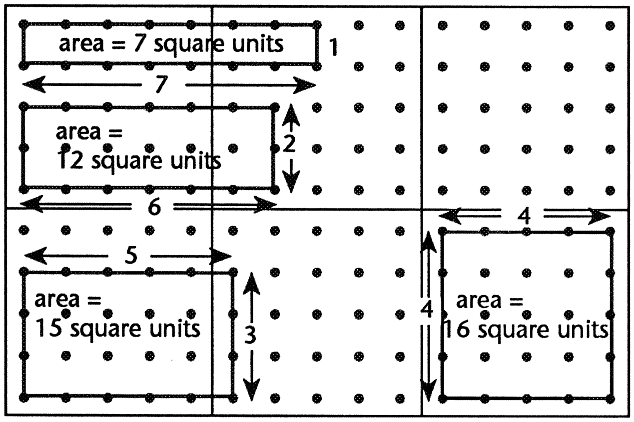


133–134 Geoboard Perimeters (continued)

Discuss the results. How many rectangles can be formed? What strategies did the children use in forming them? What observations do the children have about them? What relationships are present?

Repeat the above activity by asking the children to form rectangles that have a perimeter of 16 units. Do the same for 20 units.

Rectangles with a perimeter of 16 units.



Continue exploring in the same way, only now looking at larger perimeters:

Ask the children to imagine a rectangle that has a perimeter of 30 units. Have them work in groups of four to build such a rectangle and sketch it on Blackline 110. Discuss their results. How many rectangles did they find? What were the dimensions and areas? Have any rectangles been overlooked?

How about rectangles that have a perimeter of 50 units? Investigate.

Teacher Tips

Your children may approach these activities in several ways. For example, some may proceed in a trial-and-error manner. Others may discover that for rectangles having the same perimeter, as one dimension decreases, the other increases by the same amount. Still others may note that the sum of the dimensions of the rectangles equals half the perimeter.

Journal Writing

Write a paragraph that describes how you felt about today's activities. Did anything feel especially good to you? Did anything puzzle you?

Additional Ideas

135 *Peanuts—Sorting and Graphing*

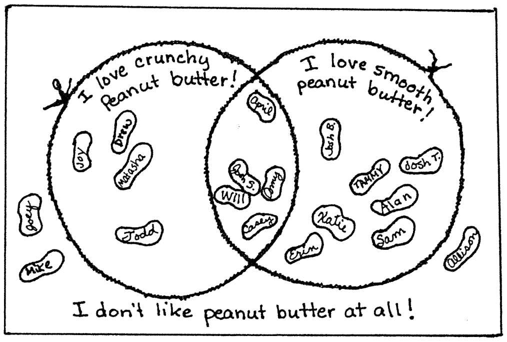
You Will Need

- Chapter 2, Sorting; Chapter 10, Data Analysis and Graphing
- two loops of different-colored yarn
- chart paper
- a 2 × 2 square of brown paper for each child
- conversation bubbles, tape and scissors

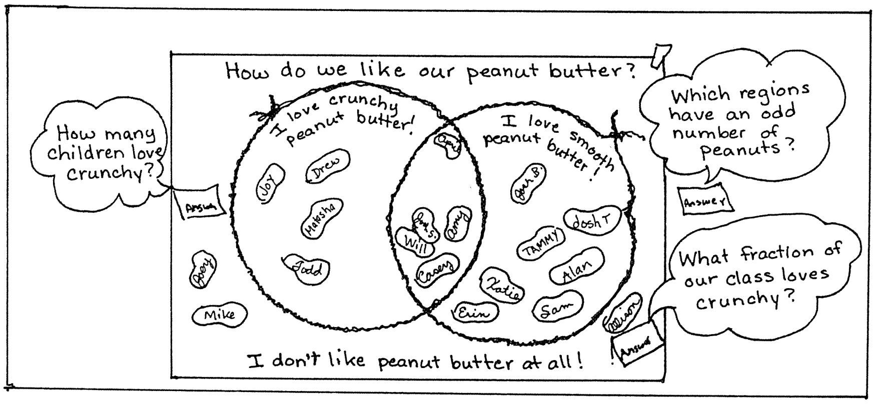
Your Lesson

Do your children like smooth peanut butter or crunchy?

Use loops of yarn to create the Venn Diagram shown below and have each child respond to this question by placing a signed peanut shape in the appropriate region.



As soon as the diagram has been completed, have the children work together to write discussion questions about the information to post on conversation bubbles.



Some questions to discuss: How many children like only crunchy peanut butter? How many like both types of peanut butter? How many like either crunchy or smooth? How many don't like either kind of peanut butter?

Suppose all of the peanut shapes were placed in a sack and one was randomly drawn out. What are the chances that the name on the drawn shape will be that of a child who likes crunchy peanut butter?

135 Peanuts—Sorting and Graphing (continued)

Use language carefully when discussing the categories. For example, children who like crunchy peanut butter includes those who like only crunchy and those who like both kinds. Children who like crunchy or smooth includes those who like only crunchy, only smooth, or both.

You will need several peanut (or peanut butter) cans or jars for Contact Lesson 138. Ask children to bring some from home.

Additional Ideas

136 *Peanuts—Measurement*

You Will Need

- peanut butter, dried milk, honey, crushed corn flakes
- mixing bowl, measuring spoons and mixing spoon
- waxed paper

Your Lesson

Are your children ready for some Big Time Measuring! Invite them to mix and *eat* Honey Milk Balls.

For this measurement lesson, let your children make Honey Milk Balls, a nutritious snack that Jane Bailey (our local Home Extension Agent) shares with our children each year. They can prepare them in class and eat them immediately.

Honey Milk Balls

1 cup dry, powdered non-fat milk

½ cup peanut butter (crunchy or plain)

½ cup honey

crushed cornflakes

Mix all ingredients in a mixing bowl until well blended. Roll the dough into balls and coat with crushed corn flakes. No cooking! Pop 'em right in the ole mouth! They can be rolled in coconut, nuts or other crunchies.

Children enjoy measuring and mixing the ingredients. We have our children bring in enough ingredients so each group of four or five can make a batch. Everyone is able to help measure, mix and eat, with plenty left to take home for samples!

Discuss questions such as: How much honey is needed to make a triple batch of balls? How much for half a batch? If a batch makes 40 balls, how many batches are needed to make 200 balls?

Additional Ideas

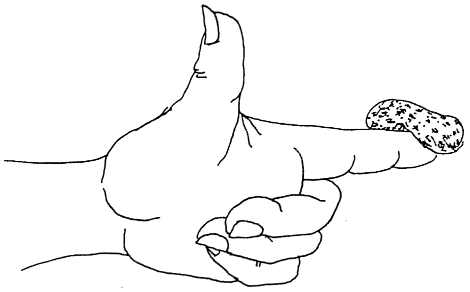
137 Peanuts—Problem Solving

You Will Need

- Chapter 10, Data Analysis and Graphing
- a peanut with two "meats" for each child
- a clock with a second hand that all can see
- paper for recording

Your Lesson

How long can your children balance a peanut on their index fingers?



Divide the class into small groups and have the groups conduct an experiment to answer the above question. They must plan how to gather and report the required information. Do they wish to practice a bit? Do they want to report the best of three trials? How will they keep track of and record the time for each trial? Do they wish to determine an average time for the group? Should the times be reported in a graphical manner?

Have groups describe their procedures in a paragraph, conduct the experiment and then share the results with the class. The class might prepare a graph that displays these results. What observations do the children have about the data? What was the range of times? What appears to be the average balancing time for the entire class?

Teacher Tips

Please see Chapter 10 for information about averaging a set of values. In this lesson, each group can represent its times by stacks of hex-a-links (perhaps using one link for every 5-second interval). These stacks can then be evened off to determine an average balancing time.

Additional Ideas

138 *Peanuts—Measurement and Problem Solving*

You Will Need

- Chapter 7, Measurement
- a one gallon cylindrical can
- a peanut (or peanut butter) can or jar for every four children
- string and scissors

Your Lesson

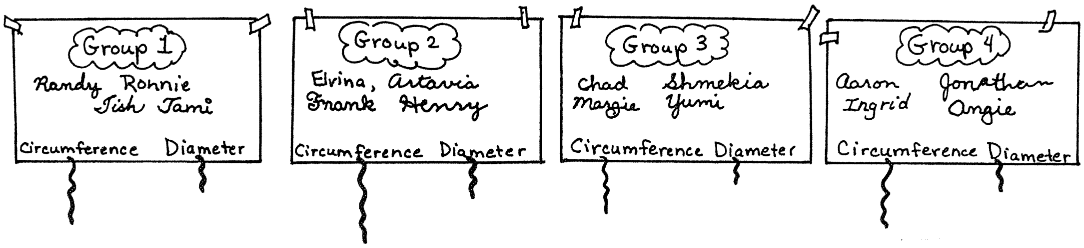
In any circle, the ratio of the circumference to the diameter is always a constant amount. This amount is represented by the Greek letter π (pi) and has an approximate value of 3.14. Today's lesson consists of an informal introduction to this relationship.

Hold up a one gallon can and tell the class you want to cut a length of string equal to the diameter of the circular base. Ask the children to suggest ways for doing this. Since a diameter must pass through the center of the base, how can the center be located precisely? Discuss the suggestions and have a volunteer measure and cut the required length.

Lay the above length of string along the circumference of the circular base. With this length as a reference, ask the class to estimate the length of string needed to measure the entire circumference. About how many times will the diameter's length fit around the circumference? Cut lengths of string to match the children's guesses and test them. The circumference will be slightly more than three times the diameter in length.

Give each group of four a peanut (or peanut butter) can, scissors and string. Ask the teams to cut lengths of string equal to the diameter and circumference of the base of their can. How do these lengths compare? About how many times longer is the circumference than the diameter?

138 Peanuts—Measurement...(continued)



Discuss the results of the above activity. Have some teams describe their measurement procedures and demonstrate how they compared the required lengths.

As time permits, have teams of four measure and compare the circumferences and diameters of other circles in the room.

Additional Ideas

139 *Peanuts—Problem Solving*

You Will Need

- different-sized jars of peanut butter
- knives or spreaders
- several loaves of bread
- a 2 × 2 slip of paper for each child

Your Lesson

One of our children's favorite peanut activities is figuring out the number of sandwiches they can make from a jar of peanut butter. They like to decide how thick to spread the peanut butter and suggest strategies for estimating the answer. For example, they might want to see how much peanut butter is needed to make 10 sandwiches and use this as a basis for making a guess. We ask them to record their estimates on slips of paper and to form a people graph that shows the range of guesses. The grand total is then determined as volunteers spread the fun by making as many sandwiches as possible (trying always to stick to the prescribed thickness) for everyone to eat and enjoy.

Conduct the above activity and discuss questions such as: How many sandwiches can be made from different sizes of peanut butter jars? What is the cost per sandwich? How many sandwiches can be spread from an ounce of peanut butter? What is the cost per ounce of peanut butter? Does the size of the jar matter?

Teacher Tips

Vary this lesson by having children discuss strategies for estimating the total number of sandwiches.

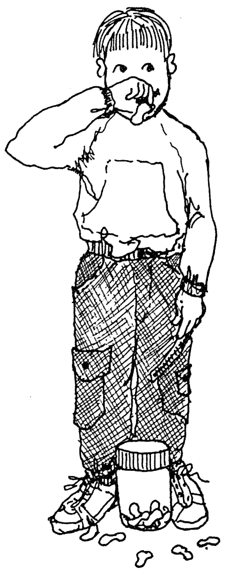
Additional Ideas

140–141 *Peanuts—Probability*

You Will Need

- Chapter 9, Probability
- peanuts
- a large jar
- a copy of Blackline 40 (Two-Column Graph) for each child

Your Lesson



These two lessons are intended to strengthen children's awareness of experimental probability. We hope they will be as enjoyable for your class as they were for ours.

Day One

Place a large jar on the floor. Tell the children that each of them will be asked to hold a peanut at a certain height and make one attempt to drop it into the jar. (We usually let the class decide how high to hold the peanut; ours chose "nose-high".)

Have children estimate the number of peanuts they think will fall into the jar. Then conduct the experiment, asking the children to keep track of the number of hits and misses on Blackline 40. Periodically, summarize the results to report the fraction of tries that resulted in hits and to see if anyone wishes to change their estimate.

Discuss such questions as: How did the results compare with the children's estimates? If the experiment were performed again, do the children think the results would be any different? Why or why not?

Conduct the experiment a few more times. Discuss the results with the class. In particular, have the children discuss how they would complete this sentence: If our class performed the experiment again, we would most likely drop the peanut into the jar _____ times.

Conclude the lesson by having the children think of ways to alter the conditions of the experiment so as to increase (or decrease) their chances of dropping the peanut into the jar. Their ideas will be the focus of tomorrow's lesson. Since you may need different-sized jars to test some of their ideas, have some brought from home.

Day Two

The main purpose of this lesson is to investigate the effect of changing the conditions of yesterday's experiment. What suggestions do children have for ways to change their probability of success in the experiment? They may suggest changing such things as the height from which the peanut is dropped, the size of the jar or the location of the jar. Have them share and test their ideas in groups of four and then report to the class.

142 *Peanuts—Data Analysis*

You Will Need

- Chapter 10, Data Analysis and Graphing
- selected graphs found in newspapers, magazines, encyclopedias, etc.

Your Lesson

Your children have had many experiences with preparing graphs of their own this year. Today's lesson asks them to examine graphs prepared by others.

Have children search through various library references for graphs related to peanuts. Working in pairs, ask them to write a brief description of one of these graphs, addressing such items as: What is the purpose of the graph? How was the data gathered and organized? What is the graph telling them? How do they feel about what the graph says?

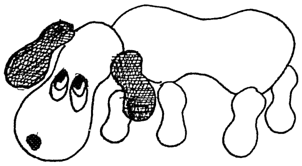
Additional Ideas

143 *Peanuts—Patterns*

You Will Need

- Chapter 3, Patterns
- paper, crayons, glue and scissors for each child
- large paper for display

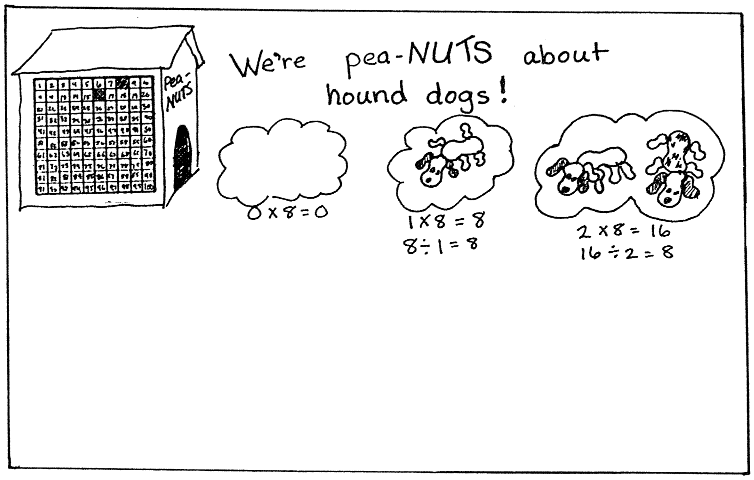
Your Lesson



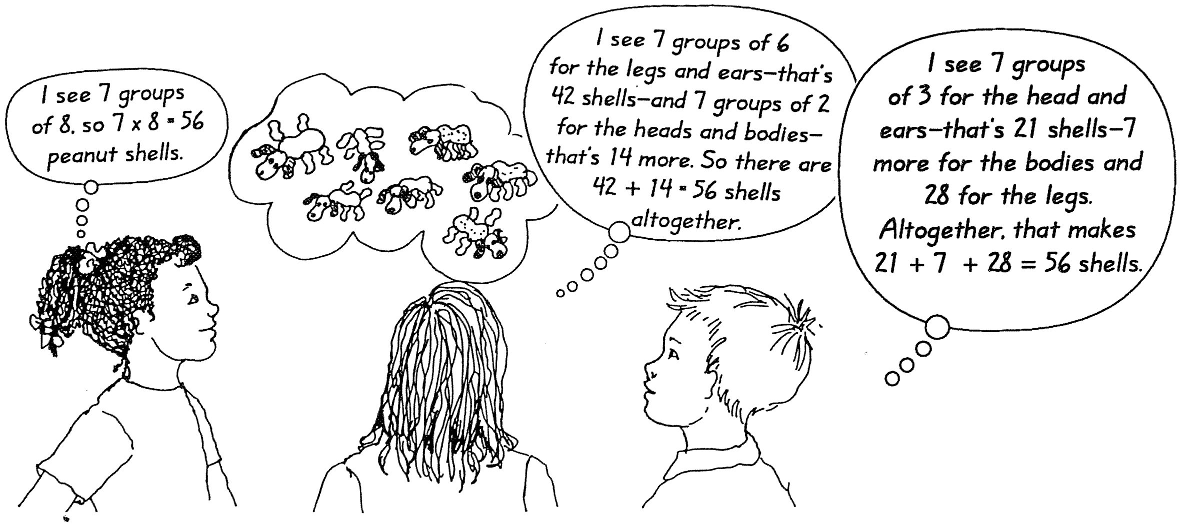
There are many ways to use peanuts to create and extend patterns. In today's lesson, peanuts motivate a discussion of multiples of 8. An exploration of the 8 counting pattern follows in Contact Lessons 144–147.

Have each child cut out 8 peanut shapes from paper and then color them. Ask them to glue their shapes together to look like a Southern hound dog.

Display these as shown:



Ask the children to think about questions, such as: How many shells would it take to make 7 dogs? How can the answer be visualized?

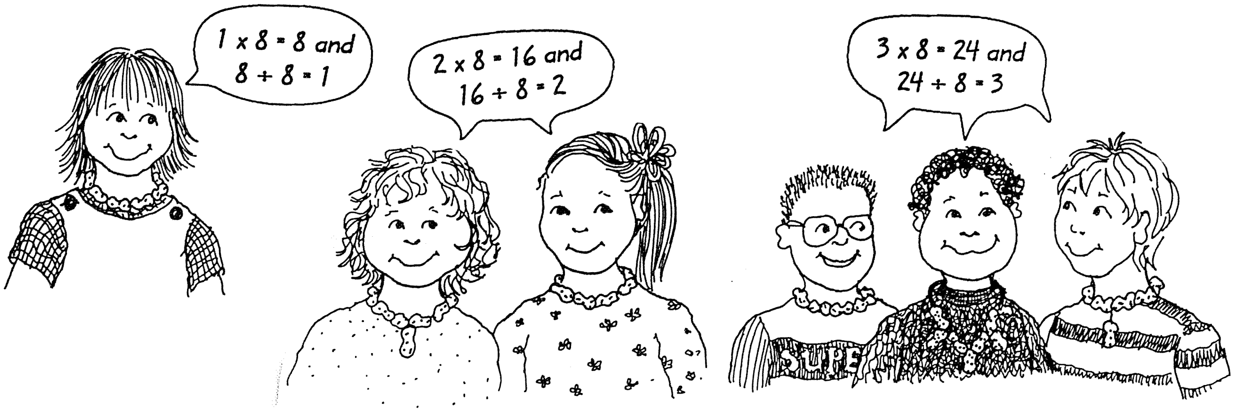


143 Peanuts—Patterns (continued)

Continue the discussion: How many shells are needed for 15 dogs? How many dogs can be made from 88 shells? Etc.

Teacher Tips

Vary this lesson by having children make necklaces that each have 8 packaging "peanuts". The children can then act out the 8 counting sequence by putting on the necklaces and forming sets of 1, 2, 3, 4, etc., people. Each group then verbalizes and writes the multiple of 8 that is represented by its necklaces.



Additional Ideas

Shapes showing 8 peanuts on a plant or 8 peanuts in a cookie could also be used to create displays that show the 8 counting pattern.

144–147 *Exploring the 8 Counting Pattern*

Your Lesson

The activities of Contact Lesson 143 motivate an exploration of the counting progression 8, 16, 24, 32, 40, 48, etc. Conduct this exploration over the next four days using arrays, patterns and song, just as you have done with the other counting progressions.

Additional Ideas

148–150 *Mystery Box—Maps; Fraction Bar War*

You Will Need

- Chapter 8, Fractions
- mystery box with a map in it
- a set of Fraction Bars for every two children

Your Lesson

As teachers of young children, we puzzle how best to guide our children to the future road of success. When lost or looking for directions, we rely on maps. It seems fitting to focus on maps that help us find our way in the world. Hopefully, these lessons (which begin with Contact Lesson 155) will help guide your children successfully down the paths of the future.

In anticipation of these lessons, place a map or atlas inside your mystery box. You might keep track of each day's questions with a 20-link paper chain, one link per question. (Do your children think they are Hot Stuff when presented with the challenge of identifying the mystery item? Their questions are likely quite sophisticated compared to those asked at the beginning of the year!)

When the contents are finally revealed, follow the *route* of the lessons for some surprising mathematical activities.

We like to provide time at this point in the year for children to strengthen their understanding of fractions by playing several versions of Fraction Bar War. Select the ones most appropriate to your class and play them over the next three days. Give a set of Fraction Bars to every two children.

Version 1

Ask each pair of children to divide their bars into two piles, giving one pile to each child. The piles are placed face down and each player turns over the top bar. Whoever has the bar showing the greater fraction wins both bars. If the fractions are equivalent, each turns over a second bar and compares those fractions, with the winner claiming all four bars.

Play continues until one player has all the bars or until some other agreed upon stopping point has been reached.

Version 2

Place as many Fraction Bars as desired face down in a pile. The bars are turned over one at a time. Each time the players attempt to name a fraction equivalent to the one shown on the bar. The player able to do this first wins the bar. If neither player does this (or if there is a tie), no one gets the bar. Play continues until all bars have been turned over, at which time the player with the most bars is declared the winner.

Version 3

Ask each pair of children to divide their bars into two piles. Players turn over the top bar in each pile and check if the bars show equivalent fractions. If the fractions are equivalent, the first player to declare it is awarded both bars. If the bars aren't equivalent, they are put aside and the next bars in the piles are examined in the same way. The player with the most bars after all have been flipped is the winner.

Additional Ideas

151–153 *Modeling Fractions With Pattern Blocks*

You Will Need

- Chapter 8, Fractions
- overhead pattern blocks
- tubs of pattern blocks

Your Lesson

Note: Magazine pictures showing solid shapes (cubes, prisms, cones, spheres, cylinders, etc.) and road maps will be needed in Contact Lessons 154 and 155, respectively. Please ask your children to bring these items from home.

In Contact Lessons 52 and 53, your children discussed several area relationships that exist among different pattern blocks. For example, the blue diamond can be covered by 2 green triangles, or 2 red trapezoids can be put together to cover a yellow hexagon. These relationships make it possible to model fractions with pattern blocks. The following activities can be used for this purpose and we suggest spending three days exploring them. Note that the activities use only the green triangles, blue diamonds, red trapezoids and yellow hexagons from a set of pattern blocks. Adjust the exercises within each activity to suit your class. As usual, these exercises are intended to be discussed in a show-and-tell manner.

Activity 1. Have the children place a blue diamond in front of themselves and tell them this block represents the number 1. Ask them to explore exercises such as:

What number is represented by the green triangle? ($\frac{1}{2}$) What number does the red trapezoid represent? ($1\frac{1}{2}$) How about the yellow hexagon? (3)

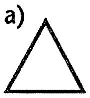
Form an arrangement of blocks that represents the number 2. (Examples are 2 diamonds, 4 triangles or a combination of 1 diamond and 2 triangles.)

Form an arrangement of 5 triangles. What number is represented? ($2\frac{1}{2}$)

Form an arrangement of blocks that represents the number $3\frac{1}{2}$. (Several answers are possible.)

Activity 2. Suppose that a red trapezoid represents 1. Explore exercises such as: What number is represented by the blocks shown here?

a)



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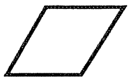

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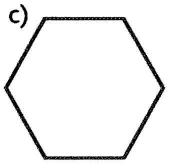

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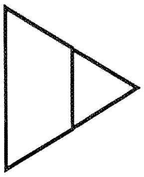

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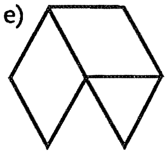
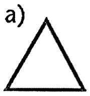


Form an arrangement of blocks that represents $1\frac{2}{3}$.

Activity 3. Suppose that a yellow hexagon represents 1. Explore exercises such as: What number is represented by the blocks shown here?

a)



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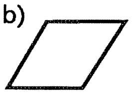

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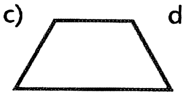

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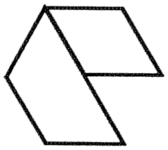

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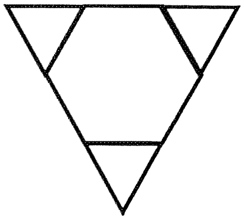


Form an arrangement of blocks that represents $1\frac{1}{6}$.

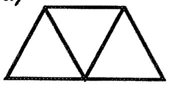
Activity 4. Create a design at the overhead (such as a hexagon formed by 2 trapezoids) and cover it from view. Tell a riddle such as, "I am a shape with 6 sides. One half of me is a trapezoid. What do I look like?" Ask children to create a design that answers the riddle. Invite volunteers to share their designs at the overhead. Uncover your design and compare it with theirs. Note that many designs are possible.

Activity 5. Suppose a blue diamond represents $\frac{1}{3}$.

Form an arrangement of blocks that represents 1.

What number is represented by the blocks shown here?

a)



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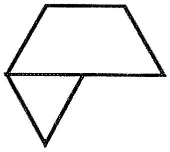

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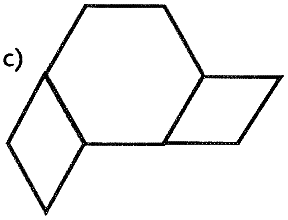
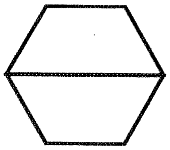


Activity 6. Suppose a red trapezoid represents $\frac{1}{4}$.

Form an arrangement of blocks that represents 1.

What number is represented by the blocks shown here?

a)



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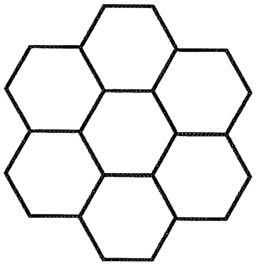

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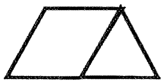


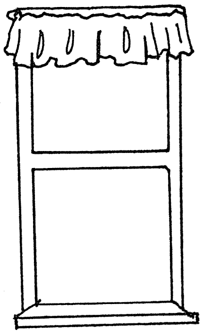
Teacher Tips	Consider adding a component to the Calendar Extravaganza in which pattern blocks are used to model fractions.
Journal Writing	At the end of these lessons, ask your children to describe how they felt while doing the activities. Have them also write about one thing they learned from the activity.
Additional Ideas	

154 *Solid Shapes*

You Will Need

- pictures of solid shapes (cones, rectangular prisms, cubes, spheres, pyramids, cylinders, etc.) brought in by your children

Your Lesson

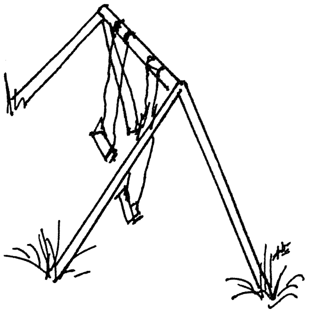


In this lesson, children examine and discuss how shapes are used in the world.

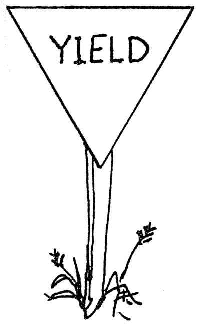
Begin by taking your class on a "shapes" hunt throughout the school and/or neighborhood. Can they find common shapes in building blocks? rafters? playground equipment? in the furnace? in the kitchen? in telephone poles? Can the children think of reasons why structures are shaped as they are?

Have children discuss the shapes in the pictures brought from home. Here are three activities that use these pictures.

In groups of four, have children sort their pictures according to the shapes that are shown.



Have each group form a pattern of shapes from their pictures. An example of this is *cone, sphere, rectangular prism, cone, sphere, rectangular prism.* We had our children assign numbers to their pattern. This example takes 3 steps to be completed, so it's a "3-step" pattern. If it were extended, pictures showing rectangular prisms would follow the sequence 3, 6, 9, 12, etc.



Allow time for creativity and fun! Some groups really took off! They presented their patterns in poem, rap and song.

Using general materials (pattern blocks, wooden cubes, hex-a-links, etc.), have the groups build replicas of some shapes in their pictures.

Homework

(Optional) What shapes can your children find in their homes? Some of our children drove their parents crazy!

155 Maps—Sorting and Fractions

You Will Need

- Chapter 2, Sorting
- a variety of road maps brought in by the children
- magnifying glasses
- chart paper divided into two columns, labeled "with" and "without"
- markers

Your Lesson

Note: Each child will need a map of a state in Contact Lesson 158. Please ask your children to bring some from home.

Begin this lesson by letting children search with a magnifying glass for interesting features on their maps. Let them talk with their neighbors about their discoveries for a period of time, then ask a volunteer to share one item of interest with the entire class.

TEACHER *Elayne, what really interesting thing did you find on your map?*

ELAYNE *I found a town named Jefferson. I'll bet it was named for Thomas Jefferson.*

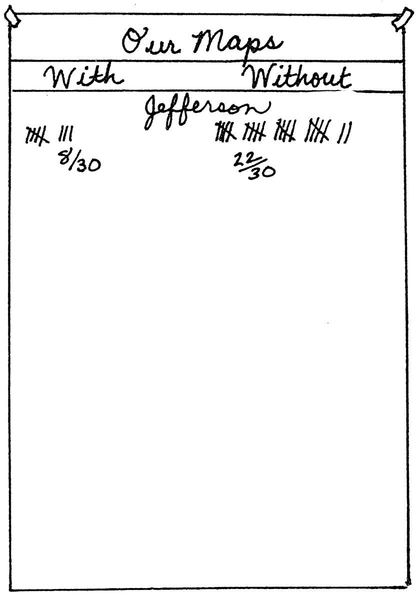
TEACHER *You may be correct. Often towns have been named for historical people. How many of you can find a town or something else named Jefferson on your map? Take a moment to look at your maps.*

VAL *That's easy! All of the towns are listed in ABC order at the bottom my map. I'll just look for Jefferson in the Js.*

GINNY *Hey, my map also has all of the towns listed in alphabetical order. Here it is—Jefferson City!*

TEACHER *If you found a town, street, river, etc., named Jefferson, will you please stand?*

Record the number of children who found Jefferson, perhaps as shown here.



Other points of interest are discussed in the same way, leaving time to examine some of the recorded information. Which items occurred frequently? Which appeared most often? Is it likely that the results would be the same if other maps were used?

Homework

A fun way to extend this activity is to have the students generate a list of items from their maps. A scavenger hunt can then take place by having each child take a map home and search for the items on the list with parental help.

Additional Ideas

156 *Maps—Estimation and Place Value Counting*

You Will Need

- base ten pieces (optional) and calculators

Your Lesson

How can the student population in the school be estimated?

Discuss different ways of answering this question. Here are some strategies that might be used:

Multiply the number of students in one class by the number of homerooms in the school.

Count the number of students that arrive on a selected school bus. Multiply this by the number of buses that arrive each morning.

At an assembly in the gymnasium, count the number of students in one section of the bleachers. Multiply this number by the number of sections that are filled up.

Determine an estimate according to each suggestion given by your children. How do the children feel about these strategies? Do they notice any problems in applying them? Which strategy seems most appropriate?

It may be helpful to determine the actual student population at this point. Students can be dispatched to gather enrollments from the various homerooms and as this information is posted, it can be totaled using base ten counting pieces or calculators.

Back to the estimates! How do they compare to the actual number of students? Were any way too high or way too low? If so, what might account for such a difference? Does averaging the estimates provide a better approximation to the actual enrollment? Are there any other observations that should be made?

A good way to conclude this lesson is to have the students use a map of the school to show each homeroom and its enrollment. As part of this, they can create a key that indicates classes of different sizes.

Teacher Tips

It is important to discuss the usefulness of each strategy that is suggested and for the children to reflect about which one(s) they would have confidence in using. They might note, for example, that the number of students on a bus will be influenced by such factors as absenteeism or bus size. Also, what about those who walk to school?

You may wish to study population maps further by discussing their uses and by having the children bring in examples to share.

Additional Ideas

157 *Maps—Fractions*

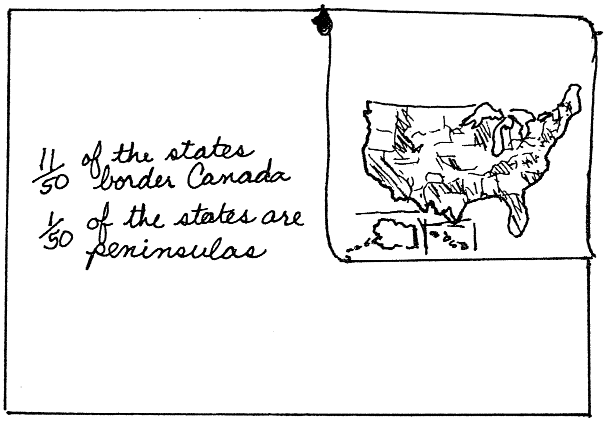
You Will Need

- classroom map of the United States
- a copy of Blackline 113 (United States map) for each child

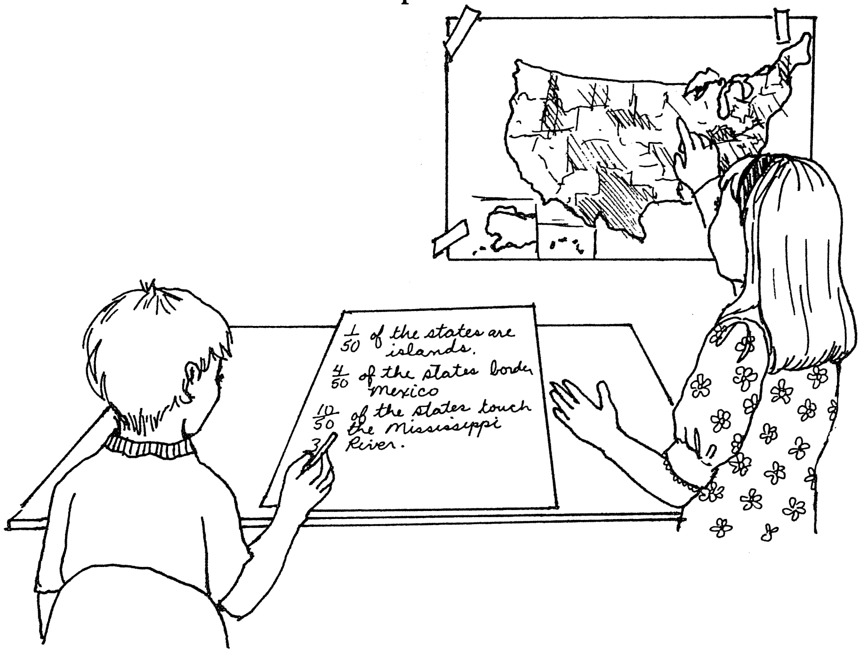
Your Lesson

What fraction of the states in our country border Canada? What fraction are peninsulas?

Have volunteers refer to the classroom map to answer questions.



Divide the class into pairs and ask each team to create additional fractions about the United States. To do this, they may refer to Blackline 113 or to the class map.



At the end of the work time, ask each team to examine their fractions. Which fraction is the largest? Which is closest to $1/2$? Which occurred most often? As time permits, invite students from each team to share some of their fractions and to demonstrate, using the classroom map, how the fractions were determined.

157 Maps—Fractions (continued)

Teacher Tips

Encourage family follow-up of this activity by challenging each family to create some unusual fractions about the United States map. Who knows—new doors might open for family discussions and trips, and fractions just may become more relevant for everyone.

Ask such questions as, "As the crow flies, what point is half way between our state and Washington, D.C.?; one third of the way between our state and the Gulf of Mexico?"

Additional Ideas

158 Maps—Geometry

You Will Need

- Chapter 11, Geometry
- a map of a state for each child

Your Lesson

The coordinate grid that appears on state maps is designed to help people locate cities easily. It also provides good examples of horizontal, vertical, parallel and perpendicular lines. Review these types of lines and the use of coordinates with the class.*

Have each child make a list of ten cities found in the index of a state map and give it to a partner. The partner records the coordinates for each city by its name and uses them to locate the city on the map.

Teacher Tips

*Some of the children may have had previous experience with the game of Battleship. We also find it helpful to conduct this review by overlaying transparencies of a coordinate grid and a school map, and then asking the children to identify areas of the building by their coordinates.

This lesson is also a wonderful activity for families to do together.

Additional Ideas

159 *Maps—Story Problems*

You Will Need

- a portion of a road map reproduced for every two children
- base ten pieces (optional)
- calculators (optional)

Your Lesson

Distribute maps to each pair of children and pose problems based on the information found in the maps.

How far is it from Frankfort to Louisville, Kentucky?

If you were hired by our governor to drive from Frankfort to Louisville to inspect flood damage there and she were reimbursing you 22¢ for each mile traveled, how much travel money would you receive?

Lisa's car usually gets 23 miles per gallon. At that rate, how many gallons of gasoline would she need to go from Cincinnatti to Louisville?

Allow time for each child to make up a problem for their partner to solve.

Additional Ideas

160–161 *Maps—Probability*

You Will Need

- Chapter 9, Probability

Version 1

- colored squares for the overhead
- two large 1–6 number cubes

Version 2

- two 1–6 number cubes for each pair of children

for both versions

- chart paper
- transparencies of Blacklines 99 (Crossing the Mississippi) and 100 (Crossing the Mississippi Recording Sheet)
- copies of Blacklines 99 and 100 for each child

Your Lesson

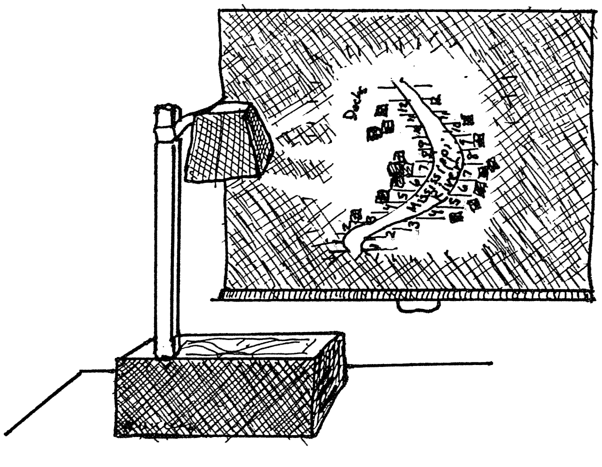
(This lesson has been adapted from *Mathematical Activities From Poland* by Jerzy Cwirko-Godycki)

Our children are particularly fond of this probability lesson, which we call "Crossing the Mississippi".

Version 1

Divide the class into two teams and display Blackline 99 at the overhead.

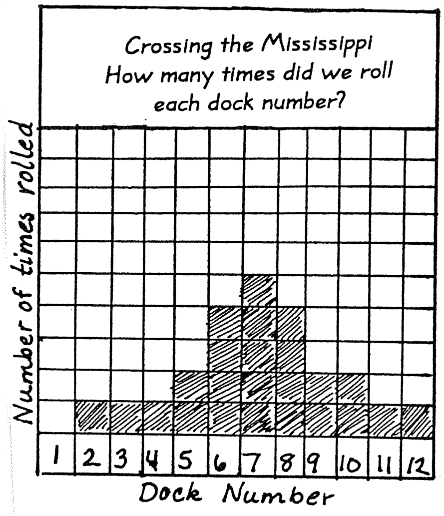
Each team is given 12 tile (or "boats") to place on the overhead in the docks on its side of the river. It is up to the team to decide how this is done. They may choose to place all of the boats in one dock or spread them out in some manner. Each team member makes a copy of the chosen arrangement as a reference for the discussion that follows the game.



The object of the game is to get all boats across the river, with play proceeding as follows:

The teacher alternately rolls the number cubes for each team. For each roll, the sum of the numbers names the dock from which one boat may cross the river. If the dock has no boat, that turn is lost.

As the rolls are made, the children construct a graph (Blackline 100) showing the number of times each sum from 1 to 12 occurred. At the end of the game, the totals are also recorded on chart paper.



How do the teams feel about their placement of boats? If the game were played again, would the first-mates encourage the captain to place the boats in different docks?

Play the game some more, providing time for teams to confer about the placement of their boats prior to each round. By compiling the results of each game, there will be enough data to motivate a good discussion of experimental probability. In particular, what might explain why certain sums occurred more often than others? Based on the data, would it be reasonable to expect a sum of 7 when the cubes are tossed? How about a sum of 10 or a sum of 1?

Version 2

An alternate way to organize this lesson is to have the children gather data by playing the game in teams of two at their desks. Compile individual results into one large class chart or graph. How do the combined results compare with those obtained individually? What might explain any differences?

Note: The rules of this game may be varied. For example, suppose the product of the number cubes determines the dock from which a boat is moved. How would the docks have to be numbered? What placements of boats would be good ones?

Teacher Tips

Theoretically, if the number cubes are randomly tossed, the sums that are possible do not have the same probability of occurring. For example, there is a greater chance of throwing a sum of 7 than a sum of 10. This can be seen by examining the following chart, which shows the different ways each sum can turn up.

160–161 Maps—Probability (continued)

numerals on die A

numerals on die B

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

From this chart, we see that the chances of tossing a sum of 7 are $^{6}/_{36}$, while the chances of tossing a sum of 3 are $^{2}/_{36}$.

The results of playing this game will vary. However, they should approximate theoretical answers, provided the number cubes are fair and a large number of random tosses have been made.

We liven the game by forming two teams, Missouri and Illinois, that are divided by an imaginary river through the middle of the room. We also keep the discussion intuitive. Which docks seem most likely to empty first? What placements of boats seem to be "good" ones?

Additional Ideas

162–163 *Mystery Box—Rocks; Geometry—Problem Solving*

You Will Need

- Chapter 11, Geometry
- your mystery box containing some rocks
- 20 tile
- Blackline 85 (Triangle Pieces for Kites) duplicated on various colors of construction paper. Each child will need at least three rows of one color of triangles and two rows of another color. We suggest at least 5 sheets each of yellow, orange, green, blue, red and violet for 30 children.

for each child:

- copies of Blacklines 86–87 (Kites)
- scissors and glue
- a sheet of 12 × 18 construction paper (provide several colors to choose from)

Your Lesson

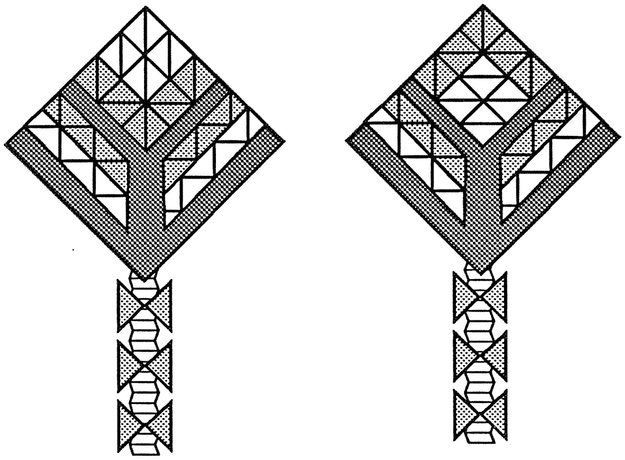
Children come to school with some built-in fascinations. It seems they all have a natural curiosity about rocks. So bring out the mystery box for one last time. Lessons related to rocks begin with Contact Lesson 170. We found these lessons rekindled our interest in rocks and gave our children some new experiences they'll long remember.

You'll find the mystery box will be met with as much enthusiasm as it was at the start of the year. Hopefully, your children are asking pretty "high tech" questions. When the rock is guessed and makes its debut before your class, begin rockin' and rollin' through this fun math unit. Keep track of the questions by forming a rectangular array of 20 tile and removing one tile per question.

Kites are usually a source of much enjoyment for children of all ages. In this lesson, your children will make kite patterns by placing triangles together in different ways. The lesson will require one hour to complete. You may wish to do it in a single session or spread it over two days.

162–163 Mystery Box—Rocks...(continued)

Begin by asking each child to choose two colors of triangles cut from Blackline 85, as described above. Have them cut out the shapes on Blacklines 86 and 87, covering the shaded sections with one color of triangles and the plain sections with the other color. Aside from cutting a few end pieces in half, they should be able to cover these sections without any overlap. The covered sections can then be cut out and glued on construction paper to form a kite design, complete with tails. Here are two possibilities:



Additional Ideas

166 Measurement—Perimeter

You Will Need

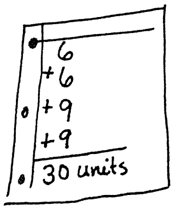
- Chapter 7, Measurement
- 7 books of varying sizes
- linear units
- blank paper and pencil for each child

Your Lesson

Note: The theme of rocks begins in Contact Lesson 170. Please ask each child to bring a handful of assorted rocks for that lesson.

Today's lesson provides experience with estimating perimeters. The main activity involves placing books (that are lying flat on the floor) in order of increasing perimeter.

Distribute linear units to your class and lay a book on the floor for all to see. Ask the children to estimate the perimeter of the book in terms of linear units. Have a volunteer(s) place units around the book's edge to determine the exact total.



For comparison, lay a book having a different perimeter next to the first one. How would the children estimate the perimeter of this second book? What strategies do they have for doing this? Some may want to lay out linear units as before; others may suggest ways of using information about the first book's perimeter.

Lay five more books on the floor. Divide your children into seven groups. Have each group estimate each book's perimeter and predict how to order the books in terms of increasing perimeter. Ask the groups to describe the thinking they used in making their predictions.

Ask each group to determine the perimeter of one of the books and place the books in the correct order. Discuss the results. How did the predicted orderings compare to the actual one? Would the order of the books be different if the perimeters were measured with some other unit of length?

Additional Ideas

167–169 *Measuring Lengths With Standard Tools*

You Will Need

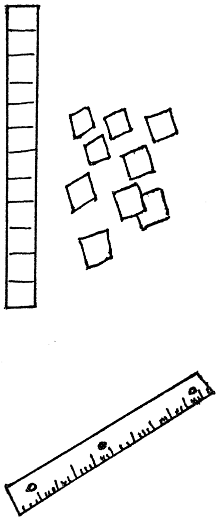
- Chapter 7, Measurement

Day 1
- individual copies of Blackline 25 (centimeter grid paper)
- crayons or markers for each child
- scissors, tape, paper clips

Day 2
- a copy of Blackline 88 (Guess and Check Record Sheet) for every two children
- chart paper

Day 3
- a copy of Blackline 88 for every two children
- rulers, yardsticks, tape measures

Your Lesson



Day 1

Today's lesson is the first of three that focus on the use of standard tools for measuring lengths. We begin by building upon your children's familiarity with centimeter grid paper.

Divide the class into pairs and assign the task of creating a centimeter tape measure. Teams can do this by cutting and taping strips from Blackline 25. They also decide how to mark the tape measure for convenient reading while measuring. Some may use different colors for each block of ten squares; others may label every centimeter or every tenth one. Our children decided the tape measure has to be long, easy to read, and able to be folded (and paper-clipped) like a carpenter's rule to take up as little space as possible. Save the tape measures for tomorrow's class.

Day 2

Select six lengths in the room to measure (e.g., a desk, floor, chalktray, door height, etc.). Form the same pairs as yesterday and ask each pair to estimate these lengths and then measure them with their tape measures, recording answers on Blackline 88. When this task has been completed, ask the teams to post their actual measurements on chart paper. Discuss the results. Were the measurements in agreement? What might explain any differences? How did the children feel about their initial estimates? Did they encounter any problems while measuring?

Theoretically, the teams' measurements should be in general agreement. There may be some differences due to errors normally associated with any measurement process.

164–165 *Geometry—Hexagon Quilts*

You Will Need

- Chapter 11, Geometry

for each child

- Blackline 64 (Pattern Block Triangles). Duplicate on three complementary colors of construction paper. Each child will need about 1/3 sheet of each color.
- a copy of Blackline 78 (Hexagon Quilt)
- scissors and glue

Your Lesson

Today's lesson integrates art, history, geometry, measurement, pattern, literature—and more! Each child will be responsible for completing one large hexagon that will become part of a class quilt.

Discuss the background of quilts. What kinds of quilt patterns are there? How are quilts related to various areas of your curriculum? Do the children have quilts that hold special meaning for them or their families? Is it necessary for all sections of a quilt to exhibit the same pattern?

Let children decide what color will be used for the light grey, dark grey and white elements, respectively, of the hexagon block (Blackline 78). Or, if they prefer, each child can decide on their own pattern of colors.

Have each child cut out their hexagon block and cover it with triangles, so there are no gaps or overlaps. Children may do this in various ways. Some may cover each section of the block with individual triangles throughout. Some may do so in a way that shows diamonds, trapezoids or smaller hexagons.

Discuss the problem of forming a class quilt from the hexagon blocks. How many rows of blocks should it have? Have many blocks in each row? Should there be a border?

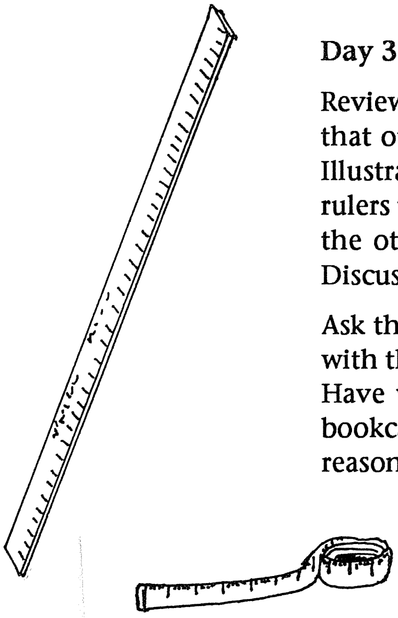
Once the quilt is made, discuss the geometry that is present. What shapes can be seen? Are any of them congruent? Is there any symmetry? Are there any parallel lines? What patterns can be observed? What is the perimeter of the quilt? Etc.

Teacher Tips

You might select colors that reflect a particular holiday, season or area of study.

Additional Ideas

167–169 Measuring Lengths With...(continued)

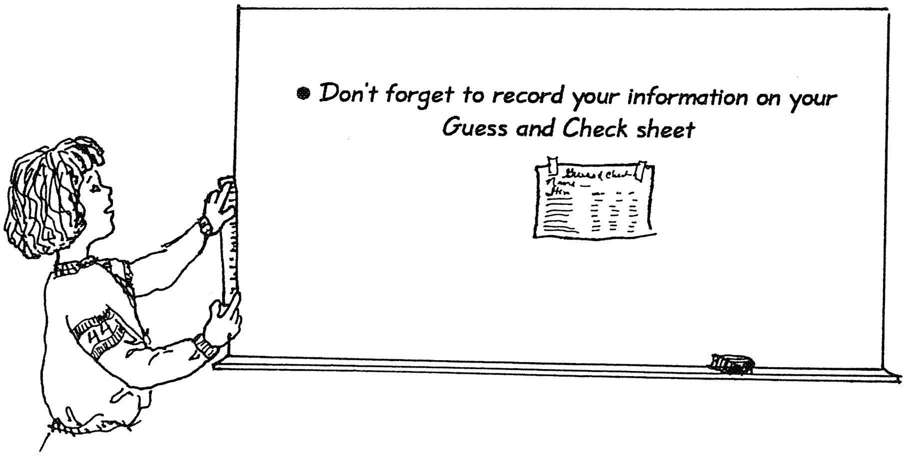


Day 3

Review the activities of the previous two days reminding the class that other units besides centimeters are used to measure lengths. Illustrate this by letting children examine foot rulers (if possible, use rulers that are marked with centimeters on one side and inches on the other), yardsticks, carpenter's rulers, or inch tape measures. Discuss the relationships among inches, feet and yards.

Ask the class to suggest ways to measure the height of a bookcase with these other tools. Which tool seems most appropriate to use? Have volunteer(s) complete the measurement. How high is the bookcase in terms of inches? in feet? in yards? What is the most reasonable way to report the answer?

Divide the class into pairs. Ask each pair to estimate and measure various lengths within the classroom. Have them pick appropriate measuring instruments for each task and record their answers on Blackline 88.



Additional Ideas

170 *Rocks—Sorting*

You Will Need

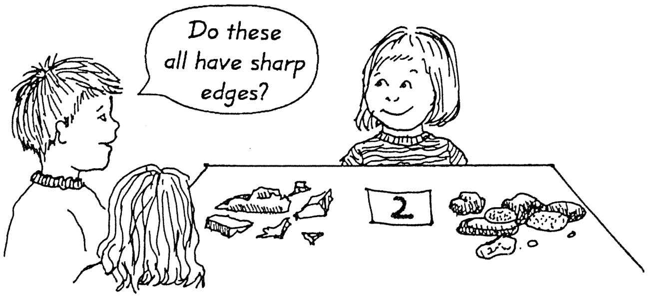
- Chapter 2, Sorting
- several rocks brought from home by children
- index cards and paper

Your Lesson

How many times has a child, with all the eagerness in the world, shown you a rock—and, to you, it looked just like any ordinary rock? After this lesson, you may appreciate the attributes of rocks in new ways.

Divide the class into teams of two and identify each with a number. Each team is to sort a handful of rocks into two piles, and record its method of sorting on one side of an index card and its team number on the other side. The card is then placed between the piles with the team number showing.

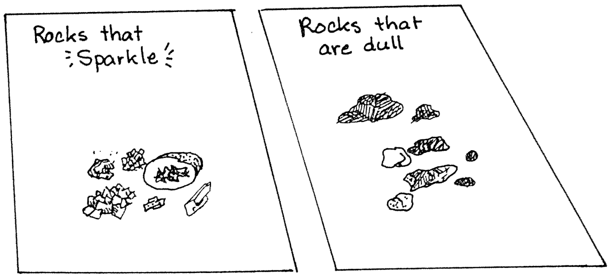
When the sorting is finished, each team will circulate, in order, to the other teams' stations. At each location, the teams examine the piles and guess the method of sorting. The teams each choose a recorder who keeps a running list of the guesses. This list is kept for future reference.



After an appropriate time, invite the students to share some of their guesses and compare them with the methods recorded on the index cards. How many times did the guesses agree with the methods of sorting actually used? What might explain any lack of agreement? Did any team have guesses that agreed more than half the time?

Teacher Tips

There is likely to be much discussion in this activity, particularly if words qualitative in nature were used to describe the sorted piles. The following illustration shows sorting criteria that could be ambiguous, since people may not agree on whether a given rock "sparkles" or is "dull".



171 Rocks—Estimation/Measurement

You Will Need

- a clear jar filled with water to within 2" of the top
- a sack of pea gravel
- a scoop (from drink mix, laundry detergent, etc.)

Your Lesson

Did you think we would conclude the monthly estimation activities by suggesting you guess the number of pieces of gravel on the school driveway? Well, you can relax a bit! Instead, we recommend estimating such rocky things as the number of pieces of aquarium gravel in a bag or trying this experiment which our children found interesting.

Start with a container almost filled with water and slowly drop scoops of pea gravel into it. What happens to the level of water? About how many scoops will be required to first displace water over the container's edge? What strategies might be used to make an estimate here?

One strategy our class devised was to calibrate the jar and see how many scoops it took to raise the water level to the next higher mark. This total was used to estimate the number of scoops required to raise the water level to each higher mark.

Add a bit of history to this activity by having your children research how Archimedes used water displacement to distinguish between pure gold and a gold alloy.

Teacher Tips

If a small jar is used, estimate the number of rocks, rather than the number of scoops, needed to displace the water over the edge.

Homework

What would happen if your bathtub were full to the brim with water—and you got in! Write about it and bring your story to class tomorrow.

Additional Ideas

172 *Rocks—Fractions*

You Will Need

- a large number cube with the sides labeled ½-½-⅓-¼-¼ and "Remove 1"
- a pile of small rocks for each team of two children
- transparency of Blackline 96 (More or Less Spinner)

Your Lesson

Your children will enjoy reviewing their year's experiences with fractions by playing the following game.

Spin the More or Less spinner to decide if "more" or "less" will win the game. Then give each team of two children a pile of rocks and roll the number cube. The fraction that shows up determines the number of rocks to be taken from the existing pile on that turn. Thus, if a team has 24 rocks and you toss ¼, then Player A takes 6 rocks for A's keeper pile. Player B then gets to play with a pile of 18 rocks. If you toss ½ on the next toss, Player B can remove 9 rocks to B's keeper pile. If you toss ¼, however, no play is possible and the person loses their turn.

Note that any time "Remove 1" is tossed, the player for that turn gets to remove one rock from the remaining pile and place it in their keeper pile. Play continues until a team has to play from a pile that has only one rock; at that time, the person with the most (or least) rocks wins. Teams that finish early can begin new games as time permits.

Teacher Tips

You can let teams set their own pace by giving each team its own number cube.

Additional Ideas

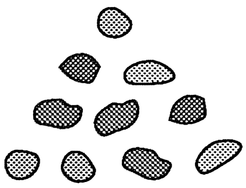
173 Rocks—Problem Solving

You Will Need

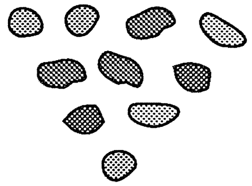
- a handful of small rocks
- base ten units for each child

Your Lesson

a)



b)

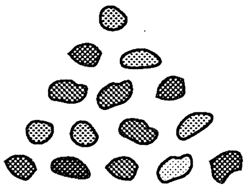


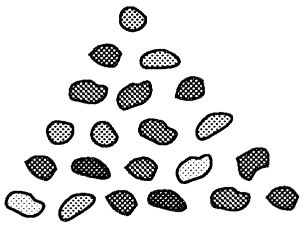
This may bring back memories from your childhood—or frustrate you to no end! On the overhead, form the arrangement of 10 rocks shown in figure a).

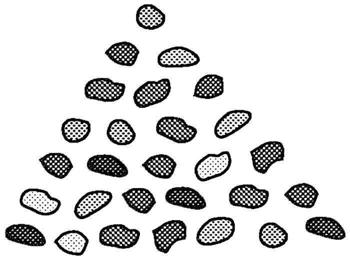
Have children make the same arrangement with base ten units. Now present them with the following puzzle: What is the fewest number of rocks (or units) one can move to make the triangle look like b)?

Invite volunteers to show how they changed the triangle. See if anyone can do it by moving only 3 rocks!

Make this puzzle more challenging by beginning with larger triangles. Ask children to change the direction of these by moving the minimum number of rocks.







Does anyone have a strategy for solving these puzzles? Can anyone predict the fewest number of rocks needed to change the direction of the third arrangement above?

Teacher Tips

These are puzzles that families at home will enjoy, so challenge your parents!

Have a guest speaker talk to your class about the probability of earthquakes in your area. The discussion will provide interesting information about the rock formations in your locale.

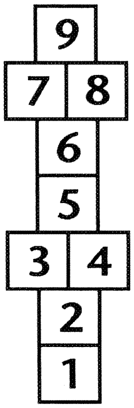
Additional Ideas

174 Rocks—Patterns

You Will Need

- Chapter 3, Patterns
- art paper and crayons

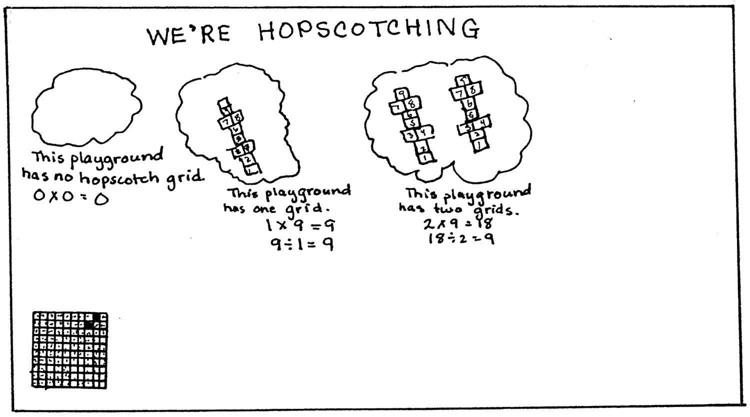
Your Lesson



Children have examined visual displays for multiples of various numbers in previous pattern lessons. This last seasonal unit gives them the chance to look at multiples of 9. Do this by building on the students' enjoyment of the game of hopscotch—Kentucky style. In this version, a grid of 9 squares is used.

The rules are the same as ever, only the grid is different. One seeks to roll a rock into square #1 and jump through the grid and back, skipping #1. This is followed by trying to toss the rock into square #2, etc.

If possible, provide time for the children to play the game. Then have the students each make a copy of the grid on art paper. Grids can then be displayed to show increasing sets of 9 squares.



Ask the children to imagine larger sets. How many squares would there be in 10 grids? How about 35 grids? How many grids can be made from 72 squares?

Additional Ideas

Two other displays you could make: slingshots with 9 rocks or 9 rocks in a clear plastic bag.

175–179 *Exploring the 9 Counting Pattern*

Your Lesson

It's time to explore the last counting pattern for the year. By now, we're sure this is "old hat" to you. Use the activities of Contact Lesson 173 to suggest ways to look at the counting progression 9, 18, 27, 36, 45, etc., in terms of arrays, patterns and song.

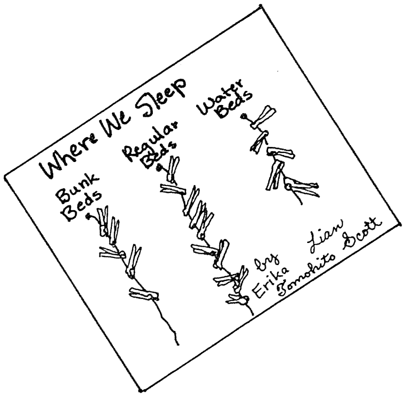
Additional Ideas

180–183 *Graphing Extravaganza*

You Will Need

- Chapter 10, Data Analysis and Graphing
- a variety of materials for making graphs

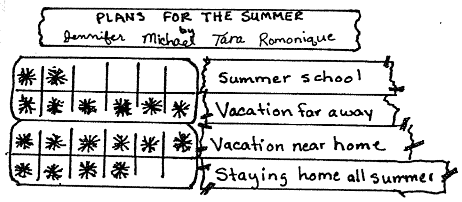
Your Lesson



One of our favorite ways of celebrating the year's end is to conduct a graphing extravaganza with our children. This four-day set of activities brings together all the graphing experiences you have done throughout the year.

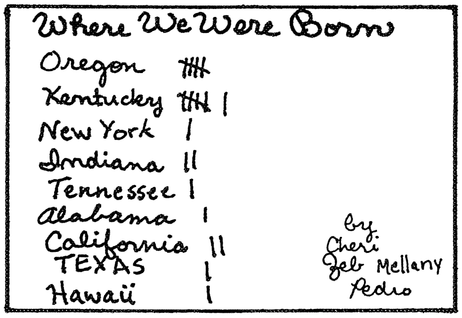
Day 1

Divide children into groups of four that will work together all four days. Ask the groups to decide what they would like to know about their classmates (summer plans, who sleeps with stuffed animals, favorite movie, etc.). After making this decision, have the groups design a graph that will be used to record information. Be sure each group indicates how this information is to be recorded. Save the graphs for the following lessons.



Day 2

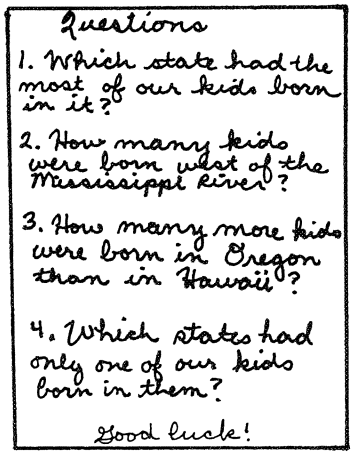
Post the graphs made during Day 1 at various stations throughout the room. Have each child visit each station and provide the information called for in the graph. Ask the originators of each graph to write some questions about the data that appears on their graph. These questions can be typed on the computer or written on a blackline to be copied for tomorrow's lesson.



Where We Were Born

Oregon	卌
Kentucky	卌 I
New York	I
Indiana	II
Tennessee	I
Alabama	I
California	II
TEXAS	I
Hawaii	I

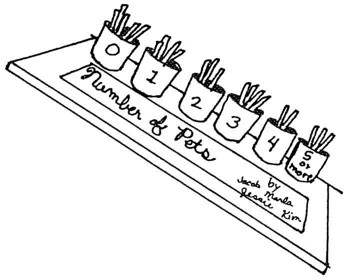
by
Cheri
Jeb Mellany
Pedro



Questions

1. Which state had the most of our kids born in it?
2. How many kids were born west of the Mississippi River?
3. How many more kids were born in Oregon than in Hawaii?
4. Which states had only one of our kids born in them?

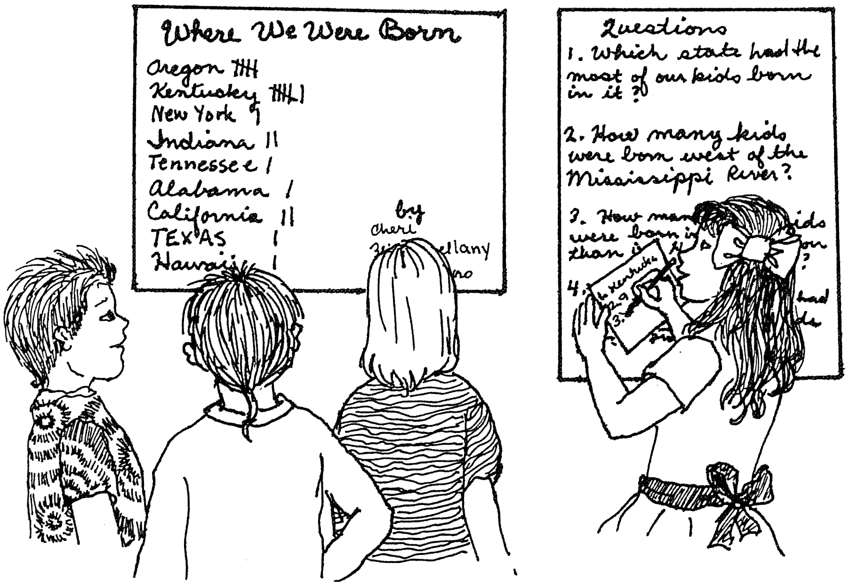
Good luck!



Day 3

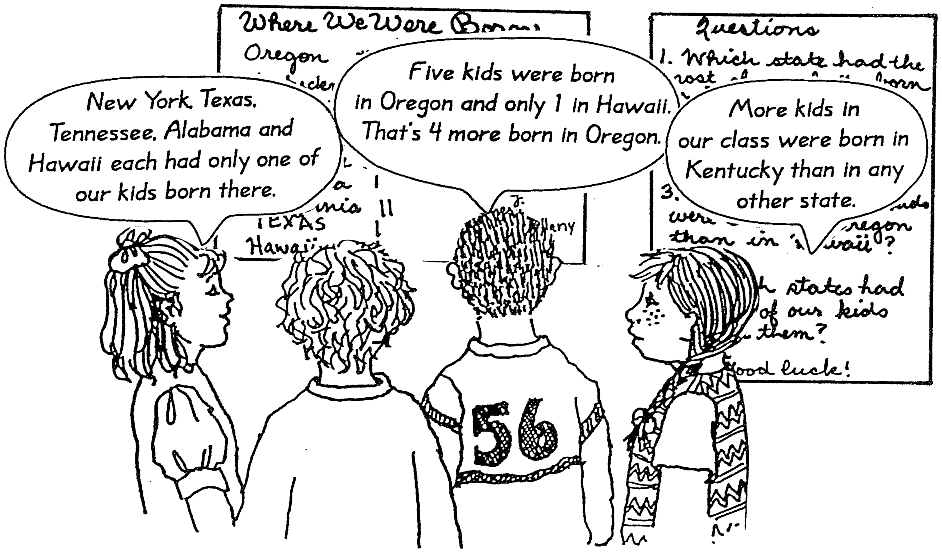
Distribute the questions that were written at the end of Day 2. Each group should have a copy of every other group's questions. Ask the groups to visit each graph and answer the corresponding questions.

180–183 Graphing Extravaganza (continued)



Day 4

Complete the graphing extravaganza by having each group summarize the data on its graph and answer the questions they composed. As this is done, the groups at their seats will check their responses to the questions.

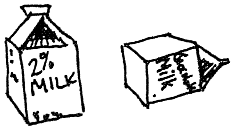


Additional Ideas

184 Problem Solving

Your Lesson

How much milk do your children consume at school?



Divide your class into groups of four and ask them to investigate the above question. Leave the method of investigation up to them, but do ask them to prepare a report of their procedures and conclusions. Have them describe their feelings about their work.

An alternate way to organize this lesson is to have each group of four respond to questions such as:

If children drink a half pint of milk at school each day, how many days will it take a child to drink a quart? a gallon?

If every child drinks a half pint today, how many quarts or gallons would the class consume today alone? How much would each child drink from this day until school is out?

How much does daily milk consumption at the school vary? How might one find this information?

If the milk company makes a nickel profit from every pint sold, what profit would it make in your school today? Why does milk cost so much if so little is profit? How many ways must the profit be shared?

Additional Ideas

185 Calendar Extravaganza Wrap-up!

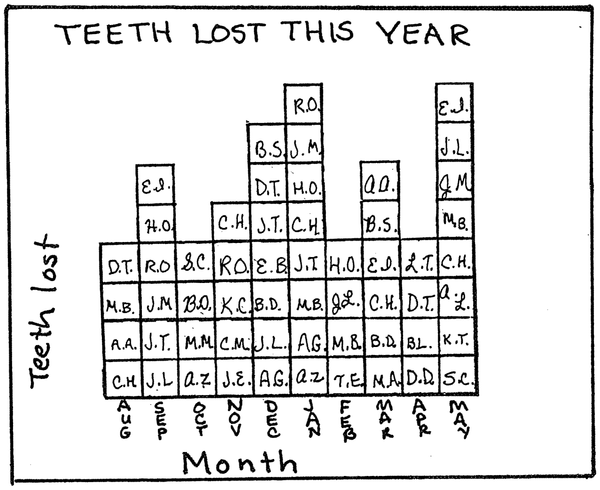
You Will Need

- your Calendar Extravaganza

Your Lesson

Take a few minutes on the last day to analyze the data on the calendar before dismantling it. For example:

Cut apart the dental floss record of teeth lost. Put each strip side by side to create a graph and see which month saw the most teeth go by the way.



Examine the number line to see the year in review. Ask questions involving mental calculations concerning some big events. For example: How many days before winter break did we get our pictures taken? Cut apart the number line and give it to your children as a record of their fun together.

Let this class determine and create a pattern for the first month of next year's calendar grid.

Add your own ideas to this Calendar Extravaganza!

Hope you had a great year!

Additional Ideas

Notes

Insight Lessons
68 – 102

68 Decimals—Introducing Tenths and Hundredths

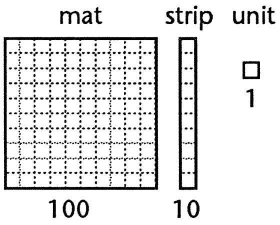
You Will Need

- Chapter 4, Place Value
- base ten counting pieces for the overhead

for each child

- a base ten counting piece unit
- two pieces of 8½ × 11 paper
- Blackline 89 (Decimal Pieces)

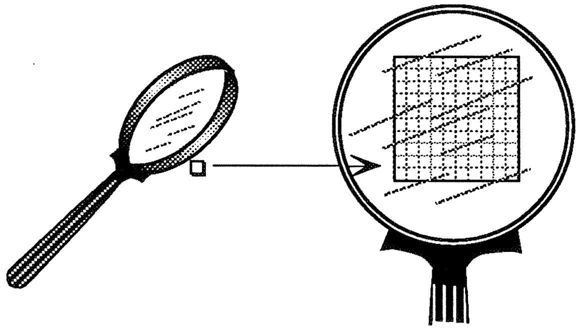
Your Lesson



(2 days) In our numeration system, the concepts of place value and positional notation used to describe whole numbers apply also to decimals (see Teaching Reference Manual, pages 31–33, for more information). It is therefore possible to model decimals with base ten pieces. This lesson illustrates how to introduce such a model to children.

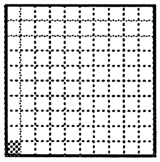
Show a mat, strip and unit on the overhead and review the patterns that exist among the base ten pieces.

Now place a unit square on the overhead. Ask your children to close their eyes and imagine looking at this unit through a magnifying glass for hidden surprises. While their eyes are closed, replace the unit with a "magnified" one in which tenths and hundredths can be seen (a base ten mat can be used for this purpose).

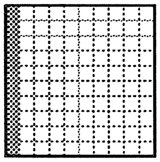


Use the magnified unit square to motivate a discussion of the following points:

The unit square has been divided equally into 100 smaller squares. Each smaller square represents one-hundredth of the unit square.



1/100 of a unit square



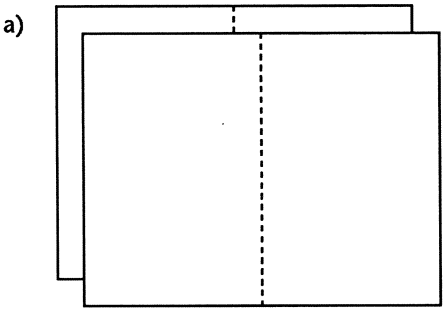
This strip is 1/10 of a unit square

Ten of the smaller squares also form a strip that is one-tenth of the unit square. Ten of these strips make up the unit square.

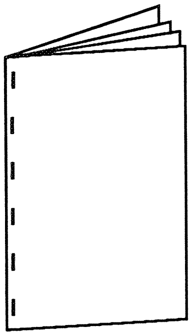
68 Decimals—Introducing Tenths...(continued)

The smaller squares may be referred to as hundredth squares. Similarly, the strips that are mentioned can be identified as tenth strips.

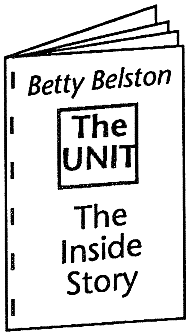
Distribute 8½ × 11 paper and copies of Blackline 89 to the class and ask your children to create a book that describes the "inside" story of the unit square.

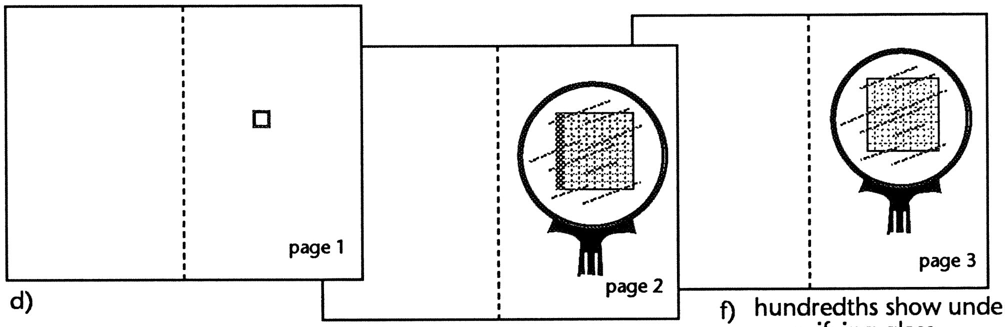


fold both pieces



fit one page inside the other and staple





e) tenths show under magnifying glass

f) hundredths show under magnifying glass

Emphasis

If the unit square is thought of as having a value of 1, then each tenth strip has value 1/10 and each hundredth square has value 1/100.

One tenth is made up of 10 hundredths.

A unit square consists of 10 tenths or 100 hundredths.

Additional Ideas

69 Decimals—Minimal Collections

You Will Need

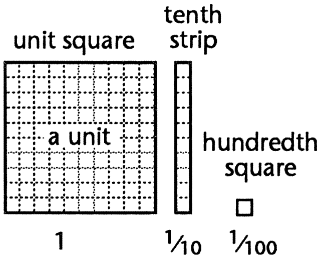
- Chapter 4, Place Value
- transparency of Blackline 35 (Decimal Collections)
- base ten pieces for the overhead
- (optional) Blackline 25 (1-cm Grid Paper)

for each child

- base ten counting pieces
- copies of Blackline 35 (Decimal Collections)

Your Lesson

(2–3 days) In this lesson, children use decimal pieces to represent decimal amounts. These pieces consist of unit squares, tenth strips and hundredth squares.



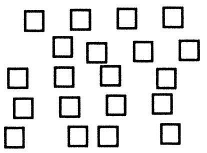
We usually seek the children's help in deciding how to make these pieces. Some suggest cutting them from centimeter grid paper, while others point out that the base ten counting pieces could be used. We generally go with the latter suggestion, since the children seem quite willing to pretend that the mats, strips and units now represent unit squares, tenth strips and hundredth squares, respectively.

Display the pieces at the overhead in the order shown above. Discuss the value of each piece and review how they are related. In particular, note that the pieces exhibit a growing pattern, where each piece is ten times as large as the next smallest one. Also, they extend the alternating square-rectangle pattern encountered in previous place value lessons.

Conduct these two activities.

Activity 1

Distribute the pieces and lay out 20 hundredth squares at the overhead. Ask the children to form the same collection at their desks. Record information about this collection on Blackline 35.



units (1)	"tenth" strips (1⁄10)	"hundredth" square (1⁄100)	total number of pieces
0	0	20	20

units (1)	tenth strips ($^{1}/_{10}$)	hundredth squares ($^{1}/_{100}$)	total number of pieces
0	0	20	20
0	1	10	11
0	2	0	2*

* minimal collection

Have children work in pairs on this problem: By making equal exchanges of pieces, what other collections of pieces also contain 20 hundredth squares?

Conduct a show-and-tell discussion of the children's findings, recording each result on Blackline 35. On the left is a complete listing of the possibilities.

Point out the collection that uses the fewest number of pieces is called the *minimal collection.* In this case, the minimal collection consists of 2 tenth strips.

Discuss the total value of these collections. Since each collection contains 20 hundredth squares, this value is the same throughout. As modeled by the collections, this common value is $^{20}/_{100}$ or $^{2}/_{10}$. It might also be reported as $^{1}/_{10}$ + $^{10}/_{100}$.

Activity 2

Ask children to set out at their desks a collection of 2 tenth strips and 17 hundredth squares. Have them record information about this collection on Blackline 35 (see first line on illustration to the left). Mention that this collection contains a total of 37 hundredth squares.

units (1)	tenth strips ($^{1}/_{10}$)	hundredth squares ($^{1}/_{100}$)	total number of pieces
0	2	17	19
0	1	27	28
0	0	37	37
0	3	7	10*

Now have children make equal exchanges of pieces to form other collections which also contain a total of 37 hundredth squares. They record their results on the blackline. The chart to the left shows a complete listing of the possibilities.

Discuss the results in a show-and-tell manner. What is the minimal collection? What is the common value of these collections?

Note that the minimal collection is identified above with an asterisk. The value of each collection is $^{37}/_{100}$.

As time permits, repeat this activity beginning with other collections of decimal pieces. This chart lists the minimal collections of other examples.

	units	tenth strips	hundredth squares
a) 16 tenths	1	6	0
b) 1 unit, 14 hundredths	1	1	4
c) 1 tenth, 21 hundredths	0	3	1
d) 9 tenths, 14 hundredths	1	0	4

Journal Writing

A creature from space has just found your decimal pieces and wants to know about them. How would you describe them?

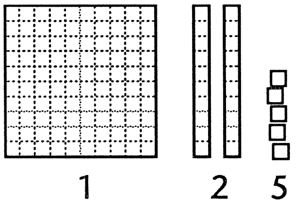
70 *Introducing Decimal Notation*

You Will Need

- Chapter 4, Place Value
- base ten counting pieces (or decimal pieces cut from centimeter grid paper) for each child
- base ten pieces for the overhead
- individual chalkboards, chalk and erasers

Your Lesson

(2–3 days)



Part One

Show this minimal collection of decimal pieces at the overhead, indicating the number of times each different piece appears.

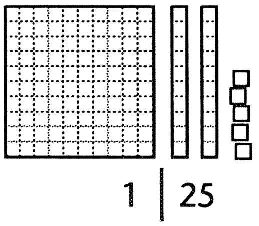
Introduce decimal notation as a means of distinguishing this collection from one that represents the number 125.

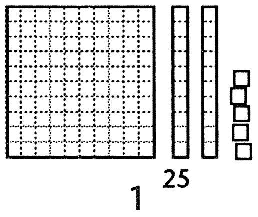
TEACHER *I'm confused. In the past these numbers represented 1 mat, 2 strips and 5 units. The number 125 meant a total of one hundred twenty-five units altogether. Here some of the pieces represent part of a unit. We need to use something to show that the tenths and hundredths are parts of a unit.*

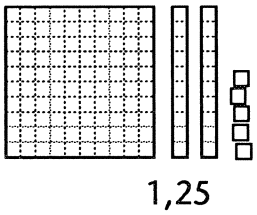
HELMUT *I see what you mean. We need something to make the numbers look different.*

TEACHER *How can we do that?*

Some ways that might be suggested:



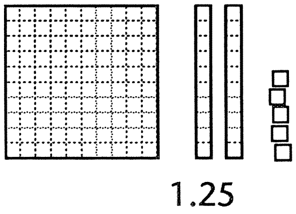




Explain that our number system uses a decimal point. It looks like a period and acts as a punctuation mark when describing numbers.

Ask the children to suggest ways to report the total value of this collection. Here are some possible responses:

a) One plus twenty-five hundredths
b) 1 + $^{25}/_{100}$
c) $^{125}/_{100}$
d) 1 unit + 2 tenths + 5 hundredths
e) 125 hundredths

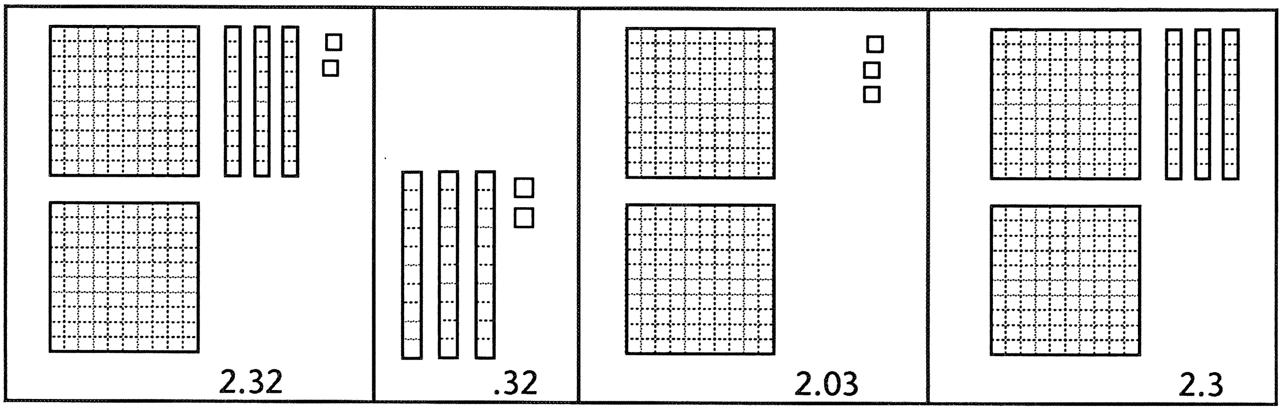


Tell the class that 1.25 is another way of describing the value of the collection. This number is commonly read as "one point two five" or "one and twenty-five hundredths."

70 Introducing Decimal Notation (continued)

Part 2

Have the children form other minimal collections of decimal pieces and record the corresponding decimals on their chalkboards. Here are some examples:



Discuss ways to report the total value of each collection. For example, here are four ways to report the value of the second collection, above:

32⁄100   .32   .3 + .02   3⁄10 + 2⁄100

Part 3

Complete the lesson by asking the children to use their decimal pieces to form the minimal collection for several numbers such as:

1.24   .13   .05   1.08

Ask volunteers to describe their thinking about each collection and the total value of each.

Emphasis

A period is used to mark the end of a sentence. In a number, the decimal point marks the end of the units and the beginning of parts of a unit.

As with whole numbers, positional notation is used when writing decimals to report the number of times each piece is used in a minimal collection.

Sometimes a zero is needed as a placeholder. A zero indicates the absence of a particular piece in a minimal collection.

Homework

Have children work with an adult to create a list of places where they have seen decimals. Have them bring the list to class tomorrow.

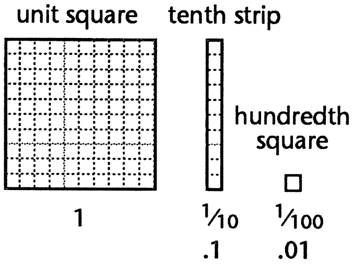
71 Relating Fractions and Decimals

You Will Need

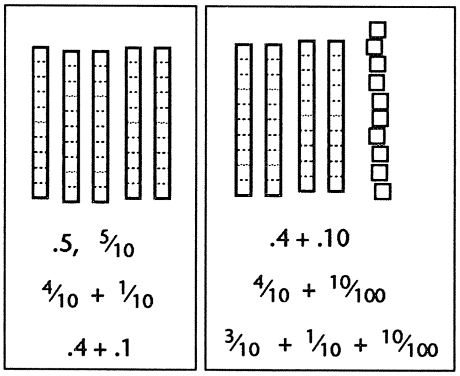
- Chapter 4, Place Value
- base ten counting pieces (or decimal pieces cut from centimeter grid paper) for each child
- base ten pieces for the overhead

Your Lesson

(2 days) Place a unit square, tenth strip and hundredth square on the overhead. Review the values of these pieces, assuming the unit square represents the number 1. Ask volunteers to then write a decimal for each piece.



Distribute the pieces and have each group of four form a collection that has a total value of $\frac{1}{2}$. Conduct a show-and-tell discussion of their work. As an example, one group might show a collection of 5 tenth strips. Another might form 4 tenth strips and 10 hundredth squares. Discuss the following questions: What decimal(s) could be used to describe the value of their collections? what fractions?



Encourage the children to share their thinking. For example, have them demonstrate why the values indicated above are appropriate. Seek out as many collections as possible. What others are there?

Repeat this activity by asking the children to form collections that have values such as:

$\frac{4}{10}$ $\frac{13}{100}$ .05 .61 $\frac{1}{4}$

Journal Writing

Sketch a picture of 3 tenth strips. How would you report the value of this collection with a decimal? How would you report it with a fraction? Explain your reasoning.

72 *Comparing Collections of Decimals Pieces*

You Will Need

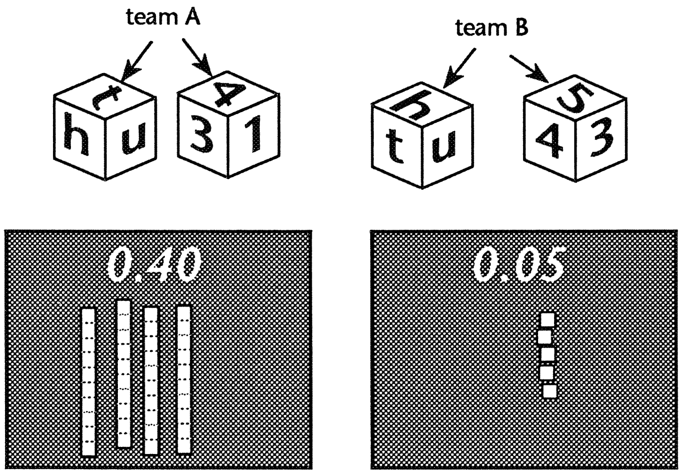
- Chapter 4, Place Value
- 0-1-2-3-4-3 number cube
- a units, tenths and hundredths cube
- individual chalkboards, chalk and erasers
- decimal pieces for each child
- transparency of Blackline 96 (More or Less Spinner)

Your Lesson

To provide practice with writing and comparing decimals, play the following version of Mine or Yours.

Divide the class into two teams.

In turn, each team rolls both cubes to determine the number of decimal pieces to place on individual chalkboards. The members of each team write a decimal that represents their collection.



The team that has more hundredths in its collection wins (or loses), as determined by a spin of the More or Less spinner.

Teacher Tips

The children need to work in pairs while playing, so they have enough decimal pieces for some of the rolls.

Additional Ideas

73 Modeling and Recording Decimals

You Will Need

- Chapter 4, Place Value
- two number cubes, 0–5 and 4–9

for each child:

- decimal pieces
- a decimal card
- individual chalkboards, chalk and erasers

Your Lesson

Here is a game that will give children practice with modeling and recording decimals. We call it Wipe-Out.

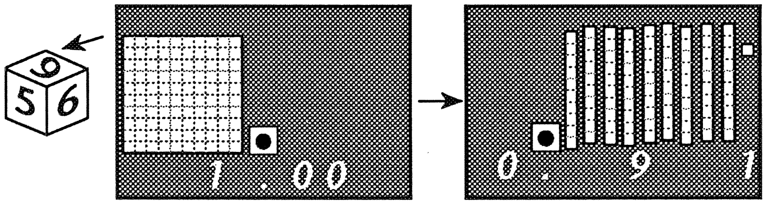
Distribute the materials listed above to each child and select two teams.

The members of each team set out a unit on their chalkboards and record its value (shown below on the first chalkboard).

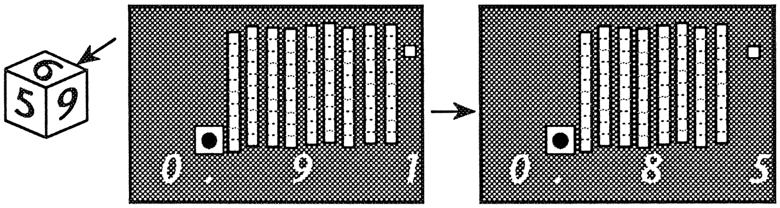
In turn, the teams select and roll one of the number cubes.

Each roll determines the number of hundredths that are removed from the team's unit. As this is done, the minimal collection of pieces is formed, trading pieces when necessary. For example, the illustrationshows three rounds of play for Team A.

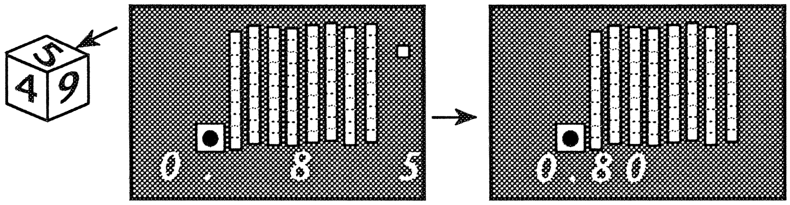
1st roll: remove 9 hundredths



2nd roll: remove 6 hundredths



3rd roll: remove 5 hundredths



The object of the game is for each team to wipe out its unit exactly and the first one to do so wins.

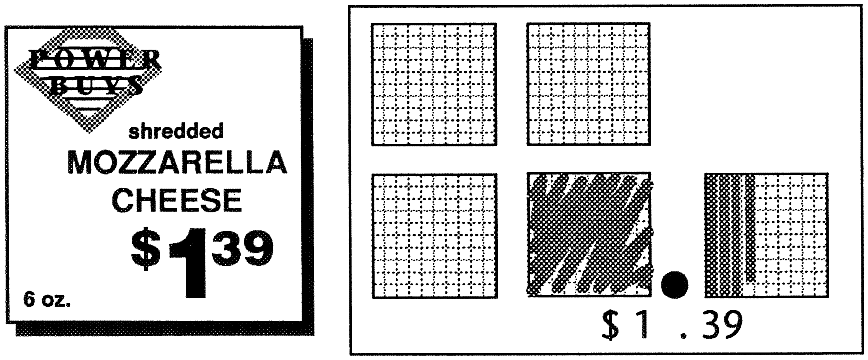
74 Money and Decimals

You Will Need

- Chapter 4, Place Value
  for every two children
- base ten decimal pieces
- base ten grid paper
- a decimal card
- glue and scissors
- newspaper, magazine or book order advertisements
- a sheet of art paper (approximately 18 × 24)
- a copy of the Blackline 116 (Money Record Sheet)
  for every four children
- money feely box (add $1 bills for this activity)

Your Lesson

(1–2 days) Distribute the materials listed above and have each team of two cut out an advertisement for an item that costs less than $5. Have them glue the advertisement on art paper and lay out a collection of decimal pieces (along with a decimal card) that models the price of the item. Have them also diagram the price on their Money Record Sheet and glue it to their display. Finally, ask the teams to represent the price with money from their feely boxes.



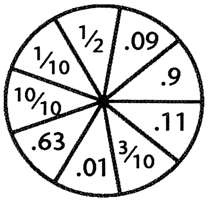
As time permits, other items may be similarly displayed. The grand total of the prices may also be estimated and calculated.

Teacher Tips

You might have each child repeat this activity using other advertisements as part of a homework or journal writing exercise.

75 Relating Fractions and Decimals

You Will Need

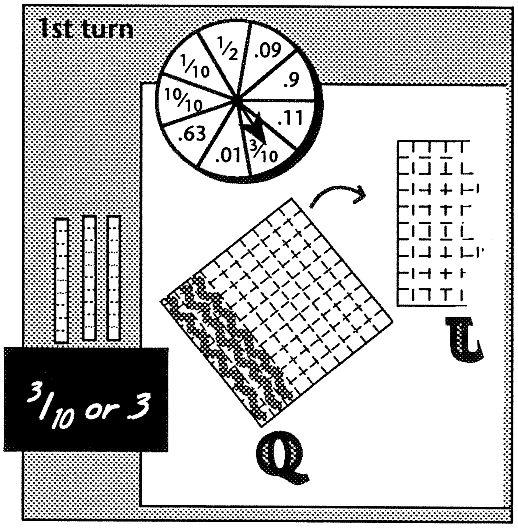


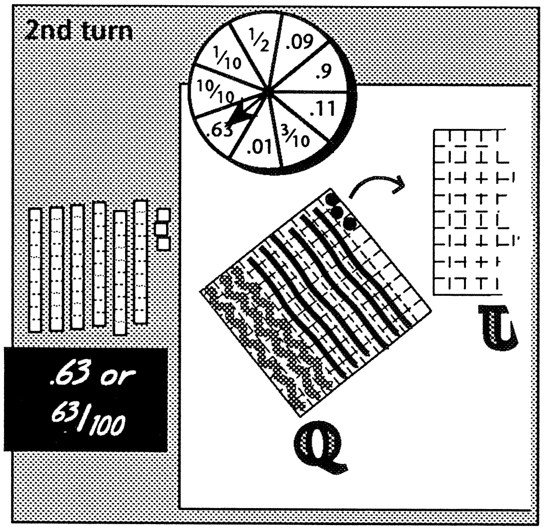
- Chapter 4, Place Value
- a copy of Blackline 37 (Quint) for each child
- yarn necklaces
- a collection of decimal pieces for each pair of children
- individual chalkboards, chalk and erasers
- transparency of Blackline 101 (Nine-Section Spinner), labeled as shown

Your Lesson

(1–2 days) The game of Quint strengthens children's awareness of how fractions and decimals are related. The mats shown on Blackline 37 each represent 1 unit and the object of the game is to completely fill (or go beyond) all 5 mats. Play the game as follows:

Divide the class into two teams by distributing yarn necklaces. The teams then alternately spin the spinner shown above. With each spin, individual team members set out a minimal collection of decimal pieces that corresponds to the number spun. On their chalkboard, they write the value of this collection as a decimal and as a fraction and shade in the amount on Blackline 37. Pictured here are sample turns for Team A.





Teacher Tips

You may wish to vary the numbers written on the spinner. Also, you might ask the teams to describe the cumulative value of their spins by a decimal and by a fraction.

Additional Ideas

76 *Multiples All Around Us*

You Will Need

- Chapter 6, Multiplication and Division
- chart paper and markers
- paper and pencil for each child
- calculators

Your Lesson

(2 days) This Insight Lesson marks the beginning of a unit on multiplication and division. The lesson focuses on the concept of a multiple of a number.

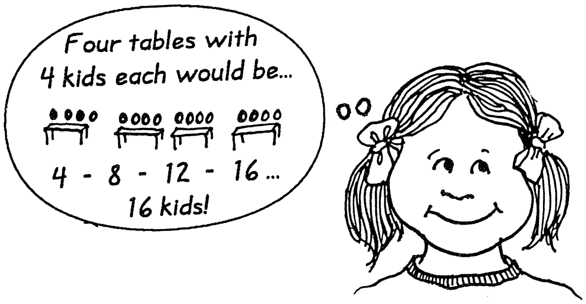
Day 1

Have the children conduct a "multiples" search throughout the classroom. Ask them to identify sets of objects that illustrate multiples of a number. Here are some things they might notice.

"Each person in the class has 2 ears." "I see 3 stars on the spine of each social studies book on the shelf." "There are 4 children at each table in the room."

Compile a list of their suggestions on chart paper and discuss them. For example, if someone observes that there are 4 children at each table, you might ask questions such as:

How many children are at 1 table? 2 tables? 3 tables? 4 tables?



If there were 10 tables with 4 children each, how many children would there be altogether at these tables?

Suppose there were 12 tables of 4 children each. Would there be more than 40 children altogether? How many would there be exactly?

What if there were 23 tables of 4 children each. How many children would there be?

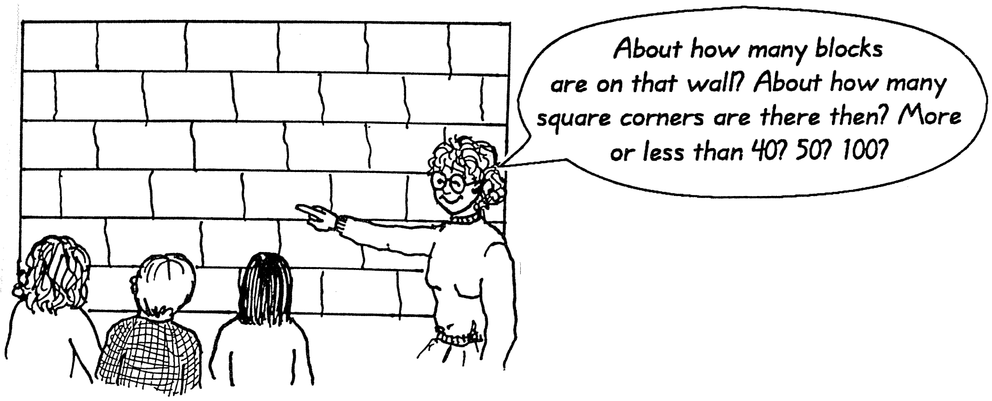
Suppose 80 children were to be seated 4 to a table. How many tables would be needed?

Your children might answer these questions in different ways. Some may be reminded of the 4 counting pattern and of the "Four Square" song. Others may suggest adding 4 repeatedly on a calculator. Still others may picture 5 tables with a total of 20 children and reason that 10 tables will have 40 children, 15 will have 60, etc.

End this day by asking the children to bring items from home that illustrate sets having the same number of objects. For example, they might find forks with 3 prongs each, hot wheel cars with 4 wheels each and maple leaves that have 5 "fingers" each. Have each child bring at least 10 of each item to display for Insight Lesson 77.

Day 2

Ask the children to extend their multiples search by looking throughout the school and its surrounding area. They might notice such things as blocks that each have 4 corners or cars that each have 4 tires. Take time to discuss some of their observations.



Have them make lists of their findings to share back in the classroom. You might compile these into a master list on chart paper.

Remind the children to bring their multiples from home for Insight Lesson 77.

Teacher Tips

This multiples search is most important because it demonstrates that multiples of numbers can be observed in the world around us and it lays the foundation for experiences with rectangular arrays and multiplication and division problems.

Additional Ideas

77 Multiples All Around Us

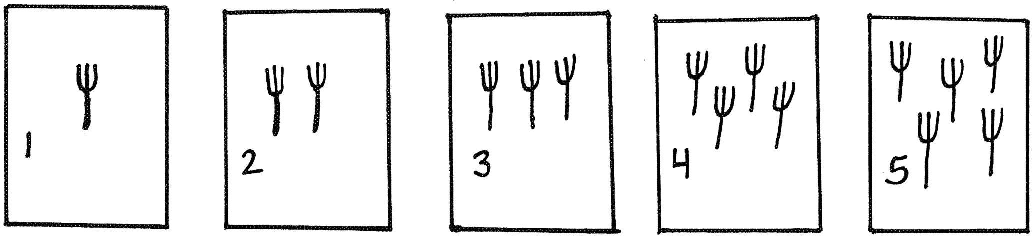
You Will Need

- Chapter 6, Multiplication and Division
- several sheets of 8½ × 11 paper for each child
- items brought from home that illustrate sets having the same number of objects (please see comment at the end of Day 1, Insight Lesson 76)

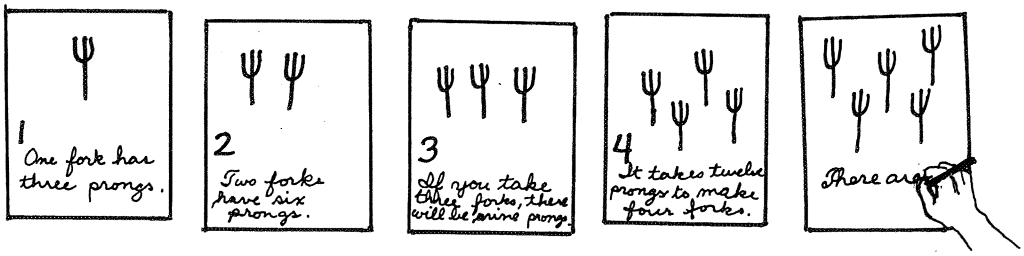
Your Lesson

(1–2 days) Complete the multiples search that was begun in Insight Lesson 76 by having your children create displays of multiples using the items from home. Have them follow these steps:

Arrange their items in order on pieces of paper, placing or drawing 1 object on the first piece, 2 on the second, 3 on the third, etc., until all are used.

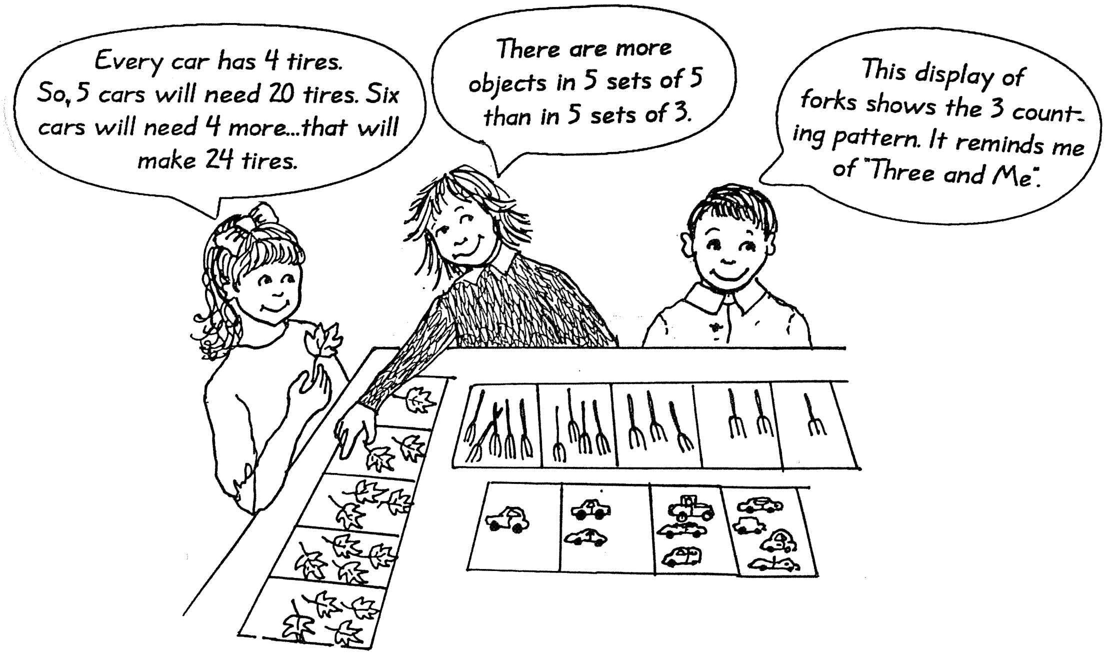


Write statements that describe their collections. Encourage the children to construct their own descriptions. Some may write verbal statements; others may record number statements that involve repeated addition or, if they are familiar with it, multiplication.



77 Multiples All Around Us (continued)

Examine and discuss one another's displays. What numerical patterns and relationships can be observed?



Save a favorite display to introduce Insight Lesson 78.

Emphasis

The children's displays of multiples illustrate the counting patterns explored in the Contact Lessons. Children usually recognize this connection and recall the activities and songs from those lessons.

Children find it helpful to make written statements that describe the growing patterns in their displays.

Allowing the children to communicate with one another about their displays broadens their awareness of collections that show multiples of a number.

Journal Writing

Before concluding this lesson, provide 10 to 15 minutes for your children to reflect about their multiples search. What did they learn from this search and what pleased them about it?

Homework

Challenge the children to extend the multiples search by making a list of additional examples seen in and around their home and neighborhood. Discuss the lists as they are brought to class.

Additional Ideas

78 Rectangular Arrays

You Will Need

- Chapter 6, Multiplication and Division
- colored squares for the overhead

for each child

- base ten units
- linear units

Your Lesson

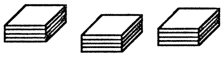
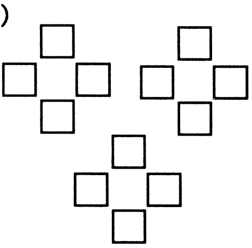
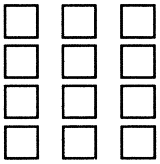
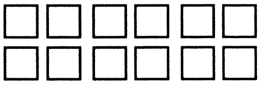
(3–5 days) In this lesson, rectangular arrays are used to model multiplication and division stories. Please refer to Chapter 6 of the Teaching Reference Manual for additional information and examples.

Multiplication

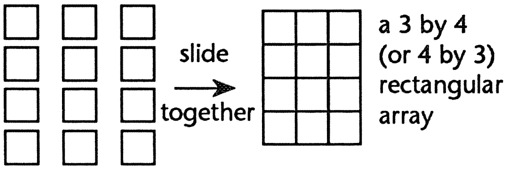
Examine the display saved from Insight Lesson 77 and ask your children to model one of the multiples with their tile.

"Larry's display shows 3 cars, each with 4 tires. I'd like you to represent these sets of tires with your tile. Imagine that each tile represents 1 tire. How many tires are there altogether?"

Several ways are likely to be demonstrated, some of which are shown here. Conduct a show-and-tell discussion of these responses. Discuss, in particular, how each child's model represents the sets of tires.

a)  "I placed my tile in piles of 4. Each tile is like 1 tire. There are 12 altogether."	b)  "I arranged my tile in 3 groups like this. Each group is a set of 4 tires."	c)  "I lined my tires up in groups of 4. Each group represents the 4 tires on a car."	d) front tires  rear tires "I see 6 front tires and 6 rear ones in my arrangement."

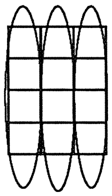
Build arrangement c) in the above illustration once more and slide the tile together to form a 3 by 4 rectangular array. (You might suggest this arrangement, if none of the children do. Note, too, that other arrays can be formed. For example, the tile in arrangement d) can be pushed together into a 2 by 6 array.)



Ask the children to build this 3 by 4 array and to show its dimensions with linear units. The array should remind them of similar ones formed in the Today's Array component of the Calendar and in the *Musical Array-ngements* counting lessons.

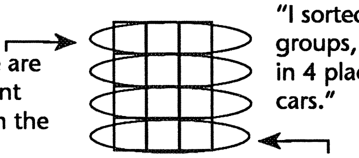
Discuss the above array. What observations do the children have about it? How does it provide information about the sets of tires?

a)



"I see 3 cars with 4 tires on each car."

b)

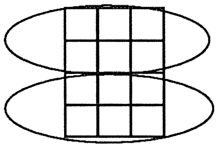


"These are the front tires on the right."

"I sorted the tires into 4 groups, since they are in 4 places on the cars."

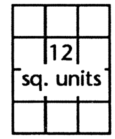
"These are the back tires on the left."

c)



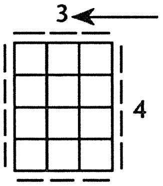
"I see the 6 front tires and the 6 back ones."

d)



"The area of this array is 12. That reminds me of the 12 tires in the display."

e)



"This dimension reminds me that there are 3 cars."

"This dimension reminds me of the number of tires on each car."

The above array can also be used to model division stories related to sets of tires. Discuss stories such as:

Jose's family bought 12 tires. They separated these into 3 equal piles. How many tires were placed in each pile?

Susan and her friends counted 12 tires altogether on their 3-wheelers. How many 3-wheelers were there?

Division

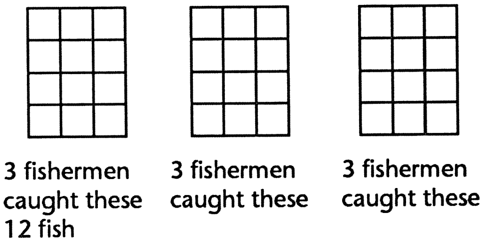
Repeat the above sequence of activities, only this time pose a division story such as:

"Yesterday, a group of people went fishing. It was such a lovely day! Everyone was very excited and spent the entire afternoon on the banks of the lake. By the end of the day, each person had 4 fish and there were 36 fish altogether. How many fishermen were there?"

78 Rectangular Arrays (continued)

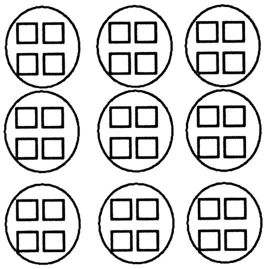
Ask the children to show-and-tell how they would model this story with tile, assuming that each tile represents a fish. Here are two possible responses:

a)



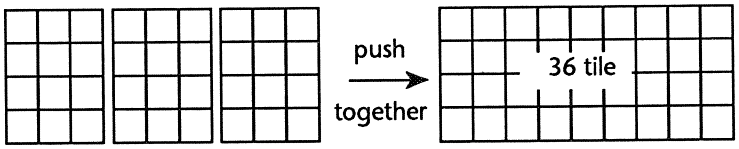
"Each tile represents a fish. I was able to arrange 36 tiles into 3 sets of 12. Each of these has 3 groups of 4, so each shows what 3 fishermen caught. There are 9 fishermen in all."

b)



"I took 36 tiles and made groups of 4. Each group shows what 1 fisherman caught. I was able to make 9 groups, so there are 9 fishermen."

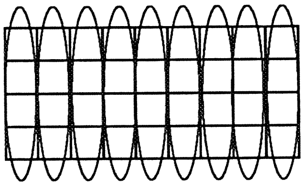
As part of this discussion, show how the 36 tile can be arranged into a 4 by 9 rectangular array. One way to demonstrate this is to push together the tile in arrangement a), above.



a 4 by 9 (or 9 by 4) array

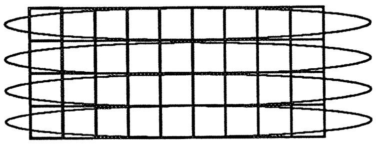
Discuss the above array. What is its area? What are its dimensions? What do the children notice about it? How does it provide information about the fishing story?

a)



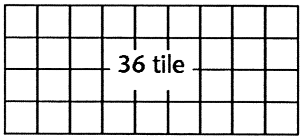
"I see 9 groups of 4. These show that 9 fishermen caught 4 fish each. So there must have been 9 people fishing."

b)



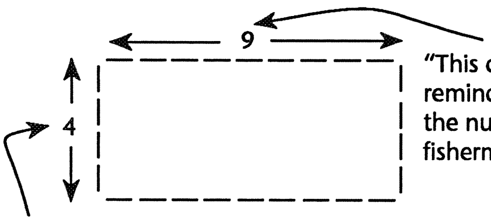
"I divided the 36 tile into 4 rows. There are 9 tile in each row—so each row shows one of the fish caught by the fishermen."

c)



"Each tile represents 1 fish. The area is 36—that shows 36 fish."

d)



"This dimension reminds me of the number of fishermen."

"This dimension reminds me of the 4 fish each person caught."

Multiplication and Division

Continue describing other multiplication and division stories. In each case, ask the children to:

Show-and-tell how they would model the story with tile.

Form rectangular arrays to model the stories.

Identify the areas and dimensions of the above arrays and describe how the arrays can be connected to the stories.

Additional examples are presented at the end of this lesson.

Before long, your children will likely want to be the storytellers. If so, turn them loose!

Emphasis

Pose both multiplication and division stories, including examples that illustrate both the sharing and grouping methods of division.

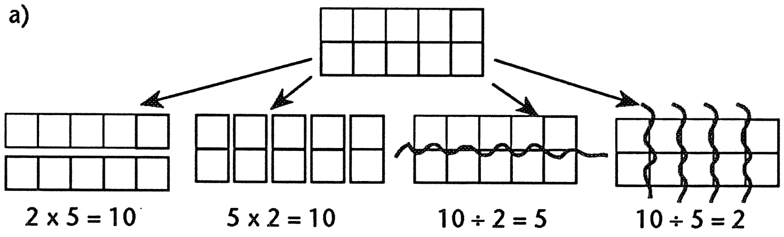
The stories provide a helpful context for learning about multiplication and division and give meaning to the children's models.

It is important to identify the dimensions and areas of the rectangular arrays that are formed in this lesson. Review the distinction between the measurements. In later multiplication lessons, your children will learn that products and factors can be modeled, respectively, by areas and dimensions of rectangular arrays.

Be alert to how your children are thinking about their models of the stories. What does each tile represent? How are they picturing the numbers used in the stories?

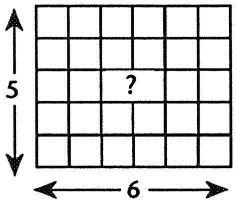
The inverse relationship between multiplication and division can be observed by examining rectangular arrays.

a)



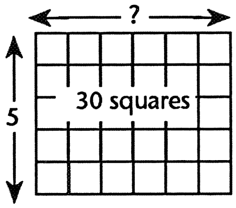
78 Rectangular Arrays (continued)

b) **Multiplication**
What is 5 x 6?
or
What is the area of 5 by 6?



5 x 6 is the number of squares in the array

Division
What is 30 divided by 5?
or
A rectangular array has an area of 30 and one dimension of 5. What is the other dimension?



30 ÷ 5 or the missing dimension

Teacher Tips

It may be helpful for children to build smaller arrays that represent parts of a story. The smaller arrays can then be pushed together into a larger one that models the entire story.

Homework

Ask each child to look through magazines or newspapers for additional examples that show multiples of a number. Have them attach their examples to paper and accompany them with pictures of rectangular arrays which show the multiples.

Assessment

Use a checklist that focuses on items such as:

Building rectangular arrays to model multiplication and division stories.

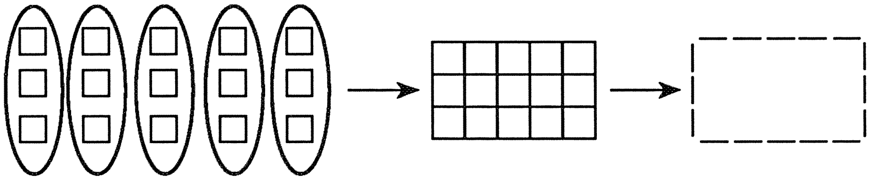
Identifying the area and dimensions of a rectangular array.

Describing the part of a multiplication or division story represented by a unit square in an array; by the area of the array; by each dimension of the array.

Additional Ideas

More examples for Insight Lesson 78

(Multiplication) Sally's display shows 5 forks with 3 prongs each. Imagine that a tile represents 1 prong. How would you use your tile to show the prongs on these forks? How many prongs are there altogether?

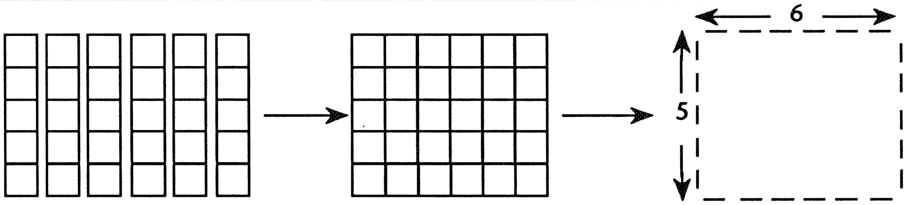


"Each tile represents a prong. I made 5 groups of 3 and used 15 tile. There are 15 prongs altogether."

"There are 15 tile in this array—so the area is 15 square units. That represents 15 prongs."

"The dimensions of the array are 3 and 5. There are 5 forks, each with 3 prongs."

(Grouping Method of Division) Joan's display shows packs of gum that have 5 sticks each. Suppose she has 30 sticks. How many packs does she have?



"There are 30 sticks of gum. If each pack has 5 sticks, there will be 6 packs. Each tile represents 1 stick."

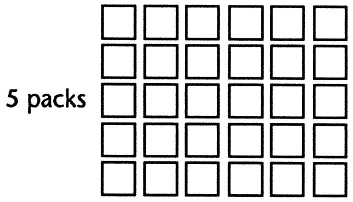
"The area of this array is 30 square units. This represents the 30 sticks of gum."

"I see 6 groups of 5. Each group shows the 5 sticks that are in a pack. So there are 6 packs."

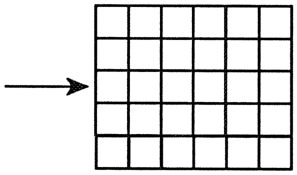
"The dimensions are 5 and 6. The 6 tells me there are 6 packs of gum. The 5 tells me there are 5 sticks in each pack."

78 Rectangular Arrays (continued)

(Sharing Method of Division) Joan's has 30 sticks of gum and she wants to divide them evenly into 5 packs. How many sticks will she have to put in each pack?

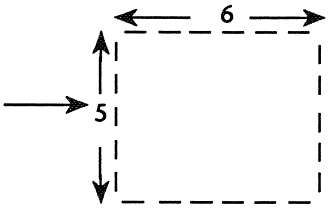


"There are 30 sticks. I kept putting 1 stick in each of the 5 packs until I used them all. Each pack got 6 sticks."



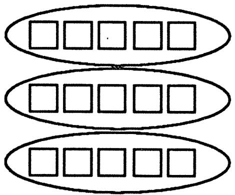
"The area is 30 square units. That shows the 30 sticks of gum. Each tile is 1 stick."

"I can see 5 rows of 6. That means that there will be 5 packs with 6 sticks each."

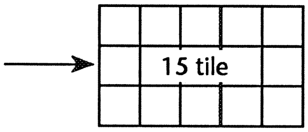


"The dimensions are 5 and 6. Here the 5 tells me there are 5 packs. The 6 reminds me that each pack has 6 sticks of gum."

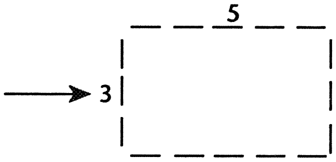
Fifteen cookies were distributed evenly among 3 children. How many cookies did each child receive?



"There are 15 cookies. I was able to make 3 groups of 5 by imagining giving each child 1 cookie at a time. Each child gets 5 cookies."



"The area of this array represents the 15 cookies. I can see 3 rows of 5. Each child gets 5 cookies."



"The dimensions are 3 and 5. I can think of 3 children getting 5 cookies each."

79 Rolling for Rectangular Arrays

You Will Need

- Chapter 6, Multiplication and Division
- translucent tile (colored squares) for the overhead
- 0–5 and 5–10 number cubes

  for each child
- base ten units
- (optional) linear units
- individual chalkboards, chalk and erasers (Version 2)
- Blackline 26 (Linear Unit Grid Paper—1-cm) (Version 3)

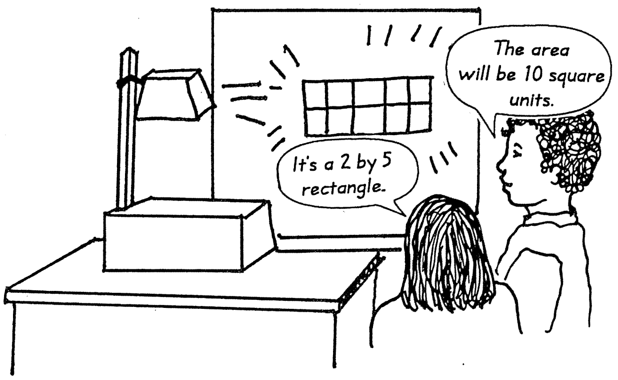
Your Lesson

During this lesson, children will form rectangular arrays having specified dimensions and write multiplication stories that can be modeled by the arrays.

Version 1 (1 day)

Roll the 0–5 and 5–10 number cubes to determine the dimensions of the array to build. As an example, if a 2 and 5 are rolled, then a 2 by 5 array is formed.

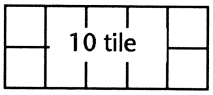
Have the children construct the above array with tile (base ten units) and invite a volunteer to build it at the overhead. Ask the children to identify the area and dimensions of this array.



Ask the children to tell multiplication or division stories that can be modeled by the array. Have the storytellers describe how the array and the stories are related. What does each tile represent? Does the area have any meaning? How about the dimensions? Here are two possibilities:

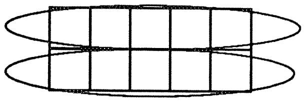
1) There were 2 boys with 5 pennies each. How many pennies did they have altogether?

a)



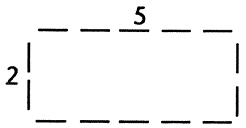
"Each tile represents 1 penny. The area is 10, so the boys have 10 pennies altogether."

b)



"I see 2 rows of 5 tile. These tell me that each boy had 5 pennies."

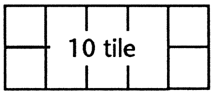
c)



"The dimensions are 2 and 5. These remind me that there are 2 boys with 5 pennies each."

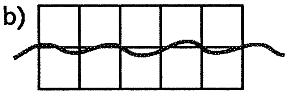
2) Together, 2 basketball players scored 10 points. Each player scored the same number of points. How many points did each make?

a)



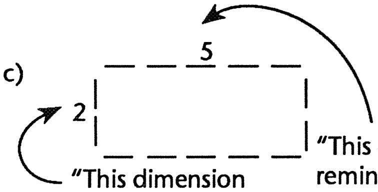
"The area is 10. It represents the 10 points that were scored."

b)



"I split the 10 tile evenly. The 2 rows showed that each player scored 5 points."

c)



"This dimension tells me there are 2 players."

"This dimension reminds me of the number of points each player scored."

Continue making rectangular arrays in this manner.

Version 2 (2 days)

Repeat the activities of Version 1, only this time, as the children build each array, have them also make a sketch of it on their chalkboards. Ask them to label the areas and dimensions of the arrays.

Version 3 (2–4 days)

Repeat the activities of Version 1, only this time, have the children make grid paper diagrams of their arrays on Blackline 26. Have them identify the area and dimensions of the array.

Emphasis

The children may suggest several ways to determine the area of an array. For a 2 by 5 array, some may see 2 rows of 5 and think of 5 + 5, while others may see 2 + 2 + 2 + 2 + 2 or some other combination of numbers.

The numbers that are rolled determine the dimensions of the arrays to be built. Your children may find it helpful to make an outline of the array with linear units prior to constructing the array with tile.

The dimensions of an array are factors of its area (see Today's Array in the Calendar). Continue to use the term "factor" in this context.

This lesson introduces multiplication by zero in a natural way. Children observe that when one of the rolled numbers is zero, they are not able to build an array.

80 Mine or Yours—Comparing Areas of Arrays

You Will Need

Version 1

- a large classroom set of Multiplication/Division Discussion Cards
- More or Less spinner (Blackline 96)

Version 2

- a set of individual Multiplication/Division Discussion Cards and a coin for every two children

Your Lesson

(2–4 days) In the following versions of Mine or Yours, opposing teams compare the areas of the rectangular arrays shown on two discussion cards. The team that has the array with more (or less) area is awarded a point. The game may be played in either a large or small group setting.

Version 1 (large group)

Divide the class into two teams and place the Discussion Cards face down in two piles. Each team draws the top card from its pile and discusses how to calculate the area of the array shown. When ready, each team shows its card to the other team and names the array and its area. The areas are compared and the More or Less spinner is used to determine which team wins a point.

Play as many rounds as time permits, using the More or Less spinner at the end to determine if the team with the most (or fewest) number of points is the overall winner.

Version 2 (teams of two)

Play proceeds exactly as in Version 1 except that the class is divided into teams of two and individual discussion cards are used. The winner of each round (as well as the overall winner) can be determined by tossing a coin, with the teams deciding which side of the coin represents "more" and which goes with "less".

Emphasis

Listen closely to the discussion that takes place throughout this activity. You will likely find children estimating and computing areas in several ways. Have them share some of these ways with each other.

Homework

Children (and parents) enjoy playing this game at home. We give the children duplicated sets of individual discussion cards and ask them to write directions for the game. Playing Mine or Yours with an adult is a pleasant way for children to learn the areas of different arrays.

Teacher Tips

At a later date, make "Mine or Yours" more challenging by having the teams compute the difference of the two areas and the winner gets that number of points. The More or Less spinner would determine the winner.

Additional Ideas

81 Matching Stories

You Will Need

- Chapter 6, Multiplication and Division
- large classroom set of Multiplication/Division Discussion Cards

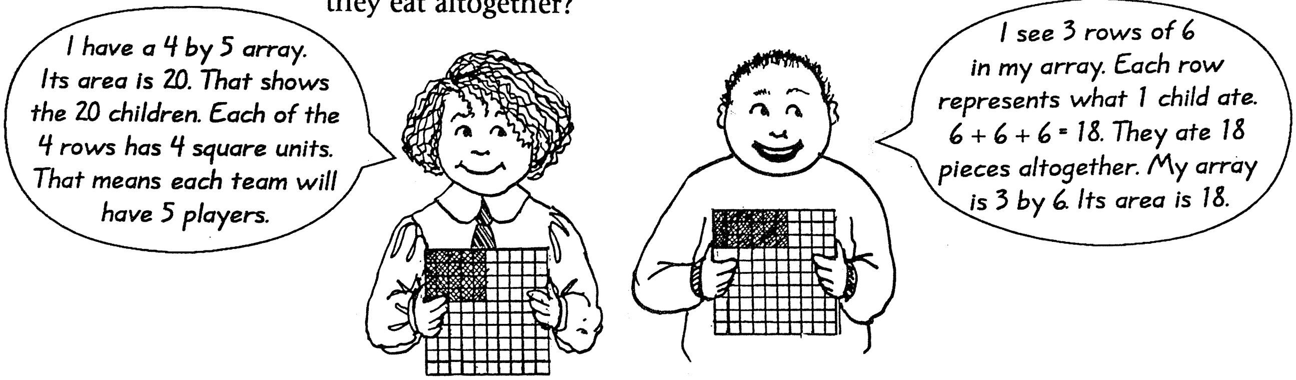
Your Lesson

(2 days)

Day 1

Randomly distribute all the discussion cards among groups of four children. As you tell a multiplication or division story, have the groups examine their cards for an array that can be used to model the story. If any group has such a card, have them discuss how it relates to the story. Here are two examples:

Three children ate 6 pieces of candy each. How many pieces did they eat altogether?



Twenty children are going to form 4 teams. They want each team to have the same number of children. How many children will be on each team?

Continue with the activity as long as time permits. Before the lesson ends, however, ask each child to write a multiplication and a division problem to share during Day 2. The factors in these stories should be 10 or less.

Day 2

Repeat the activity of Day 1, only this time use the children's stories as a source of problems. Who has a card to match their friend's story? You might allow more than one day so each child can share a story.

Journal Writing

Ask children to write a few sentences describing their feelings about modeling stories with arrays.

Homework

Send a note home suggesting that parents tell a few multiplication or division stories each evening to their children. The children can then use their home set of individual discussion cards to find arrays that model the stories.

82 *Discussion Card War—Comparing Areas of Arrays*

You Will Need

Version 1
- a large, classroom set of Multiplication/Division Discussion Cards

Version 2
- a set of individual Multiplication/Division Discussion Cards for every two children

Your Lesson

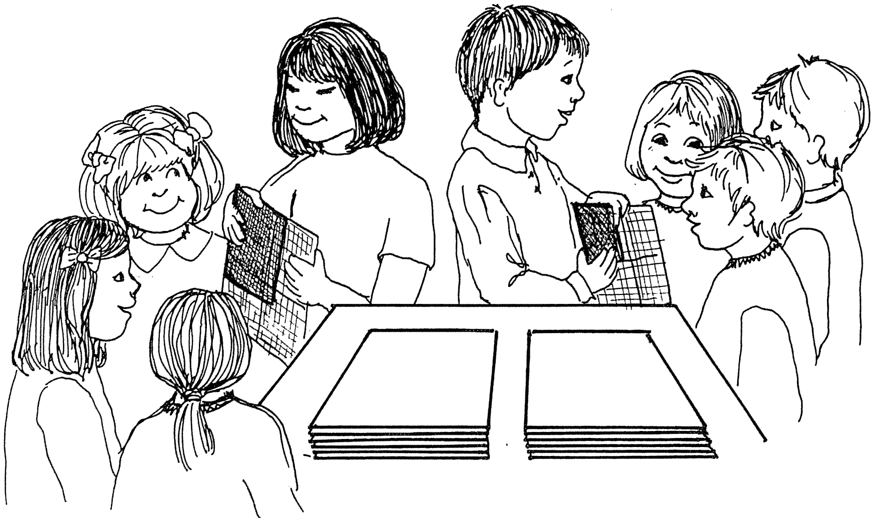
(2–3 days) Discussion Card War is similar to the multiplication version of Mine or Yours (Insight Lesson 80). It can be played in both a large and small group setting.

Version 1 (large group)

Divide the class into two teams and place the Discussion Cards face down in two piles.

Before play begins, the children decide if the winner of each round is the team whose card shows the larger or smaller area. They also decide if the overall winner is the team with the most or least cards at the end of the game.

Each team claims a stack and draws the top card in its pile. Team members discuss how to calculate the area of the array shown on the card.



When ready, each team shows its card to the other, naming the dimensions (factors) and area of its array. The winner of the round claims the cards being compared. (On any round where the selected cards have the same area, the teams draw 3 more cards each. They then compare the areas of the arrays on the third card, with the winner claiming all the cards.)

Play continues until one team loses all its cards or until time is called.

Version 2 (for teams of two)

This version is played like Version 1, except two children play against each other.

Version 3 (for teams of two)

This is played like Version 2, except no talking is allowed as children turn over their cards. Each child writes a multiplication statement for the array on their card. Children can then talk and can use these statements to compare the area of their arrays.

Teacher Tips

Discussion Card War is another game children enjoy playing at home.

Additional Ideas

83 Break 100—Estimating and Calculating Areas

You Will Need

- Chapter 6, Multiplication and Division
- a large classroom set of Multiplication/Division Discussion Cards
- individual sets of Multiplication/Division Discussion Cards
- Blackline 33 (Break 100)
- calculators, as needed

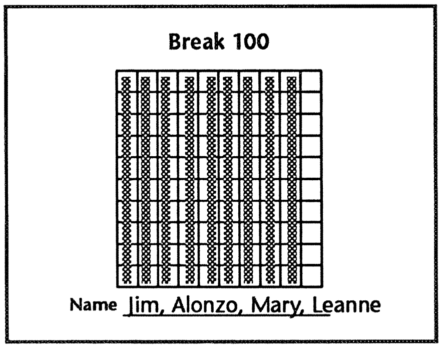
Your Lesson

(1–2 days) Break 100 is an area game that children always seem to enjoy playing both in school and at home. The game is played with teams of two to five children. In turn, each team is given the opportunity to draw as many Multiplication/Division Discussion Cards as it wishes, keeping a running total of the areas of the arrays on each card. The object is to have the sum of the areas be as close as possible to 100, without going over.

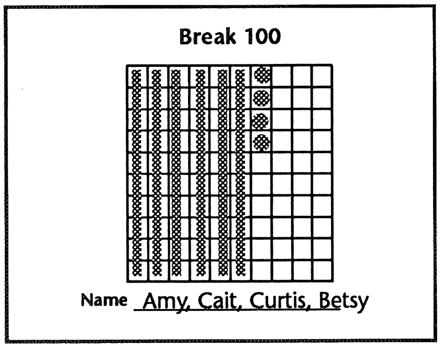
Play proceeds as follows:

Each team is dealt a card and determines the area of the array on it. The numerical value of the area is then diagrammed on Blackline 33 as shown:

Team A draws a 9 by 10 array and shades 90 units on its blackline.

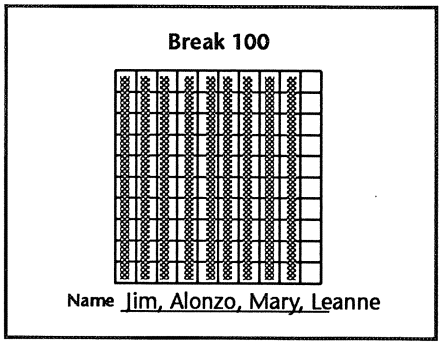


Team B draws a 8 by 8 array and shades 64 units on its blackline.



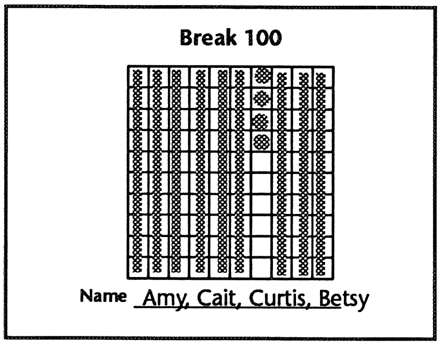
Give each team, in turn, the opportunity to draw another card or to pass. When a team chooses to draw, the area of the new array is sketched on Blackline 33.

Team A elects to pass.



Team A's area total is 90.

Team B chooses to draw and gets a 5 by 6 array. 30 more units are shaded on the blackline.



If, on its next turn, Team B elects to pass, its area total will be 94.

Repeat the second step until all teams have passed. At this point, the cards and area totals are revealed. The team with the total that is closest to 100 wins; any team whose total exceeds 100 automatically loses.

Version 1 (large group)

Break 100 can be played in a large group using a classroom set of Multiplication/Division cards. We find that teams of four or five children work well for this.

Version 2 (small group)

Break 100 can also be played in small groups, using individual sets of Multiplication/Division cards. Teams of two to four children seem to work well here.

Teacher Tips

Children use various calculating options when playing this game. The amount of mental arithmetic and estimation that takes place, even when calculators are at hand, is wonderful. As soon as a card is flipped, estimates are made regarding closeness to 100!

Homework

Like previous games, Break 100 is fun to play at home. Ask the children to write directions for it and to use the discussion cards they have at home to play the game with family members. The review of arrays, areas and factors will be enjoyable and instructive.

Additional Ideas

84 *Using Numbers and Symbols*

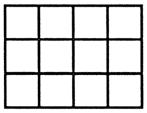
You Will Need

- a large classroom set of Multiplication/Division Discussion Cards
- colored squares for the overhead
- square tile (base ten units) for each child
- individual chalkboards, chalk and erasers

Your Lesson

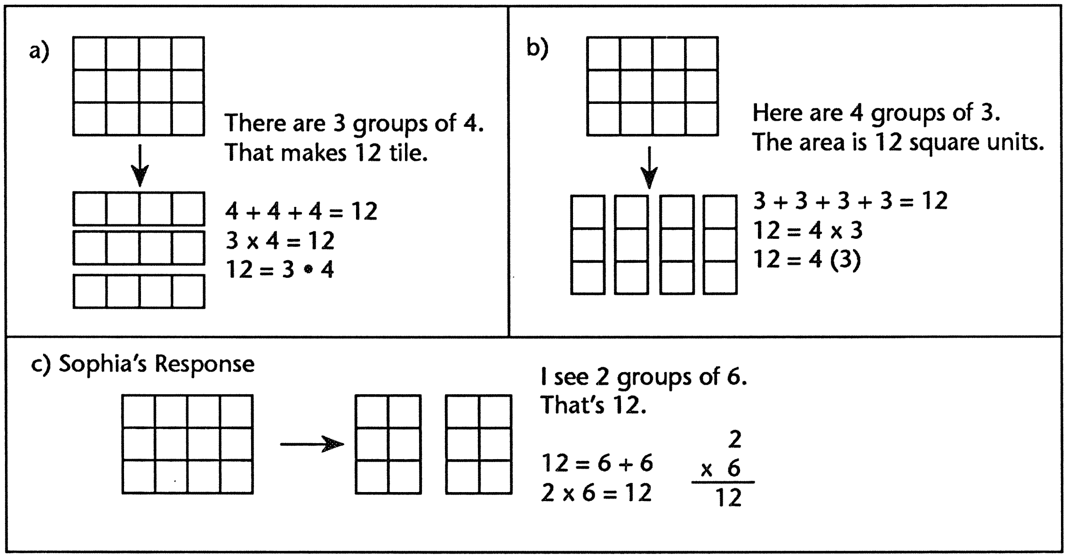
(3–5 days) This is an appropriate time to introduce the notation associated with multiplication and division. Rectangular arrays are helpful for this purpose since children can use them to attach visual meaning to this language.

Part I: Multiplication



Build a 3 by 4 array at the overhead and ask your children to do the same at their desks.

Have children suggest different ways of totaling the area of this array and write number statements that describe their thinking. Conduct a show-and-tell discussion of their methods.



a) There are 3 groups of 4. That makes 12 tile.

$4 + 4 + 4 = 12$
$3 \times 4 = 12$
$12 = 3 \bullet 4$

b) Here are 4 groups of 3. The area is 12 square units.

$3 + 3 + 3 + 3 = 12$
$12 = 4 \times 3$
$12 = 4\,(3)$

c) Sophia's Response

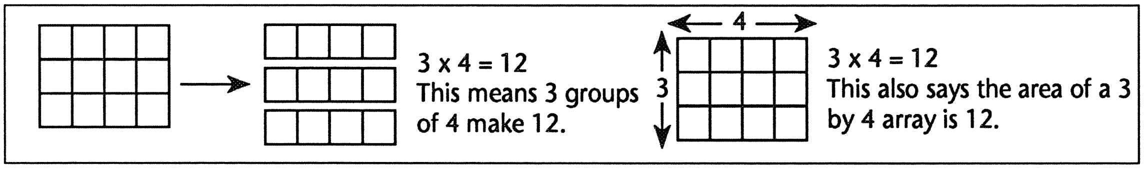
I see 2 groups of 6. That's 12.

$12 = 6 + 6$
$2 \times 6 = 12$

$$\begin{array}{r} 2 \\ \times\ 6 \\ \hline 12 \end{array}$$

Be sure multiplication statements are included in this discussion. Your children may suggest some of the ones listed above, perhaps because of their experiences with Today's Array. If not, bring them up yourself.

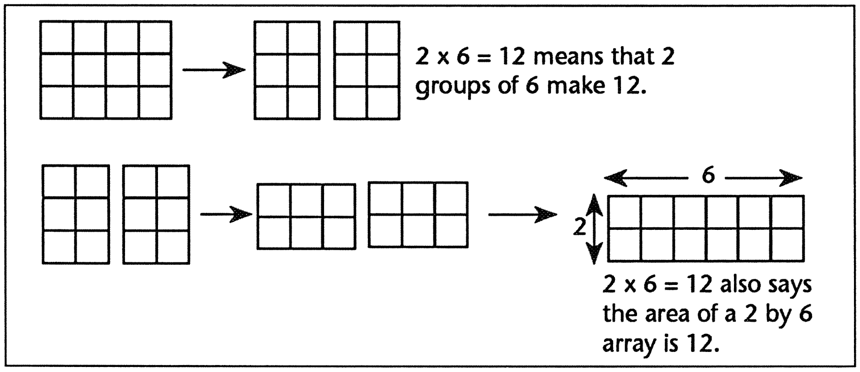
Use this as an opportunity to talk about the meaning of multiplication. Form the 3 by 4 array once more and write the statement 3 × 4 = 12 beside it. Tell the children this statement is another way of indicating that 3 groups of 4 make 12. It also reports the area of a 3 by 4 (or 4 by 3) array.



3 x 4 = 12
This means 3 groups of 4 make 12.

3 x 4 = 12
This also says the area of a 3 by 4 array is 12.

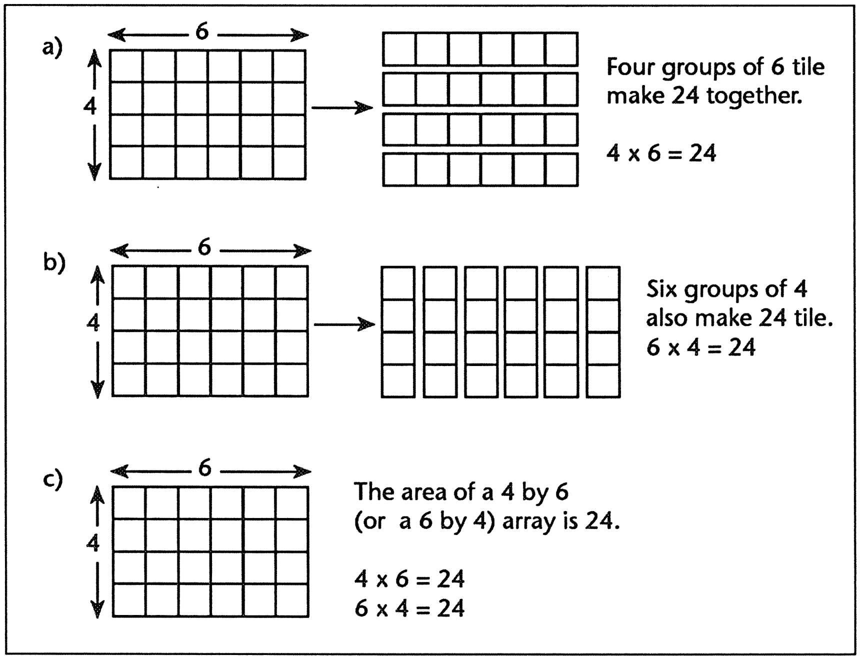
Note: Sophia's method, above, can also be described in terms of multiplication. She perceived the area of the array as 2 groups of 6. The statement 2 × 6 = 12 indicates that these 2 groups make 12. Also, as shown below, this statement also reports the area of 2 by 6 array.



2 x 6 = 12 means that 2 groups of 6 make 12.

2 x 6 = 12 also says the area of a 2 by 6 array is 12.

Conduct similar discussions about other arrays:



a) Four groups of 6 tile make 24 together.

4 x 6 = 24

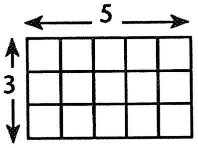
b) Six groups of 4 also make 24 tile.
6 x 4 = 24

c) The area of a 4 by 6 (or a 6 by 4) array is 24.

4 x 6 = 24
6 x 4 = 24

84 Using Numbers and Symbols (continued)

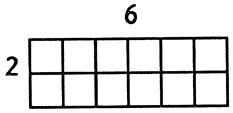
Review (or introduce) some of the vocabulary associated with multiplication. Products can be modeled by areas of rectangular arrays and factors by the corresponding dimensions of the arrays.



The area of this array is 15.

$$\underset{\text{factors (dimensions)}}{\underbrace{3 \times 5}} = \underset{\text{product (area)}}{15}$$

You may also wish to discuss different ways to indicate multiplication such as:



$2 \times 6 = 12$

$12 = 6(2)$

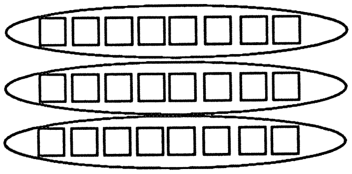
$2 \bullet 6 = 12$

$$\begin{array}{r} 6 \\ \times 2 \\ \hline 12 \end{array}$$

Part II: Division

Ask the children to show-and-tell how they would model the following story with tile, assuming that each tile represents an apple:

Dinah and her 2 friends picked 24 apples off her tree and shared them evenly. How many apples made up each child's share?



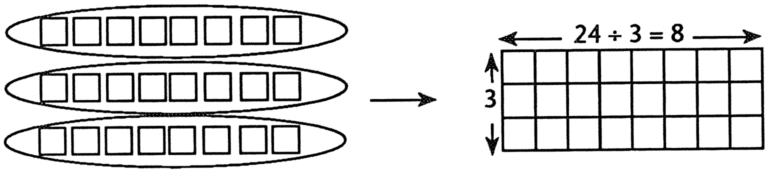
"The 24 tile can be separated evenly into 3 groups. Each group contains 8 tile. Each of the children received 8 apples."

Tell the children that the action of this story can be reported by the following division statements:

$$24 \div 3 = 8 \text{ or } \begin{array}{r} 8 \\ 3\overline{)24} \end{array}$$

That is, if 24 apples are divided evenly into 3 groups, each group will contain 8 apples.

Show how the 24 tile can also be arranged into an array that has a dimension of 3. As shown below, the other dimension of this array will be 8. The statement 24 ÷ 3 = 8 reports this other dimension.



$24 \div 3 = 8$

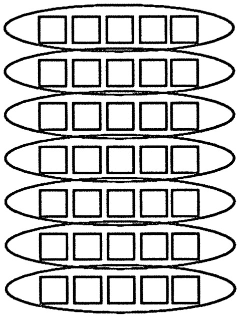
or

$$\begin{array}{r} 8 \\ 3\overline{)24} \end{array}$$

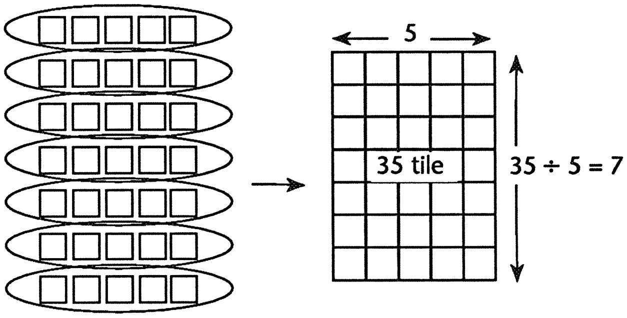
84 Using Numbers and Symbols (continued)

Repeat the above activity beginning with other division stories. Here is another example:

John put his baseball cards into piles of 5. He had 35 cards. How many piles did he make?



"Each tile represents a baseball card. The 35 tile can be placed in 7 groups, with each group having 5 cards."

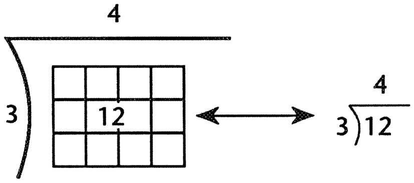


The 35 tile can be arranged into an array with one dimension 5.

7 is the other dimension.

$$35 \div 5 = 7 \text{ or } \begin{array}{r}7\\5\overline{)35}\end{array}$$

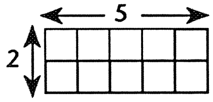
You might also wish to point out the similarities that exist between the sketch of a rectangle and the standard way of denoting division.



84 Using Numbers and Symbols (continued)

End this part of the lesson by having the children build an array and writing number statements that can be pictured in the array.

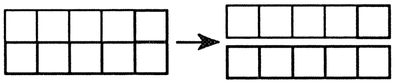
a)



The area is 10.
$2 \times 5 = 10$

$\begin{array}{r}5\\ \times 2\\ \hline 10\end{array}$

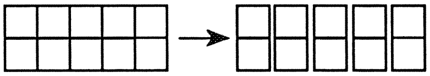
b)



There are 2 groups of 5.
$2 (5) = 10$
$10 \div 2 = 5$
$2 \bullet 5 = 10$

$\begin{array}{r}5\\ 2\overline{)10}\end{array}$ $\begin{array}{r}2\\ \times 5\\ \hline 10\end{array}$

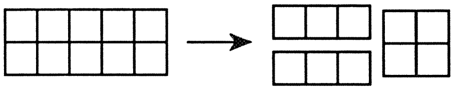
c)



There are 5 groups of 2.
$5 \times 2 = 10$
$10 \div 5 = 2$
$10 = 5 \bullet 2$

$\begin{array}{r}2\\ 5\overline{)10}\end{array}$ $\begin{array}{r}5\\ \times 2\\ \hline 10\end{array}$

d)



$2 (3) + 4 = 10$

Practice (Independent) Divide the class into small groups, giving each group some large Multiplication/Division Cards. Ask the groups to write multiplication or division stories that can be modeled by the arrays that appear on their cards. Have them also describe these stories with multiplication or division statements.

Conduct conversations with the groups about their work, perhaps using a checklist to note their progress.

Homework After a few days of practice, give each child a duplicated copy of an array of your choice. Ask the children to write number statements that can be pictured by looking at the array.

Share these statements after they are returned, perhaps creating a class chart. Did any children write the same statements? Did anyone come up with one that "gee-whizzed" everyone else?

Journal Writing Ask the children to write a brief paragraph about their work during this lesson.

Additional Ideas

85 Multiplication and Division Number Pattern Books

You Will Need

- Chapter 6, Multiplication and Division
- 12 × 18 art paper and general art supplies
- Blackline 25 (Centimeter Grid Paper)

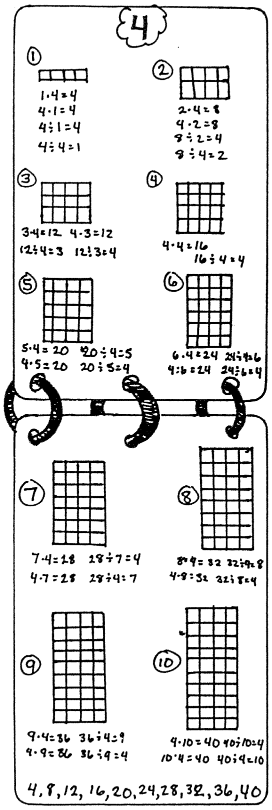
Your Lesson

(1–2 days) The object of this lesson is to create a reference book that shows the first ten multiples of the numbers from 2–10. The book relates the number patterns children have been learning throughout the year to multiplication and division.

Divide the class into teams of two to four children and assign each team the task of writing about a particular number pattern. Ask them to include arrays (cut from grid paper) that picture the multiples in their pattern. Have them also write multiplication and division statements that are modeled by the arrays. The illustration shows multiples of four.

Bind the pages into a big book that can be shared and enjoyed during the rest of the year. When finished, sing the number pattern songs to accompany each page.

This book provides a complete picture of "basic" multiplication and division facts. You might ask your children to imagine extending each set of multiples. For example, what would a 6 × 100 array look like? How many square units would it have? How about a 6 by 1000 array? or a 6 by 0?



Assessment

(Optional) Use a checklist to assess your children's ability to work together and to write number statements that are modeled by their arrays.

Additional Ideas

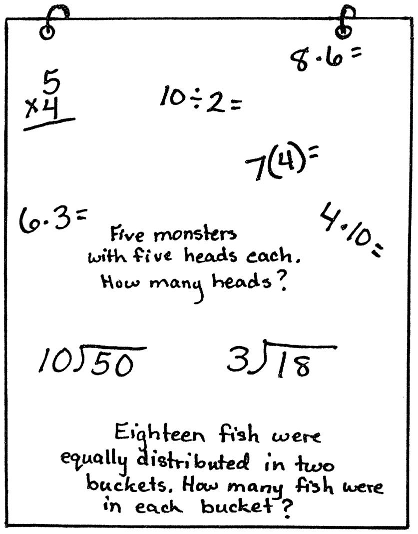
86 Teamwork—Practicing Multiplication and Division

You Will Need

- Chapter 6, Multiplication and Division
- base ten grid paper (Blacklines 8–13)
- square tile (base ten units)
- individual chalkboards, chalk and erasers
- colored squares for the overhead and base ten grid paper transparencies

Your Lesson

(2–3 days) List several multiplication or division problems (both stories and computations) on the chalkboard.

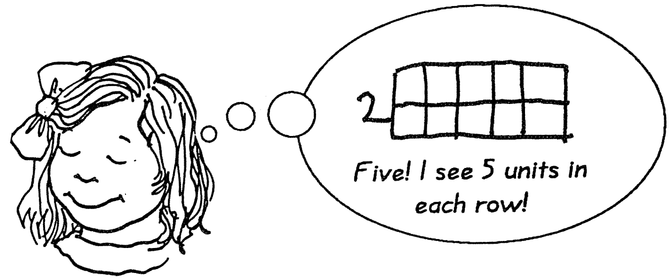


Ask each pair of children to solve these problems. One member of the team models and solves the problem with tile or pictures. The other team member writes number statements that describe the first person's thinking. Members alternate tasks after each problem.

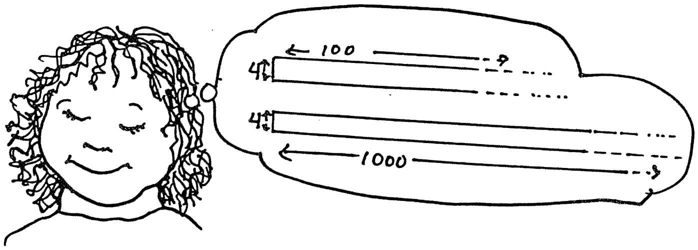
Allow time for a show-and-tell discussion about some of the problems.

You might conclude the lesson by asking your children to imagine different arrays.

TEACHER *Close your eyes. Try to imagine an array made with 10 tile. One of the factors is 2—can you see the other factor?*



TEACHER *What would a 4 by 100 array look like? Can you describe its dimensions and its area? How about a 4 by 1000 array?*



Journal Writing

Write a brief paragraph that describes what you are learning about rectangular arrays.

87 *Tell It All Story Books—Multiplication and Division*

You Will Need

- Chapter 6, Multiplication and Division

Your Lesson

(3–4 days) Have your children create a Tell It All book of multiplication and division problems. Insight Lesson 42 and the Materials Guide contain general directions for doing this.

As with your other Tell It All books, encourage children to check out this latest book for a bit of "down home" fun.

Additional Ideas

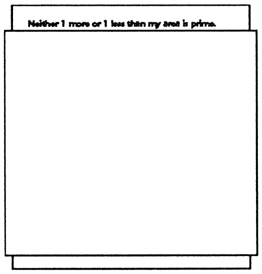
88 *Rectangular Array WhatZits*

You Will Need

- transparencies of Blacklines 111–112 (Rectangular Array WhatZits)
- transparencies of Blacklines 102–106 (Number WhatZits)

Your Lesson

Divide your class into groups of four. Show Clue 1 of the first WhatZit on Blackline 102 at the overhead.

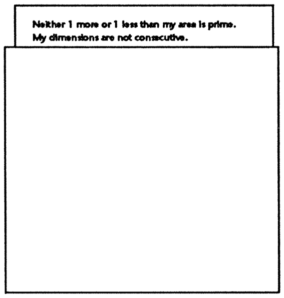


Neither 1 more or 1 less than my area is prime.

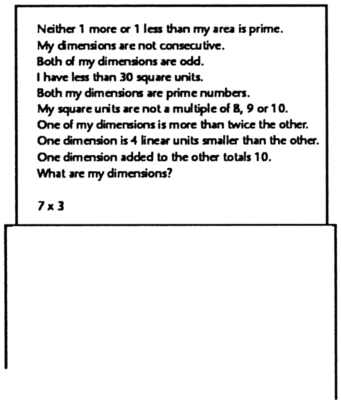
Ask your children to record numbers that might solve the puzzle.

Continue exposing one clue at a time, allowing time for each group to apply the new information and narrow the solution choices.

The last clue is the answer to the puzzle. When it is revealed, look back at previous clues and discuss them. Which were most helpful? Do they check out?



Neither 1 more or 1 less than my area is prime.
My dimensions are not consecutive.



Neither 1 more or 1 less than my area is prime.
My dimensions are not consecutive.
Both of my dimensions are odd.
I have less than 30 square units.
Both my dimensions are prime numbers.
My square units are not a multiple of 8, 9 or 10.
One of my dimensions is more than twice the other.
One dimension is 4 linear units smaller than the other.
One dimension added to the other totals 10.
What are my dimensions?

7 x 3

Repeat the activity using other WhatZits. When the supply runs out, ask your children to create some of their own.

Teacher Tips

We often like to begin the day with a WhatZit—just to get the juices flowing.

89 Problem Solving—Fitting Rectangular Arrays

You Will Need

- Chapter 6, Multiplication and Division
- Blackline 34 (Fitting Rectangular Arrays) and crayons for each child
- two number cubes: 0–5 and 5–10

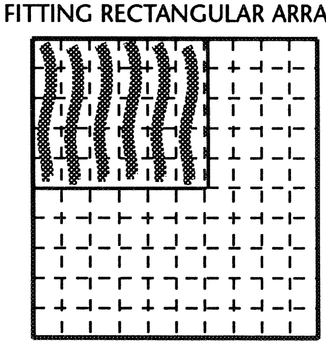
Your Lesson

(2–3 days) Fitting Rectangular Arrays is a game in which children seek to fill a 10 by 10 grid with smaller rectangular arrays. The smaller arrays are determined by tossing one or both of the 0–5 and 5–10 number cubes. There are two versions of the game.

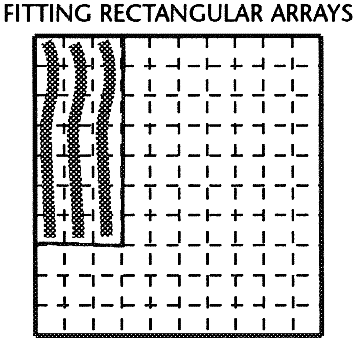
Version 1

Divide the class into two teams, pairing each child with a member of the opposing team.

Teams alternate turns, each time choosing to toss both number cubes once *or* one cube twice. The numbers that turn up determine the dimensions of an array to be shaded by each team member on a 10 by 10 grid (Blackline 34). The shaded area is checked by the partners from the opposing team.

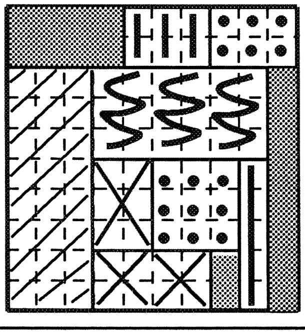


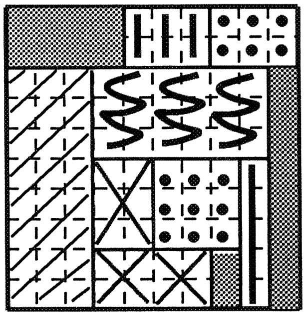
Team A tossed 5 and 6 on its first turn, so a 5 by 6 array is shaded.



Team B tossed 3 and 7 on its first turn, so a 3 by 7 array is shaded.

The object of the game is to completely shade in the 10 by 10 grid without covering any square twice. On any turn, children unable to fit an array completely within the grid lose the turn. If the arrays are shaded in different colors as they occur, a crazy-quilt effect is produced.





2 x 4 + 2(2 x 3) + 8 x 3 + 3 x 6 + 1 x 8 + 2 x 3 + 3 x 3 + 1 x 5 + 1 x 2 + 2 x 4 = 10 x 10

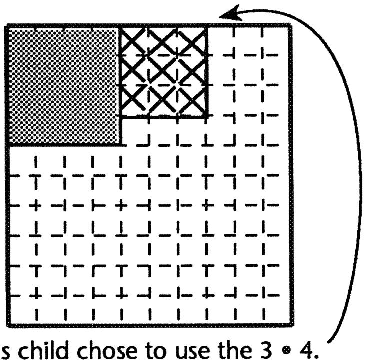
8 + 12 + 24 + 18 + 8 + 6 + 9 + 5 + 2 + 8 = 100

A team wins whenever one of its members is the first to completely fill their grid.

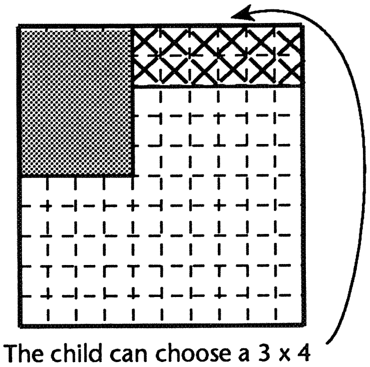
Have the children cut apart the arrays created to fill their 100 squares and write equations that report the total area of the arrays (see illustration to the left).

Version 2

This version has the same rules as above, with one exception: On any turn, the children may shade in the array having the dimensions that are rolled or any other array with the same area. For example, if a 3 and 4 are rolled, a 3 by 4 or a 2 by 6 can be shaded.



This child chose to use the 3 • 4.



The child can choose a 3 x 4 or a 2 by 6. A 2 by 6 was picked to fill this space.

Homework Ask your children to write directions for this game and then play it at home with family members. The required number cubes can be made from ¾" wooden cubes or constructed from tagboard using Blackline 3.

Journal Writing Have the children write a paragraph describing their reaction to this game. Which version was easier for them? Why?

Additional Ideas

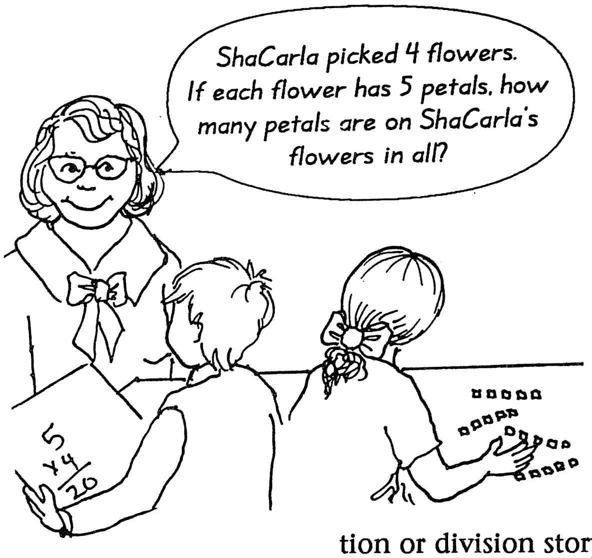
90 Mental Pictures—Back Me Up

You Will Need

- Chapter 6, Multiplication and Division
- square tile (base ten units) and base ten grid paper, as needed
- individual chalkboards, chalk and erasers
- several index cards

Your Lesson

(3–5 days) This lesson is similar to previous Back Me Up lessons (Insight Lessons 39 and 60). Its purpose is to encourage your children to explain to one another their thinking about multiplication or division problems.



Divide the class into pairs of children and tell a multiplication or division story. The "setter-uppers" model the story and its answer using tile or pictures. The "writer-downers" attempt to mentally picture the problem and use numbers and symbols to compute an answer. When team members are ready, they compare and discuss their work.

Continue in the same way with other stories, having the team members trade tasks each time.

At the end of the first or second day of this lesson, send each child home with several index cards. Ask them to write a multiplication or division story on each card, thereby providing a source of problems to use during the next few days.

Conduct the above activity using the problems the children have written on their index cards. Converse with the children about their work, perhaps using a checklist to help you assess their progress.

Emphasis

"Setter-uppers" and "writer-downers" should demonstrate their thinking to each other, showing how chalkboard statements can be modeled with pieces and vice-versa. In this way, children learn to respect the thinking of others and to view problems in several ways.

This activity promotes the formation of mental images for performing multiplication and division. The discussion that occurs is intended to relate these images to the use of tile or diagrams.

Journal Writing

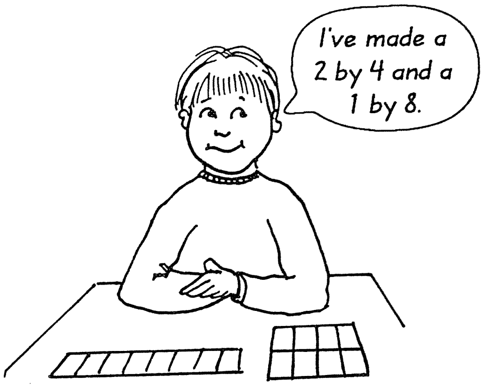
Have children complete the following sentence: Today I was pleased that I ___________.

91 Capture the Array

You Will Need

- Chapter 6, Multiplication and Division
- square tile and copies of Blackline 36 (Capture the Array) for each child
- base ten grid paper as needed

Your Lesson



(1–3 days) Your class is employed by the CIA to capture pesty Arrays who have been giving our agents fits! This is a tough assignment because some of the Arrays (products) operate under several code names (factors). An Array is captured when all of its aliases have been identified and the code names are arranged in order from smallest to largest. Some Arrays are prime, while others are not.

TEACHER *Today we are working for the CIA to help capture some pesty Arrays. To do this, we need to discover their code names.*

CHILDREN *How do we find out the code names?*

TEACHER *We will do the first one together and go after the Array whose name is really Eight. Please build all the arrays that can be made with 8 tile.*

TEACHER *How many arrays were you able to form?*

DAVID *There are 2—a 1 by 8 and a 2 by 4.*

TEACHER *Thank you. Please record that information on your record sheet (Blackline 36).*

CAPTURE THE ARRAY

given name	secret name (dimensions of rectangles)	secret code (factors)	related facts*
8	1 by 8 2 by 4		

The code names are the factors of 8. What are those factors?

SUSHIE *The factors of 8 are 1, 8, 2 and 4. These are the dimensions of the arrays.*

TEACHER *Record the factors from smallest to largest and you'll have broken the code. Eight will be captured!*

91 Capture the Array (continued)

TEACHER *Is Eight a Prime Array?*

CHILDREN *Oh! We get it! Prime Array! No, Eight is not a Prime Array because more than one rectangle can be built with 8 tile. There are more than 2 factors.*

CAPTURE THE ARRAY

given name	secret name (dimensions of rectangles)	secret code (factors)	related facts*
8	1 by 8 2 by 4	1, 2, 4, 8	Composite Not prime Even Not odd

Another Array is chosen and a new chase begins. Children enjoy going after Arrays and ask to play this game over and over.

Teacher Tips

You may wish to give your children a challenge for the day. For example, post the name of an Array (such as 56) to be captured. It is likely that some children will begin to use grid paper, calculators and mental images to help them think about the arrays that are possible.

Additional Ideas

92 *Exploring Division With Remainders*

You Will Need

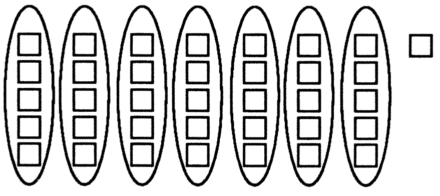
- Chapter 6, Multiplication and Division (please refer to Lesson 4 on pages 56 and 57)
- square tile and base ten grid paper
- colored squares for the overhead and markers

Your Lesson

(2–3 days) Set out 36 tile on the overhead and ask your children to do the same at their working spaces. Explain that the tile represent 36 golf balls that are to be packaged in boxes of 5. How many packages can be made?

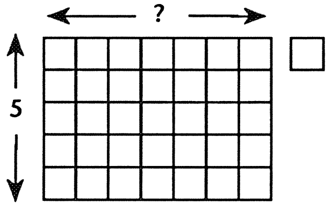
Ask the children to estimate the answer to the above question and then model the problem with their tile. Invite volunteers to share their models in a show-and-tell fashion. Two likely responses are shown here:

Terrell's Model



"I was able to make 7 groups of 5. There was 1 tile left over. Each group is like a package of balls, so 7 packages can be made. There aren't enough tile to make any more."

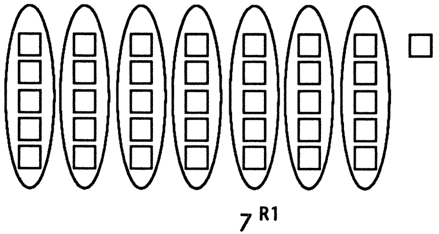
Alicia's Model



"I tried to arrange my 36 tile into an array. I made one dimension 5, since each package has 5 balls. I was able to make a 5 by 7 array, but I had 1 extra tile. So I can see that 7 packages of balls can be made."

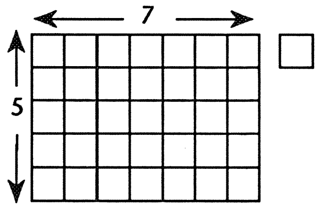
Both of the methods shown above demonstrate that 7 packages of balls can be made, with one ball left over. Here is one way to record this information:

Terrell's Model



$36 \div 5 = 7^{R1}$ $\quad \begin{array}{r} 7^{R1} \\ 5\overline{)36} \end{array}$

Alicia's Model

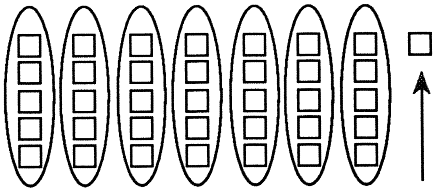


$36 \div 5 = 7^{R1}$ $\quad \begin{array}{r} 7^{R1} \\ 5\overline{)36} \end{array}$

92 Exploring Division With Remainders (continued)

A second way is depicted in the following illustrations:

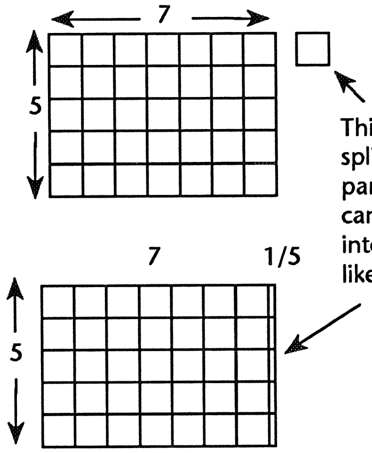
Terrell's Model



This tile forms $\frac{1}{5}$ of another package.

$36 \div 5 = 7\frac{1}{5}$ $\quad \begin{array}{r} 7\frac{1}{5} \\ 5\overline{)36} \end{array}$

Alicia's Model



This tile can be split into 5 equal parts. The parts can be placed into the array like this.

$36 \div 5 = 7\frac{1}{5}$ $\quad \begin{array}{r} 7\frac{1}{5} \\ 5\overline{)36} \end{array}$

Pose other division problems for the children to model with their tile (or with diagrams or sketches). Ask them to write division statements that describe the solutions, recording any remainders that occur as shown above.

Emphasis

In a division problem, it is common to carry out the computation to the point where the remainder is smaller than the divisor. This was demonstrated in the golf ball problem.

In Terrell's model, he formed the largest number of groups of 5 possible from 36 tiles and had 1 tile left over. The extra tile was not enough to make another group of 5.

Similarly, Alicia formed the largest rectangle possible, having one dimension of 5, from her 36 tile. Her remainder was also 1. The extra tile was not enough to form another column of 5.

When modeling a division problem with a rectangular array, the area (dividend) and one of the dimensions (divisor or factor) of the array are known. The challenge is to determine the other dimension (quotient).

Remainders may be reported as whole numbers or as a fractional part of the divisor. In the latter case, the divisor represents the "whole". This "whole" may be a group of objects (Terrell's model), or the number of tile in a column of an array (Alicia's method).

Practice

Reinforce your children's understanding of division and remainders by asking them to solve other division problems, such as:

Tabitha brought a package of chocolate chip cookies for the children on the soccer team to share after the Big Game. There were 66 cookies in the package and 17 children on the team. If each child ate the same number of cookies, how many did each eat? Were any cookies left over? If so, how could they be divided?

Challenge them to explore different ways to model the problems, including the use of arrays.

Homework

You might ask each child to write several division stories that reflect situations in their lives. Create an activity sheet for the class using these stories. This, in turn, can be completed as homework or used for independent practice.

Children also enjoy examining a number that has been cut from the newspaper. They might describe such things as an array that models the number, together with number statements that can be pictured by the array; factors of the number; an extended number pattern related to the given number.

Additional Ideas

93 *Remainders*

You Will Need

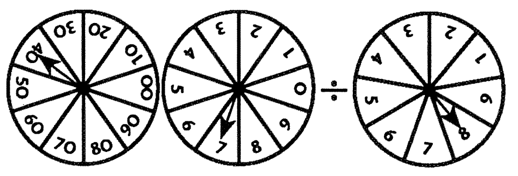
- Chapter 6, Multiplication and Division
- base ten grid paper
- transparency of Blackline 96 (More or Less spinner)
- transparency of Blackline 98 (Triple Spinner for Division)

Your Lesson

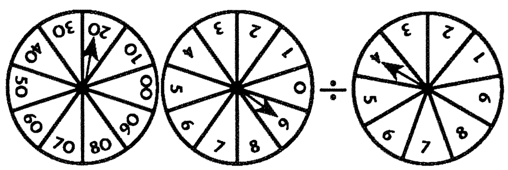
(2–3 days) The game of Remainders can be used to practice division skills. It is played as follows:

Divide the class into two teams and determine which team goes first.

In turn, each team creates a division problem by spinning the three spinners on Blackline 98. The spinner on the right determines the divisor in the problem. The two spinners on the left determine the 10s and 1s place of the dividend, respectively.

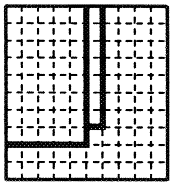


$47 \div 8 = ?$

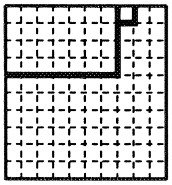


$29 \div 4 = ?$

Using grid paper, team members model their problem by trying to form an array that has a dimension equal to the divisor. Sometimes remainders will be encountered.



$47 \div 8 = 5^{R7}$



$$\begin{array}{r} 7^{R1} \\ 4\overline{)29} \\ \underline{28} \\ 1 \end{array}$$

The winner of each round will be the team with the larger (or smaller) remainder, as determined by the More or Less spinner.

Play as many rounds as time permits.

Additional Ideas

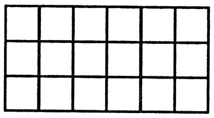
94 *Hidden Arrays*

You Will Need

- Chapter 6, Multiplication and Division
- square tile for each child
- colored squares for the overhead and markers
- base ten grid paper available

Your Lesson

(2–4 days) In the area model for multiplication, the product of two numbers is represented by the area of a rectangular array. When the numbers are large, it is often helpful to think about the array as being subdivided into smaller ones. The areas of the smaller arrays can then be summed to calculate the required product. In this lesson, your children are challenged to look for combinations of smaller (or "hidden") arrays that make up a larger one. Please refer to Lesson 2 of Chapter 6, pages 53–55, for more information about this.

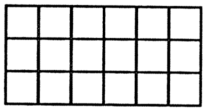


Ask your children to build a 3 by 6 array, while you do the same at the overhead.

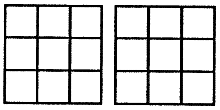
Conduct a show-and-tell discussion using the following questions:

How many tile are needed to build this array? How do they "see" the total of 18? Answers may be demonstrated by slightly separating the tile without completely distorting the 3 by 6 array.

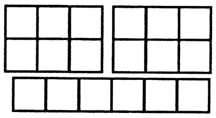
What hidden arrays can they find that might be used to find the area of the 3 by 6 array? Use numbers and symbols to describe each combination suggested.



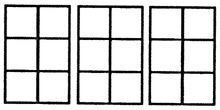
$3 \cdot 6 = 18$



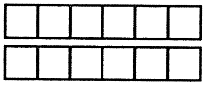
$3 \cdot 3 + 3 \cdot 3 = 9 + 9 = 18$



$2 \cdot 3 + 2 \cdot 3 + 1 \cdot 6 = 6 + 6 + 6 =18$

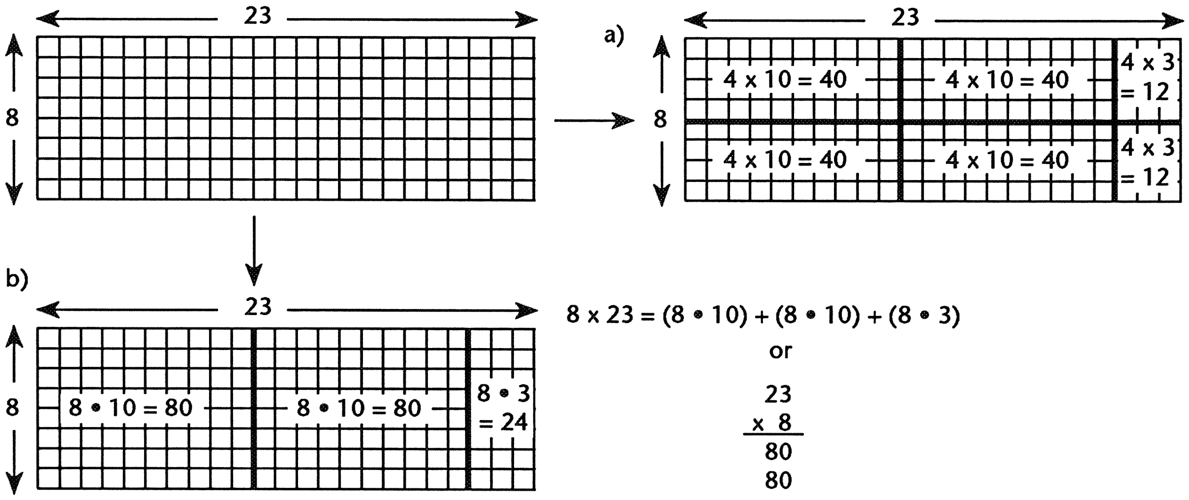


$2 \cdot 3 + 2 \cdot 3 + 2 \cdot 3 = 6 + 6 + 6 = 18$



$1 \cdot 6 + 1 \cdot 6 + 1 \cdot 6 = 6 + 6 + 6 = 18$

Repeat this activity with other rectangular arrays. Be sure to ask the children to sketch larger arrays, such as an 8 by 23. What hidden arrays can they picture?



$$\begin{array}{r} 23 \\ \times\ 8 \\ \hline 40 \\ 40 \\ 40 \\ 40 \\ 12 \\ +\ 12 \\ \hline 184 \end{array}$$

8 x 23 = (8 • 10) + (8 • 10) + (8 • 3)

or

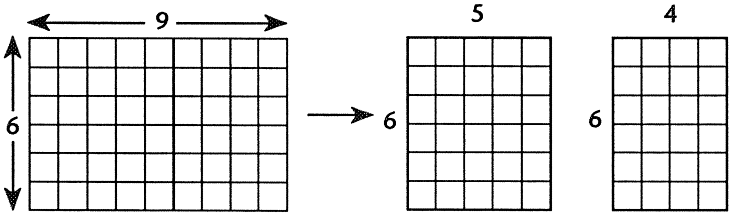
$$\begin{array}{r} 23 \\ \times\ 8 \\ \hline 80 \\ 80 \\ +\ 24 \\ \hline 184 \end{array}$$

Emphasis

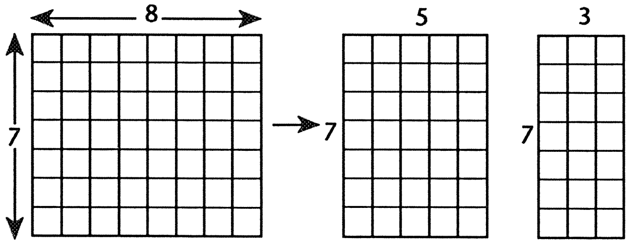
Sometimes it is helpful to calculate the area of an array by using the hidden arrays within it. This can be a very useful problem-solving procedure.

Many combinations of hidden arrays can be found within a given array.

Using hidden arrays to calculate a product illustrates the Distributive Property of Multiplication.



$$\begin{aligned} 6 \times 9 &= 6 \times 5 + 6 \times 4 \\ &= 30 + 24 \\ &= 54 \end{aligned}$$



$$\begin{aligned} 7 \times 8 &= 7 \times 5 + 7 \times 3 \\ &= 35 + 21 \\ &= 56 \end{aligned}$$

Teacher Tips

If your children can develop the ability to visualize hidden arrays when calculating products, they will be more likely to recall basic facts and apply their knowledge to problem situations.

Journal Writing

Ask your children to write a brief paragraph about their work with hidden arrays. What have they learned? What are their feelings? Do they feel challenged? excited? bored? confused? etc.

Additional Ideas

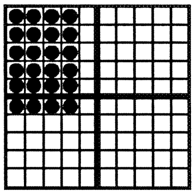
95 Hidden Arrays

You Will Need

- Chapter 6, Multiplication and Division
- a large, classroom set of Multiplication/Division Discussion Cards
- square tile and base ten grid paper

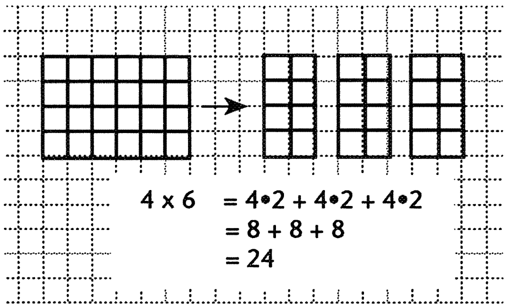
Your Lesson

(2–4 days) Continue exploring the Distributive Property of Multiplication by asking your children to look for arrays hidden within those found on discussion cards. (The cards that have areas between 18 and 45 are most useful for this lesson.)



Give each pair of children a discussion card and ask them to build with tile the array on their card.

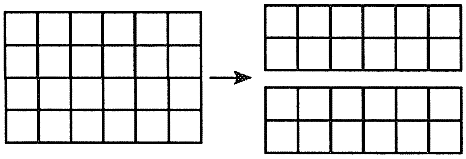
Ask the pairs to gently separate their array to show some of the hidden rectangles contained within it. Have them do this in a way that doesn't completely distort the original array. Ask them to record their work on grid paper.



$$\begin{aligned} 4 \times 6 &= 4 \cdot 2 + 4 \cdot 2 + 4 \cdot 2 \\ &= 8 + 8 + 8 \\ &= 24 \end{aligned}$$

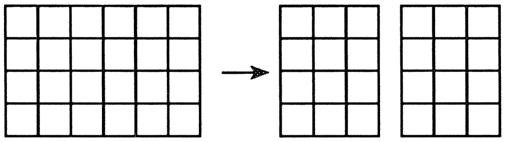
When they are ready, ask them to form the original array once more and look for other combinations of hidden rectangles. Conduct conversations with the teams about their work.

a)



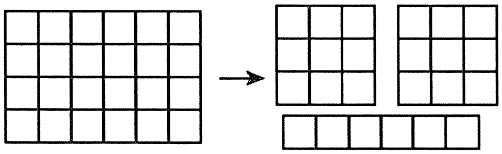
$$\begin{aligned} 4 \times 6 &= 2 \cdot 6 + 2 \cdot 6 \\ &= 12 + 12 \\ &= 24 \end{aligned}$$

b)



$$\begin{aligned} 4 \times 6 &= 4 \cdot 3 + 4 \cdot 3 \\ &= 12 + 12 \\ &= 24 \end{aligned}$$

c)



$$\begin{aligned} 4 \times 6 &= 3 \cdot 3 + 3 \cdot 3 + 1 \cdot 6 \\ &= 9 + 9 + 6 \\ &= 24 \end{aligned}$$

95 Hidden Arrays (continued)

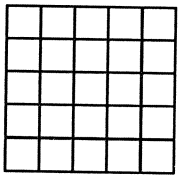
Continue this activity, each day giving the teams a different discussion card to use.

This lesson helps children develop mental images of basic facts and strengthen their number sense.

Homework

Ask children to work problems such as:

Look at the rectangular array shown below.



What are the factors? What is the area? Draw the array on grid paper and outline some hidden arrays found within the array. Write a number statement that tells about the hidden arrays you found.

Sports fans! A total of 94 points were scored by a team. This is how the points were scored: 21 points by 3-point shots; 56 points by 2-point shots; 17 points by free throws.

Draw a rectangular array that pictures the total number of points scored by 3-point shots. Do the same for the points scored by 2-point shots and by free-throws. Add the areas of these arrays to see if you obtain a total of 94.

What other combinations of shots would give a total of 94 points?

Additional Ideas

96 Hidden Arrays—Distributive Property Books

You Will Need

- Chapter 6, Multiplication and Division
- Blackline 25 (1-cm Grid Paper)
- 8½ × 11 art paper
- glue, markers, scissors, pencils and crayons

Your Lesson

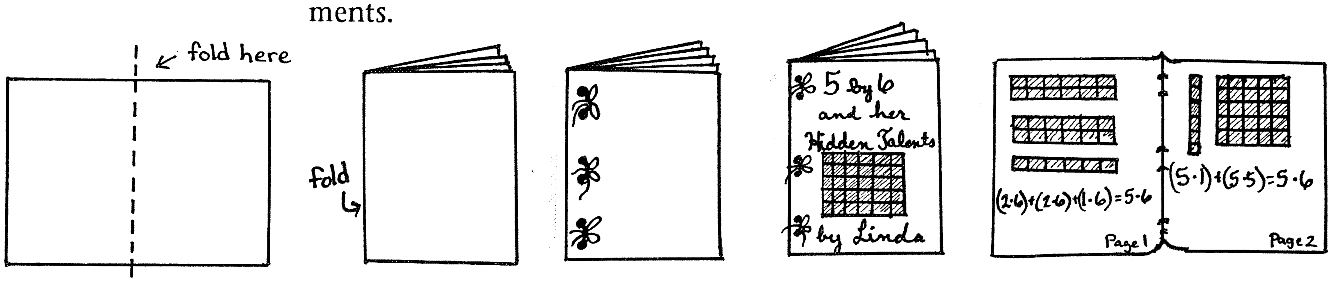
(1–2 days) Complete the exploration of hidden arrays by having your children create a book. Proceed as follows:

Ask each child to fold five sheets of art paper and then sandwich them to form a book.

Bind the books, perhaps by weaving yarn or pipe cleaners through holes punched near the folds.

Each child then chooses a rectangular array that is to be the "main character" of their book. That array is cut from grid paper, decorated and attached to the front of the book.

Each page of the book shows a different combination of hidden arrays that make up the main character and related number statements.



Keep the books at a location where the children can enjoy each other's work. In that way, they will see many illustrations of the Distributive Property.

Teacher Tips

We like to have our children write letters to each other, commending their work. We encourage them to include statements that tell what they learned about the main character and what they liked best about the book. At the end of the lesson, the children take their books home for their families to enjoy.

Additional Ideas

97 Mentally Picturing Arrays— Riddle, A-Riddle, Array

You Will Need

- Chapter 6, Multiplication and Division
- a large, classroom set of Multiplication/Division Cards

Version 1

- a set of individual discussion cards for every two to four children

Version 2

- individual chalkboards, chalk and erasers

Your Lesson

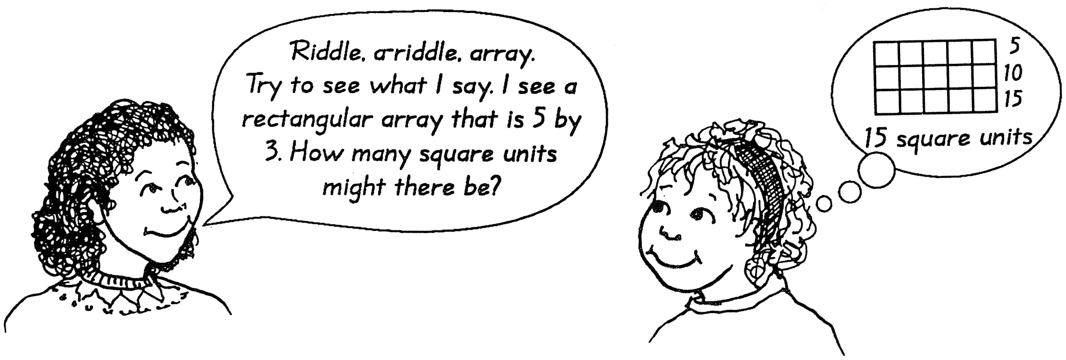
(1–2 days)

Version 1 (large group)

Do you remember as a child when your mother would play the riddle game, "Riddle, mariddle, marie. I see something you don't see and the color is..."? Riddle, A-Riddle, Array is a variation of that old childhood favorite, intended to encourage mental pictures of rectangular arrays.

Distribute the materials and play this game with your entire class.

TEACHER *(Looks at a discussion card that is out of the children's view.) Close your eyes and try to see a picture of what I see. Riddle, a-riddle, array, try to see what I say. I see a rectangular array that is a 5 by 3. How many square units might there be?*



Write your secret answer on your chalkboard. (pause) Okay, show me what you saw.

Conduct a show-and-tell discussion of the children's solutions.

The activity continues using other discussion cards, varying your descriptions. Be sure to include riddles such as, "I see a rectangular array with 20 square units and one factor of 10" to encourage "mental pictures" of division problems.

Version 2 (small group)

Each group of two to four children determines who will hold the cards and tell the riddles. Play then proceeds just as in Version 1.

Emphasis

Rectangular arrays can be pictured in several ways. Encourage your children to share the images that come to mind when solving these riddles. Were their mental pictures useful in thinking about the areas and dimensions associated with each riddle? Did they visualize any hidden arrays?

Homework

Hopefully, your children still have their individual discussion cards at home and have been playing games with them. Ask each child to choose several cards and write riddles about them to bring to class. If your children are into word processing, have each child enter their riddles into the computer. Print copies of a few to provide some fun homework assignments. Otherwise, they can write neatly on a blackline copy that can be duplicated.

Additional Ideas

98 *Number Riddles*

You Will Need

Day 1
- a classroom set of Multiplication/Division Cards
- small sets of discussion cards for each group of four

Day 2
- individual sets of multiplication/division discussion cards
- individual chalkboards, chalk and erasers

Day 3
- art supplies
- two metal rings for binding
- 5½ × 8½ sheets of paper
- a sheet of construction paper, cut in half, for a book cover
- tape
- 2 × 4 pieces of heavy paper to serve as lift-up flaps

Your Lesson

(3 days)

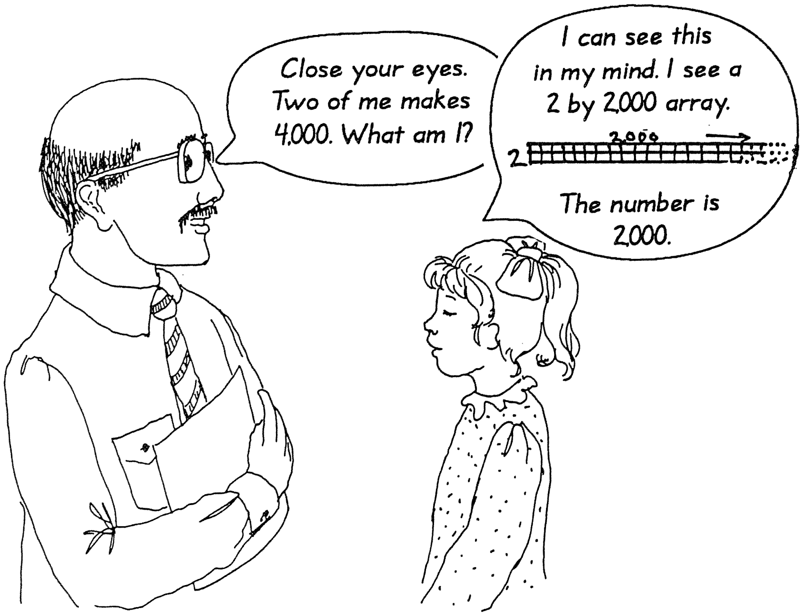
Day 1

All children love riddles and number riddles can be especially fun. Working in groups of four, challenge your children to find the discussion cards that solve the multiplication or division riddles you tell.

TEACHER *(Looks at the 3 by 4 rectangular array discussion card and hides it from the children.) Three of me makes 12. What number am I?*

COREY *Four; 3 fours makes 12. Here it is—a 3 by 4 rectangular array.*

Continue posing other riddles, eventually including ones that involve larger numbers.



98 Number Riddles (continued)

Before long, the children may want to be the riddle-tellers.

Day 2

Write this riddle—2 of what number makes 18?—on the overhead and ask your children to find the discussion card that shows an answer .

Conduct a show-and-tell discussion using this question: How can this riddle be posed to a friend using only numbers or symbols?

Some possible responses: $2 \times ? = 18$; $\square + \square = 18$; $2 \bullet ? = 18$; $2n = 18$. (Should none of your children suggest it, you may wish to discuss the use of a variable.)

Explore other multiplication or division riddles in the same way. Some examples:

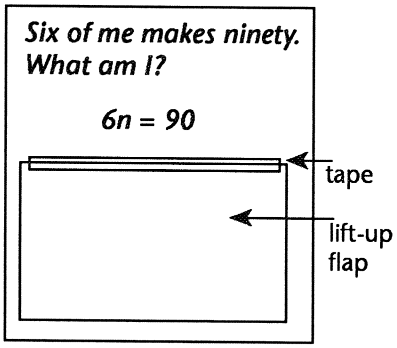
Four 8s total to this number. What is the number? ($4 \bullet 8 = n$)

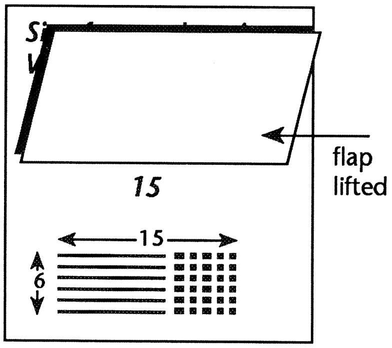
My area is 56 square units and one of my dimensions is 8. What is my other dimension? ($^{56}/_{8} = n$ or $8n = 56$)

Nine of what number makes 72? ($9n = 72$ or $^{72}/_{9} = n$)

Day 3

Challenge pairs to write one or two multiplication or division riddles for a class book. Ask the pairs to describe the riddles with words and with symbols. The solution to the riddle should be hidden under a lift-up flap and, when possible, accompanied by a model.





Read the riddle book to your children. Enjoy the solving the riddles together.

Emphasis

Riddles can be posed with numbers and symbols in several ways. Some of these ways may use a variable such as "n" to represent unknown numbers.

This lesson is intended to strengthen children's ability to form mental images of multiplication and division concepts. For example, children might visualize an array being subdivided into smaller arrays as they think about a product.

Homework

Encourage your children to check out the riddle book to share with family members at home. Perhaps family members can pose additional riddles to be shared with the class.

Assessment

You might ask children to solve a sample of multiplication and division problems. We recommend you don't time children on these problems. Conduct individual conversations with your children. Ask them to describe their thinking when solving a multiplication or division problem.

A review of Math Journals might also be helpful at this time. How are your children feeling about their work?

Additional Ideas

99 Building Rectangles

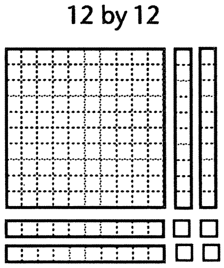
You Will Need

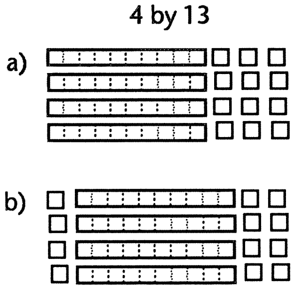
- Chapter 6, Multiplication and Division
- base ten counting pieces and grid paper for each child

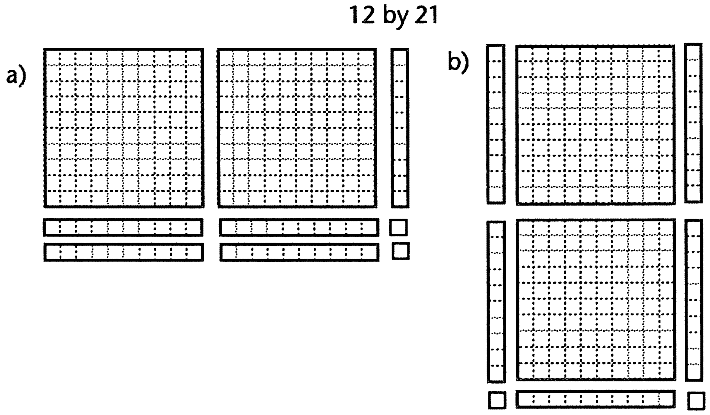
Your Lesson

(5 days) This lesson focuses on building rectangles with base ten counting pieces and observing number relationships modeled by each rectangle.

Distribute the pieces and ask each child to build a rectangle of their choice.



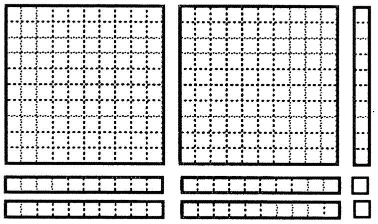




Have the children write a paragraph describing their rectangles. Ask them to include a picture of the rectangle (on base ten grid paper), the pieces used in making the rectangle, and the area and dimensions of the rectangle. As time permits, have them share some of their descriptions in a show-and-tell manner.

99 Building Rectangles (continued)

After a few days, extend the above activity by asking the children to write number/symbol statements that are modeled by their rectangles. Have them accompany each statement with a picture that shows their thinking.


12 by 21

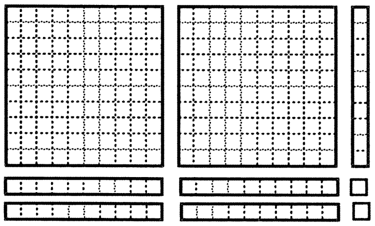
$$\begin{array}{r} 12 \\ \underline{\times 21} \\ 100 \\ 100 \\ 20 \\ 20 \\ 10 \\ \underline{+\,2} \\ 252 \end{array}$$

$$12 \times 21 = 200 + 50 + 2 = 252$$

$$\begin{aligned} 12 \times 21 &= (12 \times 10) + (12 \times 10) + (12 \times 1) \\ &= 120 + 120 + 12 \\ &= 252 \end{aligned}$$

$$\begin{array}{r} \left.\begin{array}{r} 1 \\ 10 \\ 10 \end{array}\right\rangle 21 \\ 12\overline{)252} \\ \underline{120} \\ 132 \\ \underline{120} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

Have the children complete the activity by writing multiplication or division stories that can be modeled by their rectangles.


12 by 21

Jose's Mom was buying 12-packs of soda pop for a picnic. She bought 21 packs. How many cans of pop did she buy?

Children enjoy creating the many rectangles that can be built with base ten pieces.

Additional Ideas

100 Multiplication and Division— Story Problems

You Will Need

- Chapter 6, Multiplication and Division
- base ten grid paper and counting pieces, as needed
- calculators

Your Lesson

(3 days) Ask your children to model the following story with base ten pieces or with a sketch and to share their methods with the class.

"Ethyl found a deal on soda pop! She found that 12-packs were on sale for a low, low price of $2.40 each. She decided to stockpile and buy eleven 12-packs. How many individual cans are in her stash?

Continue in the same way with other problems. Here are a few extensions of the above situation to try:

Lucky Ethyl lived in Oregon where she paid no sales tax. The price you see is the price you pay! How much did Ethyl pay for her soda pop?"

Ethyl wanted to know exactly how much each can cost. What is the cost per can?

If Ethyl were to purchase each can in a vending machine for 60¢ apiece, how much money would she need? How does this amount compare to what is needed to buy the 12-packs?

Add to the list with problems interesting to your children. In fact, invite the children to suggest some of their own. They may wish to compose a book of problems.

Additional Ideas

101 *Averaging*

You Will Need

- Chapter 10, Data Analysis and Graphing (pages 83–85)
- colored squares for the overhead
- base ten units for each child

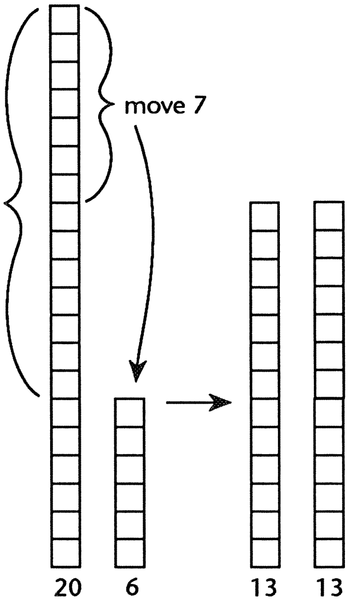
Your Lesson

(2 days) The following relay activity is a fun way to introduce the process of averaging a set of numbers. Choose a task of interest to your children. (We've used such things as carrying a golf ball in a spoon for a specified distance, walking with an eraser balanced on one's head, and writing by 10s to 100 on the chalkboard.)

Begin by lining up children into two obviously unequal teams, such as 6 children on Team A and 20 on Team B. Announce that each child will get only one turn and the winner of the relay will be the team whose players complete the required task first.

Of course, this will lead to complaints from the larger team. How can the two teams be made even? What suggestions do the children have? You might, for example, seat everyone and assign players one at a time to each team or take half the extra players on the larger team and assign them to the smaller one.

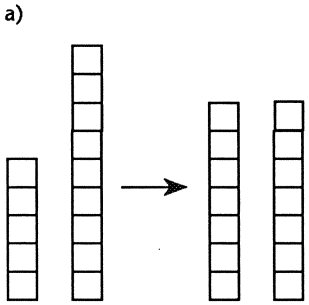
Test the suggestions and make the teams even. Have fun running the relay. Afterwards, model the process of making the teams even using columns of tile (base ten units).



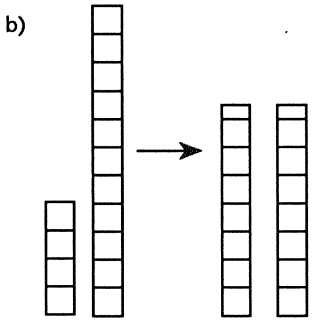
Tell the class that 13 is the "average" of 6 and 20. This is modeled by the height of the evened-off columns.

101 Averaging (continued)

Ask the class to determine the average of 5 and 9 by forming two columns of tile and evening them off (see a). Discuss their methods for doing this.



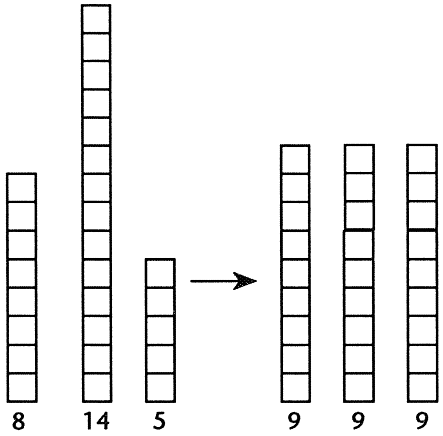
The average of 5 and 9 is 7.



The average of 4 and 11 is $7\frac{1}{2}$.

Have them find the average of other pairs of numbers in the same way (see b above).

As time permits, ask the children to find the average of three numbers by evening off columns of tile.



The average of 8, 14 and 5 is 9.

Emphasis

Finding the average of two or more numbers can be modeled by evening off columns of tile. The "evening off" process (called *averaging*) can be accomplished in several ways. The same process occurs often in life (e.g., miles per gallon of gasoline, baseball averages, etc.).

Additional Ideas

102 *Averaging—Shoot the Basket*

You Will Need

- Chapter 10, Data Analysis and Graphing
- a sheet of scrap paper and base ten units for each child
- colored squares for the overhead

Your Lesson

(1–2 days) On the average, how many times would your children "hit the bull's eye" if they threw paper wads at the waste basket? To find out, conduct the following experiment.

Divide your class into two teams and conduct 5 rounds of shots. In each round, every child is given an opportunity to make a basket. At the end of each round, ask the children to represent their successes with a column of tile (base ten units) and to discuss the average number of successes at that point. Then, after 5 rounds have been completed, have them determine a final average by evening off the 5 columns of tile for each team. Please see Chapter 10 of the Teaching Reference Manual (Lesson 3, pages 84 and 85) for the results of one such experiment.

Children might also discuss such things as where to place the basket or where children should stand. Should children shoot one-at-a-time or all at once? (This latter idea might help a shy person.)

What factors might effect the probability of successfully making a basket?

Emphasis

The average number of successes can be determined by evening off the corresponding columns of tile. The number of columns remains the same as this is done. The average height of the columns always falls between the highest and lowest heights of the original columns.

Additional Ideas

Index

Index Key: TRM – Teaching Reference Manual; CL – Contact Lesson(s); IL – Insight Lesson(s); Les – Lesson(s)

Index (continued)