OPENING EYES TO MATHEMATICS

TEACHING REFERENCE MANUAL

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Teaching Reference Manual

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Dedication

Michael John Arcidiacono

1941-1995

Opening Eyes to Mathematics is dedicated to Michael John Arcidiacono, a dear friend and inspiring teacher. His knowledge and style opened the eyes of all he touched to the joy of learning. Though he will be dearly missed, his work will live on as it enriches the lives of children and teachers, today and tomorrow.

This book is also dedicated to children—from whom we continue to learn—and to teachers — our source of hope for tomorrow.

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Vision

This program is dedicated to "opening the eyes" of children to the beauty and excitement of mathematics. We want them to discover mathematical concepts and relationships and learn to picture these concepts in their minds. We also hope they will recognize the connections that exist between mathematics and their world.

Our experience has convinced us that children can learn mathematics in a meaningful, enjoyable way and become confident in analyzing and solving problems. This happens when they can explore the subject through their own eyes, use models and manipulatives to develop mathematical understandings and try out various approaches when solving a problem. In addition, it is important that they share solutions and explain their thinking, and that they know their ideas will be valued.

Opening Eyes to Mathematics incorporates this approach to teaching mathematics. The curriculum it describes is consistent with the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*. The lessons are based on experiences familiar to children and are set in a problem-solving context. In these lessons, children are actively involved in the learning process. They are invited to take risks, to trust their own instincts and reasoning, to develop mental images for thinking about mathematics, and to communicate with one another. Instead of associating mathematics with formulas that are to be memorized and applied in a meaningless manner, they will find mathematics a lively, accessible and creative activity.

The NCTM *Standards* contain recommendations for change in the mathematics education of children. They emphasize problem solving, communication, making connections and using technology. In grades K–4, they recommend that instruction in mathematics focus on concepts, involve children in doing mathematics, and include explorations of content areas such as geometry and probability. They also call for increased attention to items like number and operation sense, use of manipulatives and calculators, writing about mathematics and justifying thinking. Less emphasis is placed on complex paper-and-pencil computation, rote memorization of rules and teacher-directed instruction. *Opening Eyes to Mathematics* shows how to implement these recommendations in your classroom.

This program has been developed with the hope of adding to your excitement about teaching mathematics. As teachers we have seen how enthusiastic children can be as they grow mathematically. We have become partners with our students, learning more about mathematics and finding more enjoyment in it. By sharing the ideas and strategies that have worked well for us, we hope to stimulate new, innovative dimensions in mathematics for you and your children.

Organization

In *Opening Eyes to Mathematics*, children use models, manipulatives and visual thinking to:

- Explore mathematics through their own eyes,
- Develop mathematical understandings,
- Construct mathematical relationships,
- Participate in mathematical discussions,
- Solve problems.

This program contains a complete curriculum that is consistent with the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics.*

Opening Eyes to Mathematics consists of three types of lessons:

- Calendar Extravaganza
- Insight Lessons
- Contact Lessons.

To assist you in teaching these, we have organized the materials for this program into four parts.

1. Teaching Reference Manual

The Teaching Reference Manual describes the philosophy of teaching and learning that embodies Op*ening Eyes to Mathematics.* It contains chapters that discuss the mathematical content, instructional goals and pedagogy related to the lessons of the program. These chapters also include sample lessons and dialogues that illustrate concept development, use of materials and ways that children might investigate and discuss mathematics.

The Reference Manual is bound separately to serve as a professional reference, answering those often asked "what, why, how, and when" questions. It contains a complete Materials Guide that lists needed materials and suggestions for purchasing, making and storing them.

The Calendar Extravaganza is also part of the Reference Manual. It consists of 20–30 minute explorations of several mathematics concepts. These explorations are related to each day's date and often connect mathematics to other subject areas. We urge you to use teacher choice in determining which components to emphasize each month. In that way you will create an Extravaganza that best meets the needs of your children. We are confident that you and your children will enjoy these daily activities.

2. Insight Lessons

Insight lessons (bound in two volumes with the Contact lessons) provide daily experiences with numeration topics and help children strengthen their sense of numbers. During Insight lessons, children learn about place value, decimals, addition, subtraction, multiplication, and division. They also develop their ability to make reasonable estimates, choose appropriate calculating options, and solve problems.

To assist you with planning, we have sequenced the Insight lessons and suggest the range of days it might take to complete each one. You may change or even skip some lessons; others you might decide to extend.

The Insight lessons are intended to be a road map that provides general directions for your children's journey through their study of numeration. We urge you to trust your professional judgement and intuition, adjusting the path of this journey so that it is appropriate for your children, taking detours or expressways as needed.

3. Contact Lessons

Contact lessons make contact with fractions, measurement, geometry, probability, data analysis and additional numeration activities. Many of these lessons are set in the context of monthly themes (such as football, apples and stamps) that link mathematics to the world of children.

Contact lessons are organized in a spiraling fashion so that, as children mature and grow in confidence, they explore concepts again and again throughout the year. There are 185 lessons that can typically be conducted in thirty minutes. As with Insight lessons, however, we encourage you to exercise teacher choice when using them. You may wish to conduct them as written or alter (or even replace) them to suit the needs and interests of your children.

Most Insight and Contact lessons are written in a brief style. Where appropriate, they contain examples, discussion topics and possible responses from children. They also include suggestions for homework, assessment and journal writing. As you prepare to teach the lessons, please refer to the corresponding chapters in the Reference Manual for general guidance.

In all three types of lessons, children use general mathematical processes such as problem solving, sorting, patterning and estimating in an ongoing manner.

4. Blackline Masters

The blacklines are numbered for easy reference and access; those needed for a lesson are listed. Transparencies may be made from Blackline Masters or purchased ready-made.

In the Insight and Contact lesson guides, we make suggestions for fitting the lessons of this program into your daily schedule. In general, we recommend that each day include a Calendar activity, thirty minutes on an Contact lesson, and thirty minutes on a Insight lesson. Since neither lesson is directly dependent upon the other, the Insight lesson does not need to immediately follow the Contact lesson in the daily schedule.

Our aim as we organized *Opening Eyes to Mathematics* was to provide meaningful sequencing in an easy-to-use format. We hope you will experience many happy days ahead!

1 Spirit

In many classrooms, learning is conceived of as a process in which students passively absorb information, storing it in easily retrievable fragments as a result of repeated practice and reinforcement. Research findings from psychology indicate that learning does not occur by passive absorption alone (Resnick, 1987). Furthermore, ideas are not isolated in memory but are organized and associated with the natural language that one uses and the situations one has encountered in the past. This constructive, active view of the learning process must be reflected in the way much of mathematics is taught. *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 10.*

Opening Eyes to Mathematics implements a constructive, active learning process for young children. By following this program, you will help your children develop a strong sense of number and number operations. Important branches of mathematics, such as geometry, probability, measurement and statistics, are included. Young people have an avid interest in exploring and applying these topics. In doing this, they also discover how mathematics is related to many other fields.

This program goes beyond presenting a set of daily lessons or a collection of teaching strategies. It embodies a philosophy of teaching and learning that recognizes mathematics as an enjoyable human endeavor and that emphasizes visual thinking as a means for understanding concepts. Let's look more deeply into this philosophy and how to make it work in your classroom.

A problem-solving approach

How do people develop interest in and become skillful at hobbies or pastimes in a way that the activities can become truly enjoyable? Whether one develops an interest in pastimes such as golf, racquetball, piano playing, bridge or reading, one initially pursues it by learning and adding skills as they go along. In fact, if people waited until they mastered the fundamentals needed to be skillful at a game like golf before actually playing a round, most would never play at all. And one often jumps into a new activity without first learning all its rules. Can you imagine someone trying to learn bridge by memorizing a manual of bidding and playing conventions before actually playing a hand? Most likely, the person would not only fail to learn the game, but also quickly lose interest.

Applying this viewpoint to learning mathematics suggests a learn-by-doing, problem-solving approach. Children actively explore situations that develop mathematical insights, even before they have the skills needed to analyze those situations in traditional ways. Building on familiar experiences, they begin to construct and explain their mathematical understandings using their own words as encouraged by the NCTM's curriculum standards.

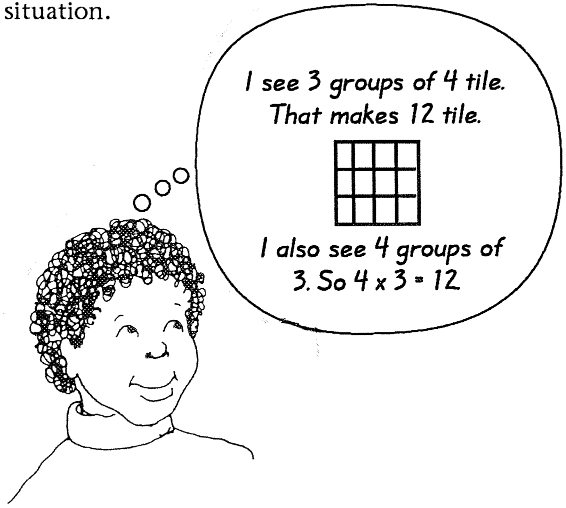
This problem-oriented context for learning mathematics promotes mathematical thinking. As children investigate problems, they gain experience with making and testing conjectures, deciding if an answer or procedure makes sense and justifying solutions. They hone their critical thinking skills and become confident, independent learners.

"A major goal of mathematics instruction is to help children develop the belief that they have the power to do mathematics and that they have control over their own success or failure. This autonomy develops as children gain confidence in their ability to reason and justify their thinking. It grows as children learn that mathematics is not simply memorizing rules and procedures but that mathematics makes sense, is logical, and is enjoyable." NCTM's Standards, p. 29.

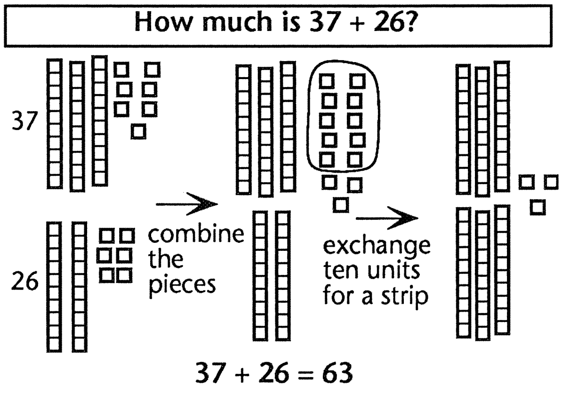
Learning mathematics in this way, children begin to see how mathematical ideas fit together and can be applied in problem situations. They start making connections between mathematics and other subjects.

Visual thinking

Opening Eyes to Mathematics draws upon children's ability to think visually: to use models, manipulatives and diagrams to analyze problems and to form useful mental images of mathematical concepts and processes. For example, by representing multiplication with a rectangular array of square tile, the underlying structure is revealed. After arranging tile in these rectangular arrays to solve multiplication problems, children can later "see" these arrays in their minds and use the visualization to think through a new problem situation.



In a similar way, place value concepts and number operations can be effectively modeled with base ten counting pieces.



Visualizing mathematical ideas helps children organize and retain information and provides them with a powerful tool for solving problems. Using this style of thinking, children will find a variety of ways to solve problems and represent concepts, ways that seem natural and sensible to them. (See the following sample lessons.) They eagerly share their thinking and discoveries. The resulting discussions broaden the students' view of the problem situation and help them see various strategies for solving it.

Classroom environment

The caring, loving atmosphere we all strive for in our classroom underlies the spirit of this program. By creating a child-centered environment that is non-threatening, a place where it is safe to risk, you will find your children more willingly and confidently participating in activities. They will share their ideas enthusiastically, knowing their ideas will be accepted warmly. They will look forward to each day's lesson. Learning mathematics will be exciting and fun.

Such a happy, nurturing environment encourages children to respect and appreciate one another. Many lessons in this program have children work together in groups. Ideally, each child will be a contributing member of the group, seeking help or offering assistance as needed. Decisions about problems and solutions are made cooperatively and ideas are shared in a positive manner.

Encourage your children to express their feelings about classroom mathematical activities and about their own work. As the teacher, you can acknowledge these feelings without passing judgment. As children find that you value their ideas and as they successfully work through problems, they more willingly try new approaches. They begin to see that mathematics can be user-friendly. Writing regularly in a journal enables children to reflect on their own learning process. Children will become clearer about how they learn, what most often causes them difficulties, what emotions and feelings come forward, and how insights into a problem may suddenly pop into their heads.

"Students will perform better and learn more in a caring environment in which they feel free to explore mathematical ideas, ask questions, discuss their ideas, and make mistakes. By listening to students' ideas and encouraging them to listen to one another, one can establish an atmosphere of mutual respect. Teachers can foster this willingness to share by helping students explore a variety of ideas in reaching solutions and verifying their own thinking. This approach instills in students an understanding of the value of independent learning and judgment and discourages them from relying on an outside authority to tell them whether they are right or wrong." NCTM's Standards, p. 69.

The role of the teacher

Your role in this program is critical as you facilitate the learning process in a number of specific ways. The following sample lessons illustrate several instructional practices we find effective. As you read, note the use of manipulatives, the interactions among children, the questioning strategies of the teacher, the different ways that children look at the mathematics, and the children's presentations at the overhead.

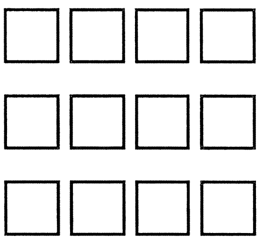
SAMPLE LESSON 1

(In this lesson, children use square tile to form rectangular arrays. These arrays are useful for modeling multiplication and division concepts.)

TEACHER *Larry's display shows 3 cars, each with 4 tires. I'd like you to represent these sets of tires with your tile. Imagine that each tile represents 1 tire. How many tires are there altogether?*

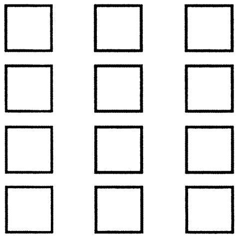
TEACHER *(After a short time has elapsed.) Would someone like to show their model at the overhead?*

ISIAH *(He comes to the overhead.) Sure! I see 3 groups of tile with 4 in each. Each tile represents a tire. There is a group for each car. There are 12 tires altogether.*



TEACHER *Thank you, Isiah. Does anyone have another arrangement to share at the overhead?*

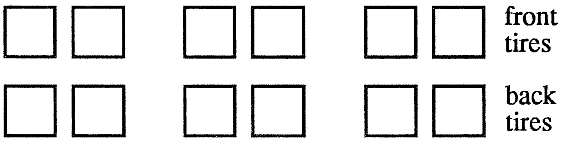
MONICA *I saw the same groups but I arranged them differently.*



I also see 4 groups of 3 in this arrangement—that's still 12 tires.

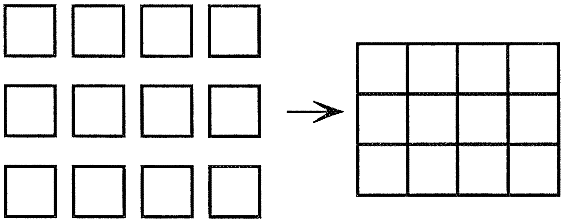
ISIAH *Mine and Monica's are alike—turn mine and you have hers!*

JOSH *I have another way. I placed the tile like this. I thought of the tires like they are on a car. See these are the front tires and these are the back ones.*



I still see 3 fours or 12 tires altogether.

TEACHER *That's nice, Josh. Now, I'm going to build Isiah's way again at the overhead and would like you to do the same at your desks. Look at what happens when we move the tile together like this.*

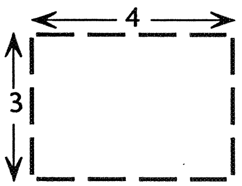


MORRIS *We get a rectangular array. It looks like one that was made when we counted by fours.*

TEACHER *Yes. It's also like the ones made in Today's Array. Can you still see the 3 groups of 4?*

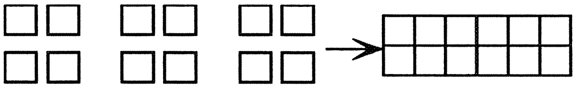
ALICE *Yes! And if I turn my head sideways, I see 4 groups of 3 as well. Its area is 12 square units.*

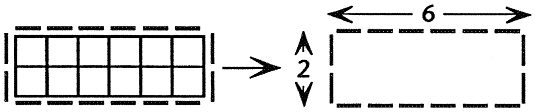
NATALIE *(She uses linear units to show the dimensions.) The dimensions are 3 and 4. It's a 3 by 4 array.*



TEACHER *We can also call it a 4 by 3 array.*

JOSH *Look, you can make a rectangular array out of my way, too! (He lays out his model again at the overhead.) I can push the tiles into a rectangle like this.*

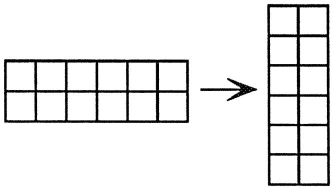




TEACHER *Yes, you have just made a 2 by 6 rectangular array with your 12 tile. What can be seen in Josh's array?*

JOSH *Well, I think of the top row as the 6 front tires and the bottom as the 6 back ones. I see 2 rows with 6 in each.*

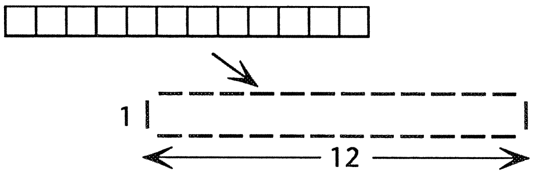
ANDY *Turn Josh's array like Monica did earlier. (He does so at the overhead.) Now I see 6 groups of 2.*



I guess that's like pairing a front and a back tire together in each group.

JUDY *Hey! I made another one! May I show? (She comes to the overhead.) I arranged my tile like this at first. I thought of each of these groups as a car with 4 tires.*

Then I pushed them together and I got a long, thin rectangle.



I guess this could be called a 1 by 12 array! That's like laying all the tires out in a line.

The children discuss further Judy's array and then model other multiplication and division stories with their tile.

SAMPLE LESSON 2

(Children work together to solve a multiplication problem. Strategies for estimating and then determining the answer are discussed.)

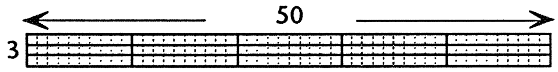
TEACHER *Our grade is going to have to raise enough money to go on a field trip at the end of the year. It's going to cost each child $3 dollars and there are 54 children. About how much money will we need to collect?*

The children work in groups for a short time.

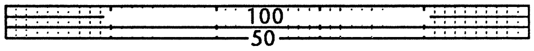
TEACHER *Could we please have some group reports?*

GROUP 1 *We decided that more than $150 are needed. There are about 50 children. Instead of paying the $3 all at once, each child could pay $1 at a time. Every time they do this, it gives $50 They'd do this three times. That gives 50 + 50 + 50 = 150.*

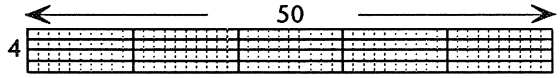
GROUP 2 *We thought of a 3 by 50 rectangle. There will be 3 groups of 5 strips. That's 3 times 5 or 15 strips. Each strip is like $10. Ten of them will be $100. Add in the other 5 tens and there will be $150. This will not be enough money since there are 54 children, but it will be close!*



There are 15 strips. Ten represent $100 and the other five $50. That's $150 altogether.



GROUP 3 *We estimated that about $200 would be collected. There are about 50 kids and if each pays about $4, that would give $200. We chose 4 because we could imagine 4 × 50 or 20 strips in a rectangle. Ten strips make 100, so 20 of them will make 200.*



20 strips or $200 in all

We aren't sure if that is more or less money than is needed. We used fewer kids but a larger price!

TEACHER *These are all interesting methods. Would we need to know exactly how much money is to be collected?*

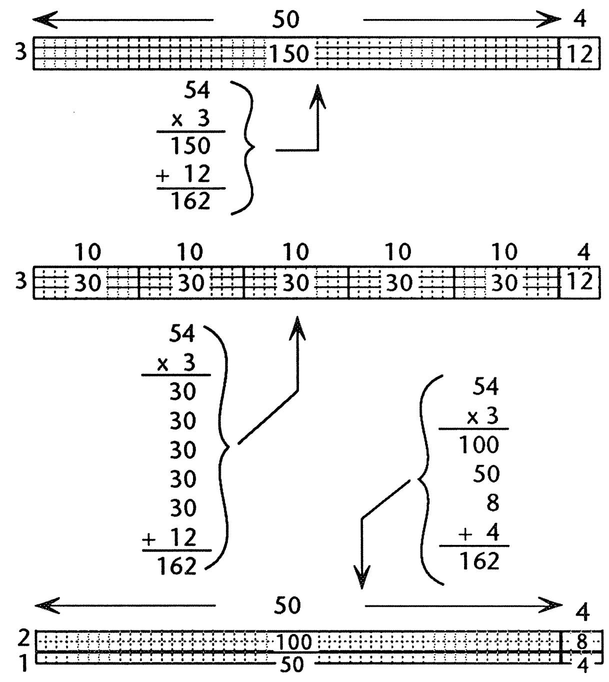
JUANITA *Well, I think so. That way you'd be sure each* *person will be able to go.*

MYRNA *Couldn't you just check each person's name off as they paid? That way, you'd know that everyone's paid, but you wouldn't need to count all the money.*

LING *I think the people who are getting the money would want to know the exact amount. That way they'd be sure they got the right amount.*

TEACHER *What is the exact amount? See if you can find some ways to calculate it.*

The lesson continues with the teacher encouraging the children to find as many ways as they can for answering the question. As time permits, these ways are shared and the exact answer is compared with the previous estimates. Some possible methods:



In these lessons, the teacher

- engaged children in the use of models and manipulatives to represent and think about mathematics concepts and procedures.
- used questioning strategies designed to elicit children's thinking and perceptions about mathematics. Some of the questions that might be used for this purpose are: What do you see? How does that work? Why is this so? Can you prove your observation? When have you seen this before? Where else might you observe this in our world? Can you find another way?
- used both small group and whole class activities.
- focused on the understanding of concepts.
- emphasized communication about mathematics.
- allowed children to make observations and share, in their own words, the mathematical principles involved.
- encouraged and acknowledged various views of concepts and procedures. Seeing there are many valid ways for thinking about mathematics, children can confidently use those that make the most sense to them.

"A variety of instructional methods should be used in classrooms in order to cultivate students' abilities to investigate, to make sense of, and to construct meanings from new situations; to make and provide arguments for conjectures; and to use a flexible set of strategies to solve problems from both within and outside mathematics. In addition to traditional teacher demonstrations and teacher-led discussions, greater opportunities should be provided for small-group work, individual explorations, peer instructions, and whole-class discussions in which the teacher serves as moderator. These alternative methods of instruction will require the teacher's role to shift from dispensing information to facilitating learning, from that of director to that of catalyst and coach." NCTM Standards, pp. 125 and 128.

Teaching in this way is exciting! As your children learn and enjoy mathematics, you will be learning and enjoying it, too. Grow right along with them! There may be times when you don't know an answer or when children will share unexpected views or procedures. These can be opportunities to learn with your class, to take some risks yourself, or to demonstrate that it is okay to ask for help. Let children see that adults don't have all the answers and help them develop a healthy view of learning as a lifelong process.

Presentations at the overhead and making assessments

Show-and-tell demonstrations by children at the overhead projector are an important part of this program. This is an effective way for children to share their thinking and to see a variety of approaches to a mathematical problem. Encourage students in a friendly fashion to share their ideas in this way. For children who are shy or reluctant, ask if they will allow you to share their approaches at the overhead. This lets them know you value their work and their feelings.

These presentations also provide a basis for assessing the children's thinking. In general, we recommend that assessment be an ongoing part of your instruction. For a detailed discussion of assessment techniques, please refer to Chapter 12 in this manual.

The role of the child

In *Opening Eyes to Mathematics*, children are engaged much more actively than in traditional programs. As illustrated in the sample lessons, they work cooperatively to investigate and model mathematical situations. At various times they are involved in the following processes:

- Visualizing, sketching, representing, imaging;
- Discussing, writing, listening, explaining;
- Analyzing, conjecturing, predicting;
- Reflecting, constructing meanings, making connections, inventing procedures;
- Generalizing, justifying, evaluating;
- Exploring possibilities, applying and extending knowledge.

As your children explore mathematics in this way, they will develop greater understandings and become more aware of the options they have for solving problems. They will discover relationships that help them think about mathematics in a broad, meaningful way, rather than as a collection of isolated skills.

Implementing the lessons

The lessons of this program are based on experiences familiar to children and are generally set in a problem-solving context. There are three types of lessons:

- Calendar Extravaganza activities that use each day's date to motivate brief explorations of mathematical concepts.
- Insight Lessons that focus on numeration topics and on development of number sense. In these lessons, children learn about place value, decimals, addition, subtraction, multiplication and division.
- Contact Lessons that make contact with fractions, measurement, geometry, probability, data analysis and additional numeration activities. Many of these lessons are set in the context of monthly themes (such as football, apples and stamps) that link mathematics to the world of children.

In all three types of lessons, children use general mathematical processes such as problem-solving, sorting, patterning and estimating in an ongoing manner.

These lessons are presented in a sequence that has been successful for us. However, you can best judge how to adapt them for your class. Some lessons you may wish to change or even skip; others you might want to extend or explore further. We encourage you to adjust the pace and nature of the lessons so they fit your class. Observing the work of your children, reading entries in their math journals, and relying on your teaching instincts can help here.

The lessons are designed to meet the needs of all children. They are based on the philosophy of teaching and learning described in this chapter. The general intent of each activity is described. Where appropriate, discussion questions, points to emphasize, possible student responses and suggested extensions are included for your guidance. You and your children are the ones who actually create each lesson, however! The lessons' flexibility and open-ended nature allow you to decide what problems and examples to explore, how much practice is needed, and how to organize the lesson. We urge you to personalize each activity.

The mathematical content, instructional goals, and pedagogy related to the lessons are described in this reference manual. As you read and prepare each lesson, please refer to the chapter(s) that correspond to it. These chapters also include sample lessons and dialogues that illustrate concept development, use of materials and ways that children might investigate and discuss mathematics.

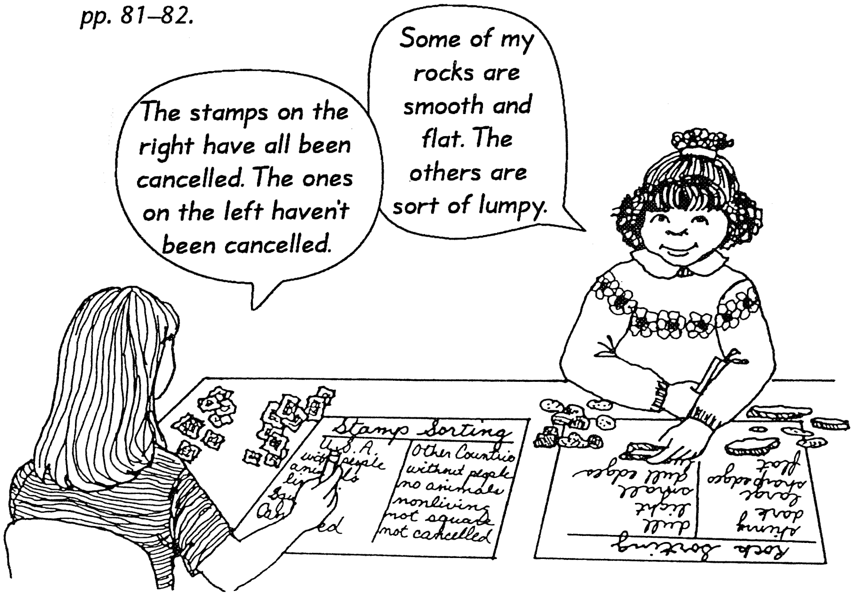
In the classroom examples discussed in this manual, the responses of children have been idealized. This has been done to indicate the richness of ideas that can be discussed and the various ways children think about mathematics. Undoubtedly, your children will come up with many interesting thoughts of their own. Also, feel free to include your ideas. If you wish to discuss a certain relationship or point, which hasn't been brought up by a child, you might do so by saying, "Here's another thing that I noticed" or "Here's another way of looking at this problem."

Conclusion

We are sure that you and your children will enjoy exploring mathematics in this active, process-oriented way. This is mathematics education for the future. Our society is changing so rapidly that children need to become independent thinkers to meet the challenges of the years to come. They are more likely to become such creative thinkers if they learn mathematics in a meaningful way, exploring many options for analyzing problems and developing solutions.

2 Sorting

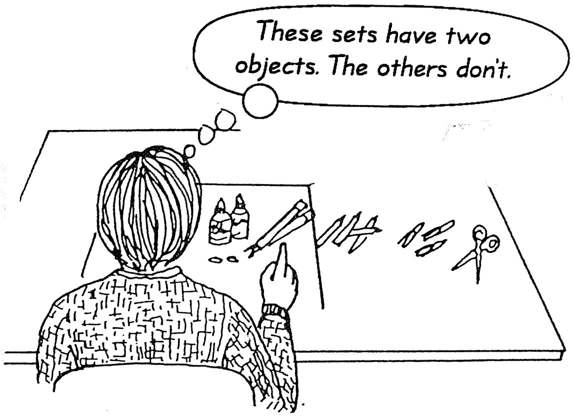
The seeds of logical thinking are planted as students learn to describe objects or processes accurately and to elaborate their properties, similarities, differences and relationships. Students should be encouraged to explain their thinking in their own words. *NCTM's Curriculum and Evaluations Standards for School Mathematics, pp. 81–82.*



People regularly classify information and objects. At work, cashiers sort coins, workers in a hardware store arrange nuts and bolts by size, and scientists typically sort experimental data for the purpose of forming hypotheses. In the home, clothes are separated prior to washing, silverware is grouped by type and addresses are filed alphabetically.

Sorting adds structure to a cluttered room, glove compartment, daily routine or problem-solving situation. It is also a process that is used in various disciplines of study, such as science, literature, music and language.

Young children have had many experiences with sorting objects both in school and out. For example, some may have arranged baseball card collections by teams or players. Others may have organized their toys or books in ways that make them easy to find. They have also learned about numbers and shapes by classifying sets of objects.



The sorting activities of this book add to these experiences and foster mathematical thinking. These activities strengthen children's ability to organize information, use logical reasoning, identify the attributes and properties of objects, and look at problems from many perspectives.

Description of the activities

Sorting is an underlying theme of mathematics and in this program becomes an integral part of many lessons. For example, when graphing information, it often helps to sort the data first. Sometimes, sorting is the main objective of a Contact lesson; two of which are described below. In the first lesson, the children sort information they have gathered while trying to deduce the contents of a mystery box. In the second, they identify sorting criteria that can be used to separate flags into two sets. In the illustration below, the teacher has drawn a tiger. With each question, one part of the tiger is erased.

Contact Lesson 1: Mystery Box Sorting

You will need

- Chapter 2, Sorting
- a mystery box that contains an apple
- chart paper
- markers

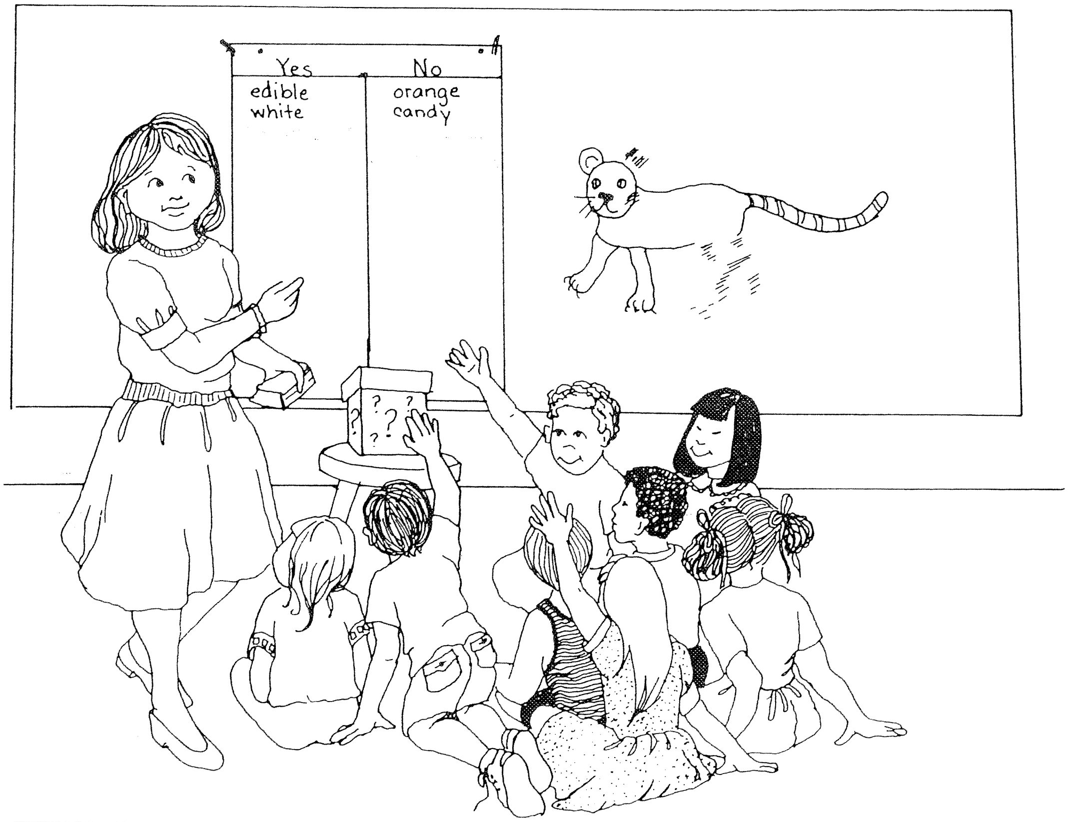
We like to introduce each Contact theme (except for Getting to Know You) by placing an object that represents the theme into a Mystery Box. The children then ask yes-or-no questions about the box's contents. The information they obtain is recorded on chart paper. Only a specified number of questions are permitted on any one day; if the quota is reached before the object is identified, the children must wait until the next day to try again. So, start this activity a few days before the other lessons related to the theme are scheduled.

The mystery item this time is an apple. The Apples theme begins with Lesson 11; we are sure it will provide you and your children with many appeel-ing mathematical experiences! After all, an apple a day keeps the doctor away and is certainly the way to a teacher's heart. This theme opens the hearts of children to a love of mathematics as well!

Let the questions begin! You might keep a tally by drawing a 12-to-15-part design on the board. As each question is answered, erase one of the parts. When all the parts are gone, the questioning stops for the day.

Sometimes children find it hard to ask questions about the attributes of the object. Instead they may try to guess the contents directly by asking, "Is it a picture?" "Is it a block?" They may require examples of helpful questions, such as, "Is it small?" or "Can it be found in school?" They may also need to be reminded that even "No" answers provide useful information.

One way to help children improve their questioning and reasoning skills is to periodically review the clues that have been gathered. Then pose new questions based on this review. Another way is to lower the daily quota for questions to only 9 or 10. You might note which questions are *guesses* of the contents and which refer to *attributes* of the object.



Teacher Tip: Dramatize the quest for the answer with some theatrics. We pretend the box contains the secret of the year and sing this ditty, "I know something you don't know, he-he-he-he-ha-ha! I know what is in this box, he-he-he-he-ha-ha! Bet you can't guess it! He-he-he-he-ha-ha!" Of course, the children are now determined to guess!

If the day's quota of questions is exhausted, we still keep the box and information chart on display throughout the day for reference purposes. This, along with the children's curiosity, will prompt continued discussions both at school and at home about what is in the box. (Sometimes, when the questioning has gone on for a number of days, parents even catch us in the grocery store and demand, "Just what *is* in that box?") On the next day, review the previous day's questions and draw a new figure to be erased bit by bit. Have fun! Do it just before lunch or whenever an extra five minutes appear in your schedule!

Continue as many days as needed. When the box is finally opened, you will find exploring mathematics with apples as easy as apple pie!

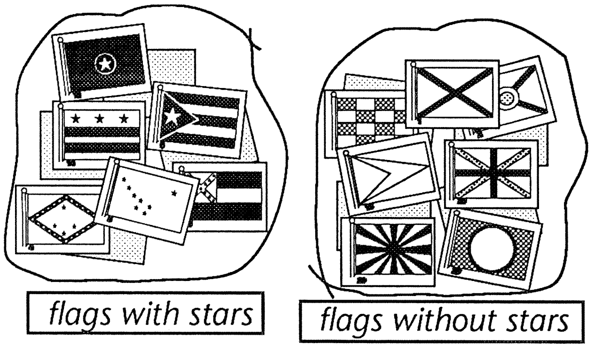
Contact Lesson 76
Flags Sorting (Venn diagrams)

You will need
- Chapter 2, Sorting

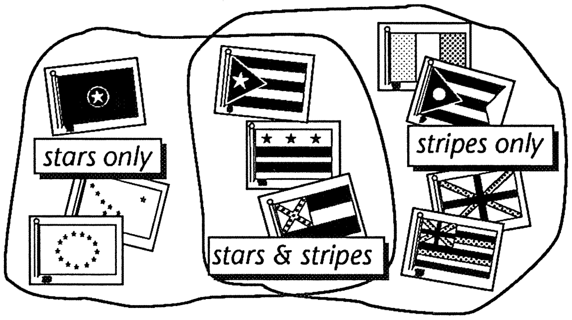
For every group of 4 children
- set of flags cut from Blacklines 71 and 72
- two loops of string

You may wish to begin this lesson by examining your state flag and discussing its history.

Give each group of four children a set of flags and two loops of string. Challenge each group to place its flags inside the loops in a way that separates the flags into two different sets. Have them give each set a title that describes how it was formed.



Have the groups share some of their sorting procedures. Some may have selected ones that require overlapping sets and these may need to be discussed further.



Teacher Tips: Vary this lesson by specifying the criteria for the children to use when sorting their flags. For example, ask them to devise a way to place the flags with stars in one loop and those with stripes in the other. In this case, the students will have to overlap the loops in order to represent both sets.

Another variation is to graph information about the flags. One possibility is to have each group randomly select twenty flags and make a graph that highlights some of their features. Each of the flags is marked on the graph using its number.

stars	30	5	6	4	11	3	10	14
circles	2	9	17	11	3	10	21	
rect-angles	25	30	15	14	6	3	10	
other	21	9	8	2	3	30	15	6

How many flags appeared in all 4 rows? In 3 of the rows? Does any row have twice as many flags as some other row? What fractional part of the 20 flags have rectangles? Is it likely that all groups will have the same graph?

Encourage children to incorporate fractions into their sorting lessons when appropriate. After the flags are sorted into groups, the fraction reflecting that division can be written on a chalkboard beside the flags.

In lessons like these, children learn to examine a situation from many points of view. They also learn to carefully examine and define the attributes of the objects being sorted. Sometimes they may use sorting criteria that cause disagreement. This can happen if they attempt to use attributes that are not clearly defined. "Pretty" is an example of such an attribute, since what is pretty to one child may not be pretty to another.

"Big" is another example of an ambiguous attribute that sometimes comes up when sorting shapes. It is important to discuss these should they occur and to encourage children to be precise.

Other kinds of sorting lessons

You may wish to vary the nature of your sorting lessons. Some variations were suggested in the Teacher Tip of Contact Lesson 76 above. Here are some additional activities that our children have found interesting.

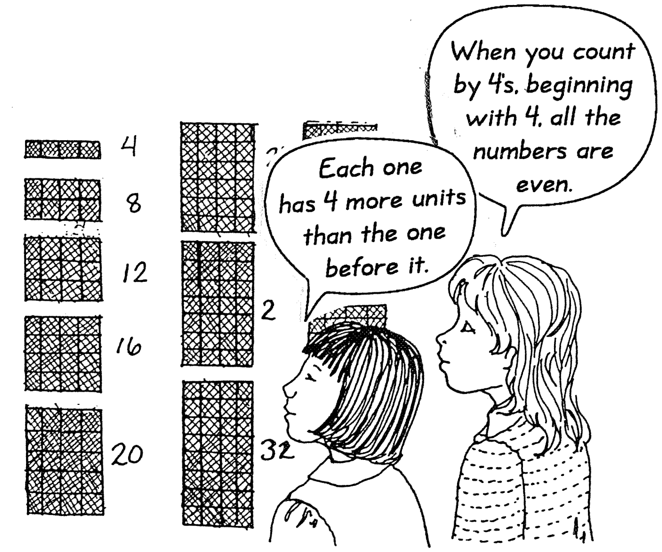
Conduct a sorting extravaganza during which the children are challenged to sort materials from a junk box in as many ways as they can in an allotted period of time. This always proves to be an activity that brings out their creativity.

Have a mystery sorting lesson! Give each group of four children in your class a collection of objects to be sorted. Have each group make a record of its sorting strategy and leave the sorted objects at its workstation for the others to see. Groups then circulate and try to identify the strategy used at each location. A lively discussion is sure to follow when guesses and master plans are compared!

Sort the children themselves! Think of a way to separate the class into two groups: brown eyes/not brown; first names that end in Y/those that don't, etc. Then begin assigning each child, one at a time, to the appropriate group and periodically give the class a chance to identify your sorting criteria. We like to do this in an entertaining manner using Silent Sam, a "critter" that is posted on the chalkboard as a signal that no one can speak. Then with exaggerated clicks of our fingers and charade-like mimes, we direct a few children to each group. After a while we invite the children to point to where they think a selected child should go. When Silent Sam is removed, the children can share their observations and attempt to identify how the groups have been formed. (For more on "Silent Sam", see Insight Lesson 1, Lessons, vol. I, p. 117.)

3 Patterns

Patterns are everywhere. Children who are encouraged to look for patterns and to express them mathematically begin to understand how mathematics applies to the world in which they live. *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 60.*



Many people enjoy mathematics because of the patterns they find in it. For them, the richness and beauty of the subject are reflected in the regularities observed in numbers and shapes. Mathematics becomes powerful for those who make a habit of searching for underlying patterns. We can help young people share in this appreciation of mathematics and grow in their mastery of the subject by designing classroom activities that focus on patterns.

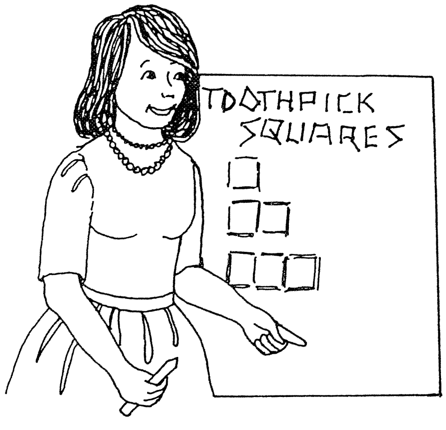
Patterns occur naturally in the everyday lives of young children and make their world predictable. They have many experiences with patterns both in and out of school. These include tying shoes, drinking through a straw, responding to a smile, playing games, counting, and creating designs.

This program seeks to broaden these experiences with pattern activities that help children deepen their understanding of mathematical concepts and relationships, become better problem solvers and understand connections that exist among mathematical topics.

Extending patterns to solve problems

For example, children might visualize the extension of a pattern to solve a problem.

TEACHER *Look at the rows of toothpick squares that I have glued to this chart.*

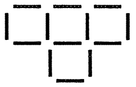


I see a pattern in the rows and am picturing the fourth row in my mind. What do you think the fourth row looks like?

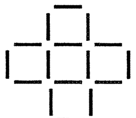
SUE *I think the squares will continue in a line, so the fourth row will have 4 squares that are joined together.*

The fifth row will have 5 squares.

JULIO *I have another way. I am thinking that the next square will be built below the middle square, so the fourth row will look like this.*



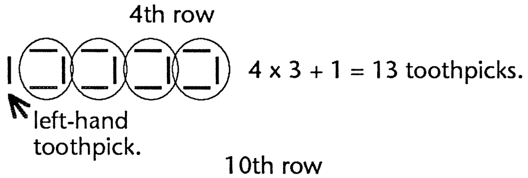
In the fifth row, 1 more square will be added on top of the middle square.

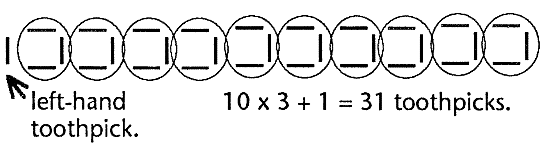


TEACHER *Those are both interesting arrangements. Suppose we think about Sue's way for the moment. If she continues building rows, can anyone describe what her tenth row will look like and tell how many toothpicks she will need for it?*

MEIKO *Let's see... I think she will still make the squares go together in a line and there will be 10 of them. In each row, I see 3 more toothpicks added on—the fourth row has 13 toothpicks, so just start with 13 and count by threes until there are 10 squares: 13, 16, 19, 22, 25, 28, 31. I predict that 31 toothpicks will be needed.*

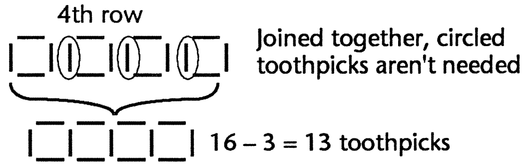
GEOFF *I see the left-hand toothpick and then 3 toothpicks for every square in a row. In the fourth row, the number of toothpicks is 4 × 3 + 1 or 13.*

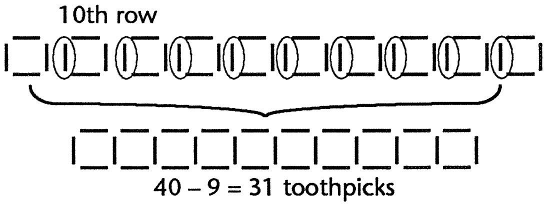




I think that the row with 10 squares will need 10 × 3 + 1 or 31 toothpicks.

NATHAN *In the fourth row, I see 4 separate squares made with 16 toothpicks. When they are pushed together, 3 of the toothpicks aren't needed. In the tenth row, you could make 10 separate squares with 40 toothpicks; when they are pushed together you won't need 9 of the toothpicks, that leaves 31.*





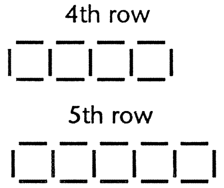
The toothpick example illustrates three aspects of patterning activity:

- identification—children recognize and identify patterns.
- extension and prediction—the students are able to extend a pattern and describe its behavior as it is continued.
- application—a pattern is used to motivate a solution to a problem.

Discovering mathematical relationships

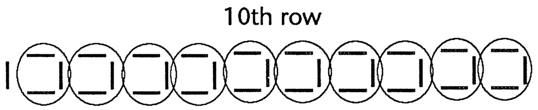
It also shows how a pattern can provide a context for discovering mathematical relationships. Here are ways the children might describe Sue's method of forming squares.

BOB *Each row has 3 more toothpicks than the row before it.*



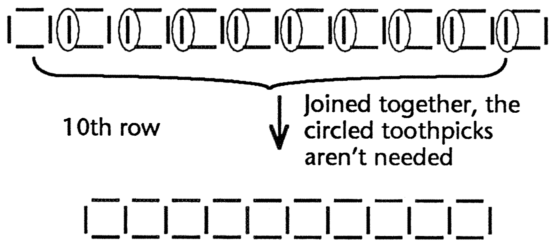
The 5th row needs 3 more toothpicks than the 4th row.

MELONY *To count the number of toothpicks in each row, multiply the number of squares in the row by 3 and then add 1.*



There are 10 squares in this row. The number of toothpicks is 3 x 10 + 1 = 31.

MARIA *If the squares are separated in a row, the number of toothpicks is 4 times the number of squares. When the squares are pushed together, you* *can take away 1 toothpick from every square but the first one.*



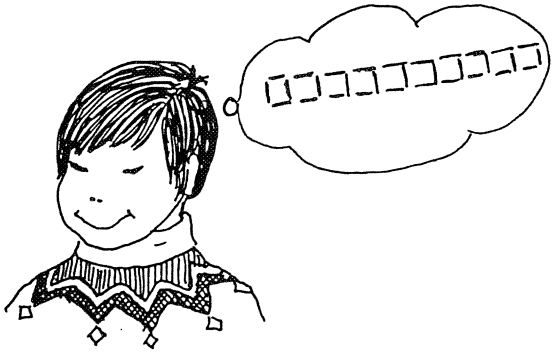
The number of toothpicks in the 10th row is 10 x 4 – 9 = 31.

All characteristics of patterning are important in the study of mathematics and are woven into many of the lessons of this program. Provide your children with many opportunities to engage in activities of this type.

Extending a pattern with mental imagery

In the toothpick example, the children were simply asked to describe the tenth row. You may question how many of the previous rows they need to see or build first.

A major goal of this type of patterning activity is encouraging students to form mental images of the rows in the sequence. Ideally, they will make predictions about the tenth row based on what they "see" in the first four rows and the intervening rows will not have to be built. This may vary with the child, however; some may need to build additional rows.



Note that the type of imaging called for may be extended: What does the 20th row look like and how many toothpicks does it use? How about the 33rd row? How many squares will be in the row that uses 91 toothpicks?

Sometimes when you ask the children to identify and extend a pattern you have in mind—as in the toothpick example—you find they are unable to do so or they suggest other patterns that seem to fit. This calls for a decision on your part. In the example, the teacher chose to discuss Sue's arrangements. If some other suggestion looks particularly interesting (such as Julio's), that too can be extended and explored. On the other hand, if a child suggests a pattern that you do not wish to consider, you can simply say, "That's a nice pattern, but not the one I had in mind." Or, as the teacher in the example appropriately said, "Let's look at Sue's method for now." In any case, each child's thought should be acknowledged.

Finally, if the children are unable to guess a pattern that you have in mind, and you wish to discuss it, we suggest that you tell them what it is and carry on with the extensions.

Let students originate patterns

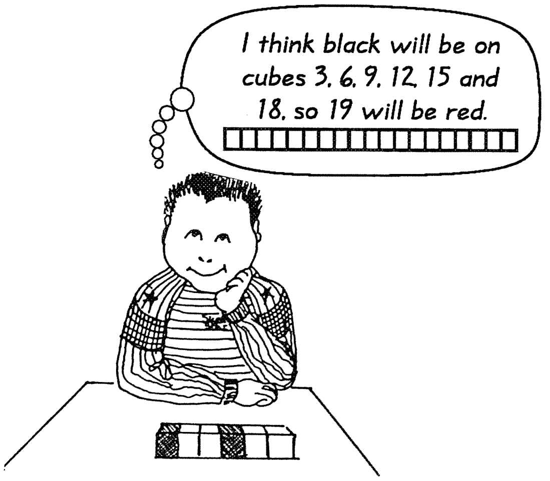
Allow for times when the children can create their own patterns. This will stimulate the creativity of the students and offer them greater ownership over their patterns. The children will also observe the varied approaches that exist in mathematics. Be ready to ask questions appropriate to each pattern and be prepared for unexpected ideas and answers!

Here are two types of lessons in which students work independently with patterns.

LESSON 1

Ask each child to create a pattern with different objects and then answer some related questions. For example, suppose a child makes a red-red-black-red-red-black pattern with hex-a-link cubes. Ask some questions such as:

What will be the color of the 19th cube in your pattern? How about the 157th cube—what will its color be?



If you extend your pattern out to 1000 cubes, about how many of the cubes will be red?

If you copy your pattern on a hundred's matrix, what numbers do you predict will be colored black?

What other things can you say about your pattern?

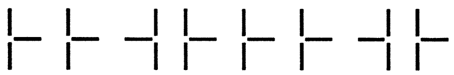
The child's thinking and observations about this pattern can be recorded in a journal or shared with the others in the class.

Here are examples of other patterns that children have created. (Questions that could be asked about the patterns are included.)

Betsy's arrangement of pattern blocks

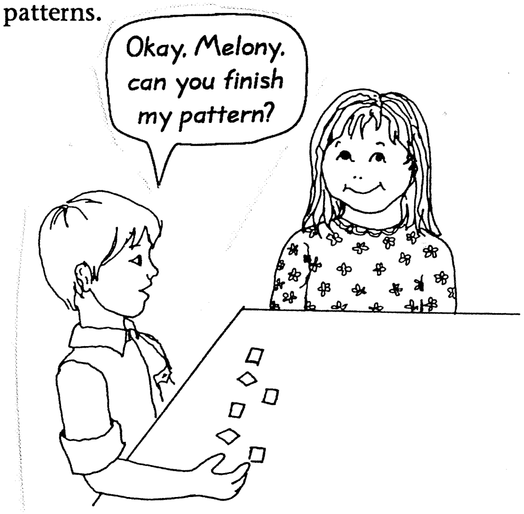
What block will be in the 29th position?
If the pattern is extended through 100 steps, about how many of the blocks will be squares?

Josh's toothpick pattern



If this AABA pattern is continued, describe the toothpicks that appear in the 20th step?
About what fraction of the items will have the horizontal toothpick pointing left?

We recommend that children also be given opportunities to replicate and extend each other's patterns.

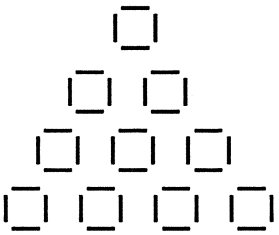


LESSON 2

Present the children with selected mathematical investigations in which they might use patterning as a problem-solving tool. Consider the toothpick example once more:

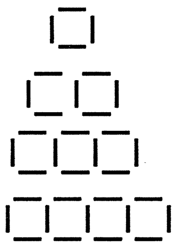
TEACHER *Each of you has toothpicks, glue and construction paper. I'd like you to make rows of toothpick squares according to some pattern and glue them on your paper. Put 1 square in the first row, 2 in the second, 3 in the third and 4 in the fourth.*

CAMERON *I made squares separately. Each row uses 4 more toothpicks than the row before it.*



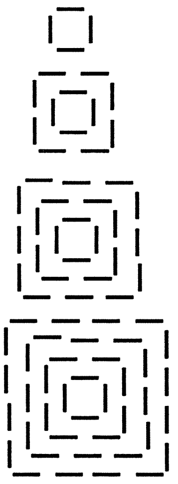
number of squares	*number of toothpicks*
1	4
2	8
3	12
4	16

PAOLO *I joined my squares in a line. Each row needs 3 more toothpicks than the one before it.*



number of squares	*number of toothpicks*
1	4
2	7
3	10
4	13

ZACH *I made my squares inside each other. Each square is bigger than the ones inside it.*



number of squares	number of toothpicks
1	4
2	12
3	24
4	40

TEACHER *I like your pictures. If you each continue building rows of squares according to your pattern, how many toothpicks will you need to make 10 squares?*

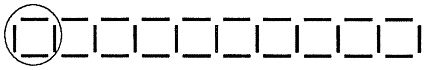
CAMERON *That's easy! Each square uses 4 toothpicks, so I will need 10 × 4—40—toothpicks for 10 squares!*

10th row

10 x 4 = 40

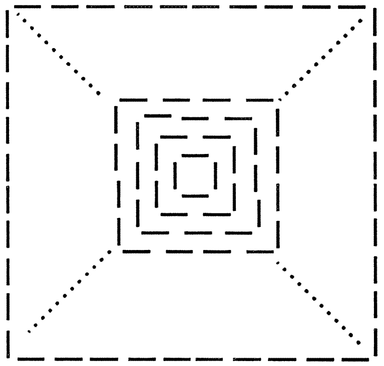
PAOLO *Wait a minute! I need 4 toothpicks for the first square, but only 3 for each of the others. So if I build 10 squares, I will need 4 for the first and 3 for each of the other 9. Altogether, that's 4 + 27 or 31 toothpicks! I don't need as many!*

10th row



4 + 9 x 3 = 4 + 27 = 31.

ZACH *My squares keep getting bigger. I am going to need a lot of toothpicks! Let's see, the smallest square has 1 toothpick on a side, the next one has 2, and then 3...I think the tenth one will have 10 on a side. So the number of toothpicks all together will be 4+8+12+16+20+24+28+32+36+40. That's 220 toothpicks! I'll really need a lot!*



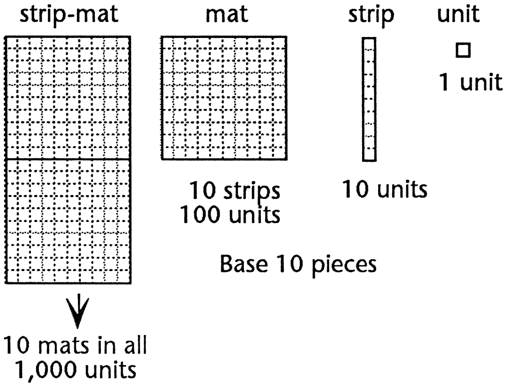
220 toothpicks in all.

Develop mathematical concepts with atterns

In *Opening Eyes to Mathematics*, many chapters and lessons make use of patterns in the development of mathematical concepts. The examples that follow provide insight into how this can be accomplished.

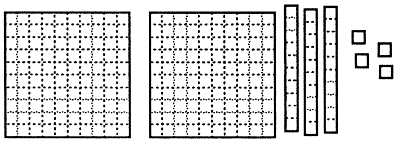
EXAMPLE 1

One of the most helpful applications of patterns is in the development of numeration principles and operations. This is done with the help of the counting pieces shown here.



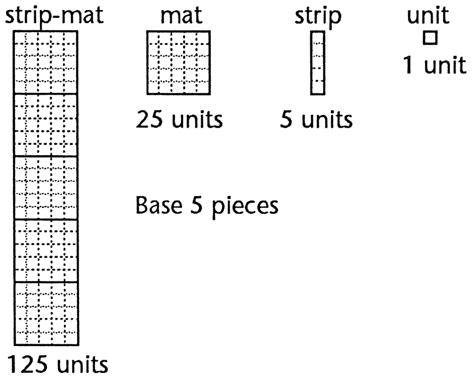
These are called base ten counting pieces because each one, other than the unit, has 10 times as many unit squares as the piece to its right. They can visually represent the place value, grouping, and regrouping aspects of our number system.

Notice that the number of unit squares in the pieces grow according to powers of 10 and represent the place values of our numeration system. The pieces also form a visual, alternating pattern (from right to left) of "square, rectangle, square, rectangle, ...". Thus 234 units can be pictured in this way:

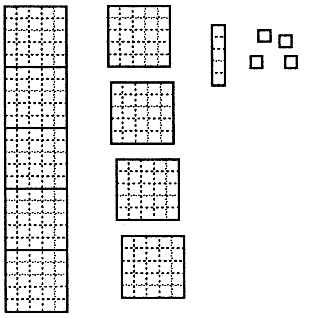


200 + 30 + 4 =
234 units in all

Other counting pieces can be constructed with similar patterns. For example, base five counting pieces have the same "square, rectangle, square, rectangle, ..." pattern. Notice that each piece, other than the unit, has 5 times as many unit squares as the one to its right.



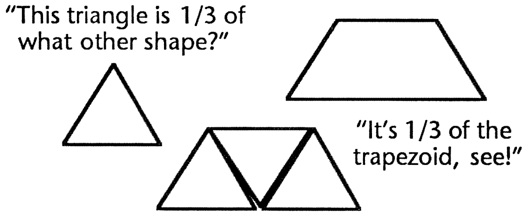
Using these pieces, 234 units can be represented in this way:



125 + 100 + 5 + 4 =
234 units in all

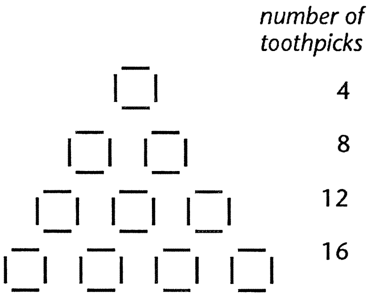
EXAMPLE 2

The relationships that exist among different pattern blocks can motivate informal experiences with fractions and areas.



EXAMPLE 3

Children can grow in their understanding of number concepts and relationships by working with number patterns in various settings. In the illustration below, toothpicks are used to picture multiples of 4. These multiples are also sketched on a hundred's matrix.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

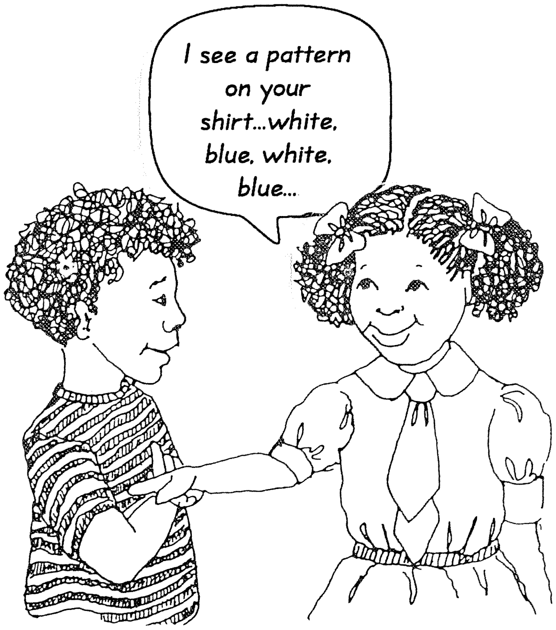
Building a sense of pattern

If you are unsure of what experiences your children had with pattern in earlier grades, lead the students in a few simple snapping, clapping, chanting, counting or movement patterns. Are the children able to join you once they get a feel for the pattern?

Here are some other things to look for:

Can the children

- identify and extend a visual pattern that is presented to them?
- create their own patterns?
- recognize number patterns?
- recognize patterns in their world such as patterns in clothes, science, health or animal life?



If the children don't exhibit a sound sense of pattern, incorporate more activities as described above. Although they may seem playful, such activities contribute in important ways to the students' mathematical development.

Although a few of the Contact lessons make patterns the main objective, we recommend patterns not be isolated, but be used to help children grow mathematically and to solve problems.

Encourage children to look for patterns beyond the classroom. The possibilities are limitless! In the world around us, patterns appear in

- music: rhythm, ABA themes, choruses.
- the environment: ocean tides, phases of the moon, blossoms in the spring.
- art: Van Gogh's use of long brush strokes side by side, the mixture of colors, the arrangements of flowers.
- design: placement of bricks to create a wall, weaving a seat of a cane bottom chair for strength as well as beauty.

You'll be pleasantly surprised how easily children's eyes can be opened to pattern!

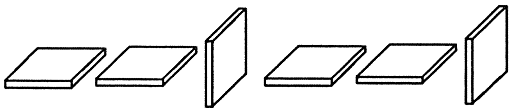
When working with patterns, you can use materials you have on hand, as well as those listed in the Materials Guide. We also use objects related to the Contact monthly themes.

Sometimes children will shy away from creating patterns with materials such as popsicle sticks, tile or toothpicks because they seem to have such few attributes. However, with some encouragement, they will respond to the challenge of going beyond the obvious qualities of these objects. A classroom incident illustrates this point.

Our children were asked to create patterns with beige, 1" ceramic tile.

TEACHER *(at a time when the children seem to have run out of ideas) Look again at your tile. Does anybody see a way we could use them to make a new pattern? (Silence followed as the students considered the problem.)*

JANET *Oh, Mrs. Head! That's easy! I can just lay my tile in different ways! If I stand one up on its edge, I can make a flat, flat, standing-up pattern!*

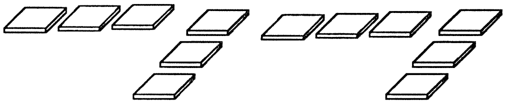


CHILDREN *That's great! We can do that, too!*

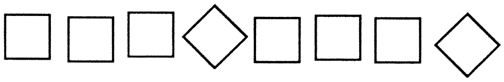
LESTER *Oh, I know one! I can stack them in different piles.*

The first child in this dialogue was not normally a class leader and, in fact, had a severe learning disability. However, it was her example that opened the eyes of the others to different possibilities for patterns. Here are other samples of children's work.

a)



b)



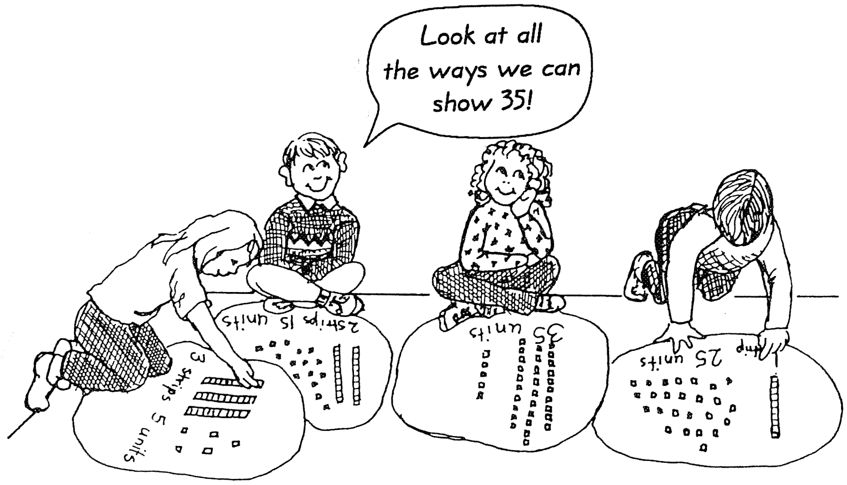
Summary

This foundation of pattern is essential to developing mathematical understanding. Pattern work contributes greatly to students' understanding and mastery of mathematics. How powerful it is to experience the patterns of multiplication in a variety of hands-on or visual settings.

Mathematics holds more meaning for those children who recognize its patterns. In this way, children become masters of the subject and make it work for them.

4 Numeration: Place Value

Understanding place value is another critical step in the development of children's comprehension of number concepts. NCTM's *Curriculum and Evaluation Standards for School Mathematics, p. 39*



What can be done to help children understand place value concepts?

This is a question teachers frequently ask. They know that an understanding of place value concepts is vital if children are to work confidently and successfully with numbers. And teachers realize the difficulty students often experience when learning these concepts.

In answer to the question above, the activities described in this chapter will enable children to develop a visual model for representing place value concepts. This model provides a way to look at the overall structure of our number system. The major goal of the activities is to help children form mental images of numbers, images that will aid them in constructing and thinking about numerical relationships and in later work with number operations.

We have patterned these activities after those used in the *Math and the Mind's Eye* materials. The numeration models used in those materials are versatile, readily used and understood by children, and easily recalled as pictures in the mind. We are confident that your children, like ours, will enjoy learning in this way.

In these activities, place value concepts are first explored by manipulating counting pieces. Diagrams and sketches are then used to connect the discoveries made in these explorations with the mental imagery and the use of symbols that occur later. A distinguishing feature of the teaching approach used in *Opening Eyes to Mathematics* is this emphasis on experiences that show graphically the interrelation between the physical models, the sketches, the imagery and the symbols. Once such connections have been established, children have several options for representing numbers: physical models, diagrams or sketches, and mental pictures. They can then select the one that is most helpful in a given situation.

Allow plenty of time for children to work through these activities. We have found the comprehension of place value cannot be rushed. Time spent now will bring ample dividends later, so enjoy exploring with your class! You can be assured that when everything is completed, your children will have a deeper understanding of place value and will be well prepared to solve numerical problems.

Number system characteristics: Positional notation and grouping by tens

In order to fully understand our number system, children need to realize that:

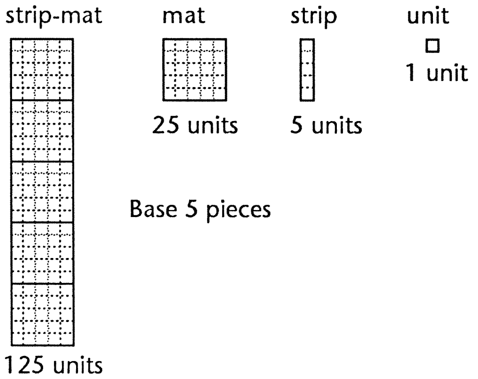
- Numbers are written using positional notation. This means that the position of a digit in a number determines its place value. That place value is a power of 10, since our procedure for counting makes use of groupings by tens. For example, in 3,405 the place values of 5, 0, 4 and 3 are 1, 10, 100 and 1,000, respectively. In this number, the 4 represents 4 hundreds, whereas in the number 47, the 4 stands for 4 tens.

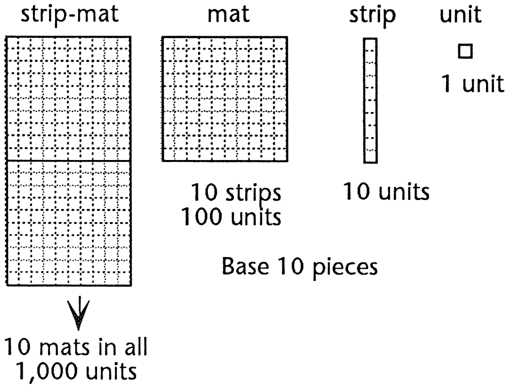
- The use of place value and positional notation gives meaning to numbers. Thus, 3,405 means 3000 + 400 + 5 and 47 means 40 + 7. This makes it possible for people to communicate with numbers in an understandable way.
- Whole numbers can be written in *expanded form*, such as 732 = 700 + 30 + 2 or 732 = 7 hundreds + 3 tens + 2 ones. An expanded form of a number provides information about the place value of each digit.
- The groupings used within the system are related to one another. Ten ones make a single group of 10; 10 tens make a group of 100; 10 hundreds make a group of 1,000, etc.
- In order to distinguish between numbers, it is sometimes necessary to use 0 as a placeholder. For example, 3,405 means three thousand four hundred five whereas 345 means three hundred forty-five.

Let's look at how we might help children construct these ideas.

Modeling numbers with base five and base ten pieces

The place value lessons of this book make extensive use of the counting pieces shown here.





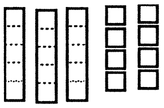
Notice how the pieces within each set are related. In the base five collection, each piece, other than the unit, has 5 times as many unit squares as the piece to its right. Each piece in the base ten collection has 10 times as many unit squares as the piece to its right. Both sets of pieces also form a visual, alternating pattern (from right to left) of "square, rectangle, square, rectangle,...".

With these pieces, children model numbers and explore the grouping aspects of our number system. Examples 1 and 2 illustrate this.

EXAMPLE 1

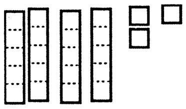
How can 23 units be shown with base five pieces?

One way of answering this question is to form a collection of 3 strips and 8 units. This collection requires 11 base five pieces.



15 + 8 = 23 units in all

By now replacing 5 of the units by a strip, a collection of 4 strips and 3 units is formed. This collection also represents 23 units but it uses only 7 base five pieces.



20 + 3 = 23 units in all

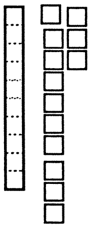
Similar exchanges of the pieces lead to other answers. The chart below summarizes all the possibilities.

Strips	Units	Number of Pieces Used
0	23	23
1	18	19
2	13	15
3	8	11
4	3	7*

EXAMPLE 2

How can 23 units be shown with base ten pieces?

There are three ways to answer this question. One way is to form a collection of 1 strip and 13 units. This collection uses 14 base ten pieces.



10 + 13 = 23 units in all

The other two answers can be formed by exchanging pieces (each strip is equivalent to 10 units).

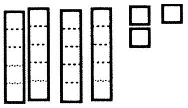
Strips	Units	Number of Pieces Used
0	23	23
1	13	14
2	3	5*

It is important to keep in mind that all the collections in these examples represent 23 units. The collections can be formed from one another by making equal exchanges (or trades) among the pieces. In making these exchanges, children gain familiarity with the regrouping that is used later when computing with numbers.

Minimal collections

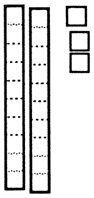
Let's look a bit further at the collections discussed in Examples 1 and 2.

In each example, the collection marked with an asterisk is special—it uses the fewest number of pieces to represent that particular number. This starred collection is called the *minimal collection.*



strips	units
4	3

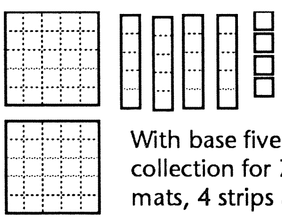
With base five pieces, the minimal collection for 23 units consists of 4 strips and 3 units.



strips	units
2	3

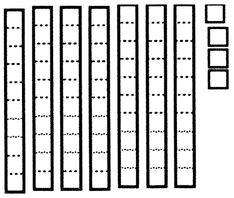
With base ten pieces, the minimal collection for 23 units consists of 2 strips and 3 units.

The following chart shows the minimal collections for 74 using these counting pieces.



mats	strips	units
2	4	4

With base five pieces, the minimal collection for 74 units consists of 2 mats, 4 strips and 4 units.



mats	strips	units
	7	4

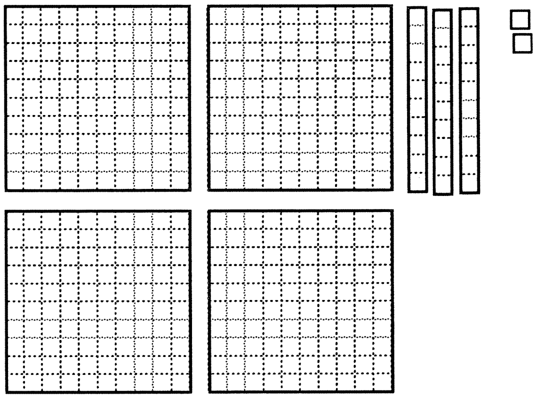
With base ten pieces, the minimal collection for 74 units consists of 7 strips and 4 units.

Notice that, when base ten pieces are used, the number of strips (or tens) and units in the minimal collection for 23 corresponds, respectively, with the digits in the tens and ones place of the number 23. The same is true for the minimal collection for 74. Children can therefore use these collections to think of (and picture) these numbers in terms of place value.

Children soon learn that any whole number is readily pictured by its minimal collection of base ten pieces. Further, they learn that the digits of the number tell how many times each different piece is to be used in this collection. This connection reflects the fundamental concept related to positional notation. Here are a few more examples.

EXAMPLE 3

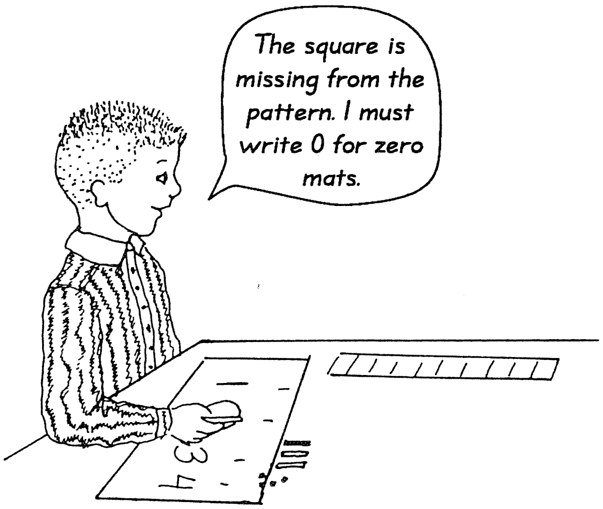
Using base ten pieces, the minimal collection for 432 units is 4 mats (or hundreds), 3 strips (tens) and 2 units.



EXAMPLE 4

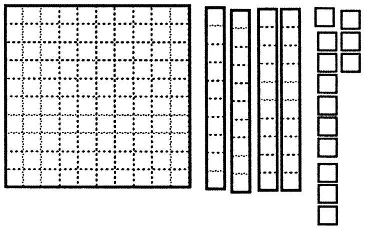
Using base ten pieces, the minimal collection for 1,034 units is 1 strip-mat (thousands), 0 mats (hundreds), 3 strips (tens) and 4 units.

Example 4 demonstrates that the use of 0 as a placeholder can be thought of visually. A 0 indicates the absence of a particular piece in a collection. Children often use the "square, rectangle, square, rectangle..." pattern mentioned earlier as a reminder of this.

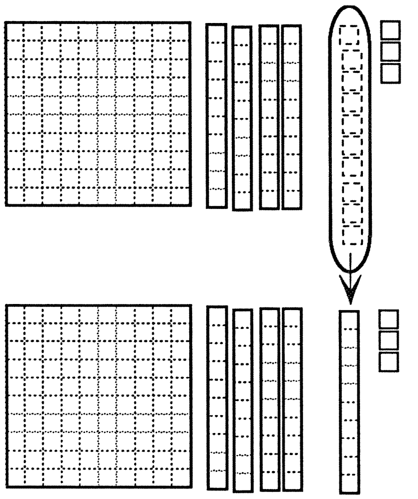


EXAMPLE 5

Consider the collection of base ten pieces shown here.



This collection shows a total of 153 units. It is not the minimal collection, however, since 10 of the units can be replaced by another strip.

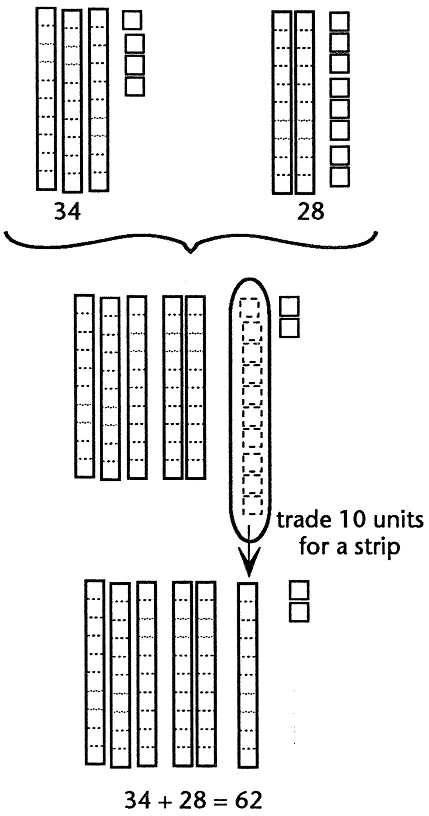


With base ten pieces, the minimal collection for 153 units consists of 1 mat, 5 strips and 3 units.

This example illustrates that, with base ten pieces, no more than 9 of any one piece will ever be required in forming a minimal collection. Any group of 10 or more of the same piece can be traded for the next larger size.

Examples 1–5 show how the underlying ideas related to place value and regrouping can be modeled by counting pieces. This type of modeling is important—it is a key to understanding these concepts!

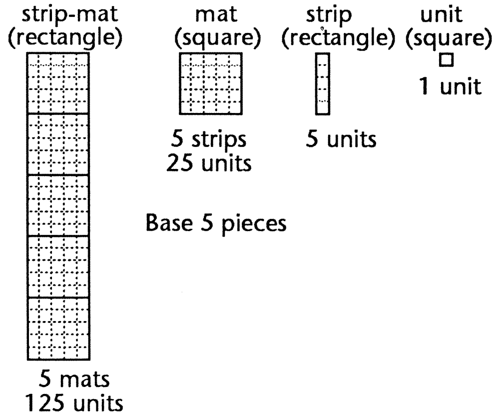
These ideas will later play an important role when children operate with numbers. Consider the problem of adding 34 + 28 as an example. As shown in the following illustration, the required sum can be found by combining collections of base ten pieces that represent 34 and 28, respectively and then trading to form the minimal collection.

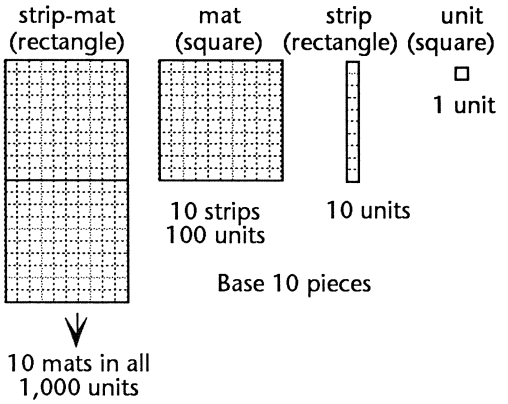


A role for base five pieces

Since our number system has a base of 10, most of the numeration activities of this book use base ten pieces (or diagrams of them). Let's not overlook the base five pieces, however. There are several reasons why it is important for children to represent numbers with these as well.

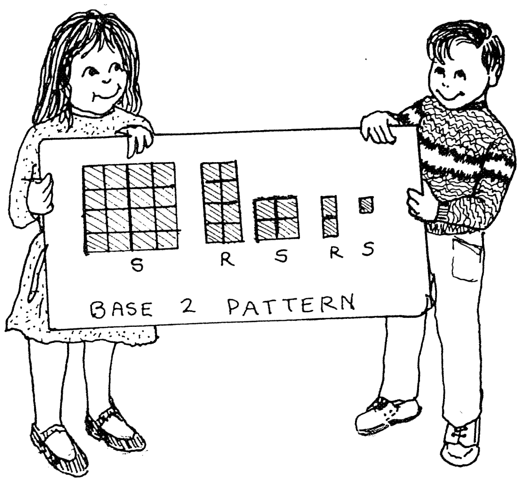
- Base five and base ten pieces are structured in similar ways. The different pieces within each set grow in a predictable way, one set involves grouping by fives, the other by tens. The pieces of both also alternate between squares and rectangles.





- Because only five units are needed to make a strip, children experience regrouping frequently when working with base five pieces. This experience prepares the way for the regrouping required in our base ten system.
- Children are more likely to understand the nature of minimal collections if they have had opportunities to think about them in different settings. They discover, for example, that no base five piece is ever used more than four times in a minimal collection. Similarly, no base ten piece is ever used more than nine times in such collections.

We begin our study of numeration with activities in which children model numbers with base five pieces. Do not be concerned that these activities will be confusing to children. They are conducted very informally and without any special notation. We have found that they lead naturally into working with our base ten system and enhance children's understanding of it. Our children even enjoy the challenge of creating base two, three and eight pieces from grid paper and we have included some lessons that use these.

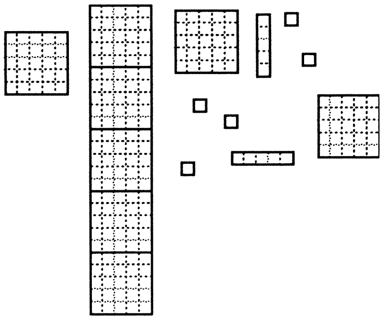


In this age of computers, numeration systems that use different bases are important. Explorations with various kinds of counting pieces can help prepare children to deal with these bases.

Beginning with base five pieces

We introduce base five pieces to our class with a period of "free exploration", just as we would with any new materials. During this time, the children become familiar with the pieces and begin to notice how they are shaped and related to one another. The dialogue below summarizes some of the discussion that takes place afterwards. During this time the children share their observations about the pieces and begin to form mental pictures of them. They also explain how the pieces remind them of pennies, nickels and quarters.

The teacher displays a collection of base five pieces on the overhead.



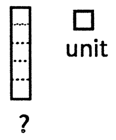
TEACHER *What can you say about these pieces?*

IMALDA *I see different shapes. It looks like there are large and small rectangles and squares.*

ZACHARY *(He comes to the overhead and holds up a strip.) Right! It looks like someone forgot to cut the 5 little squares apart in this piece. There are 5 of the little squares hooked together. It's kinda neat, because I can pick up 5 squares at once.*

DOMINIQUE *You know—that reminds me of pennies and nickels. The pennies would be the same as the little squares. A nickel would be 5 pennies just like this long piece would be 5 little squares.*

TEACHER *All of you have made some nice observations. Now let's name the pieces. The little squares are called units. What are the 5 units which are hooked together like?*



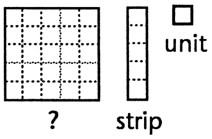
IMALDA *It's a row, a row of 5 units.*

SALLY *It looks like a train to me.*

TEACHER *I'm going to suggest we call it a strip. Does that describe it for you?*

FRED *Sure! It really looks like a strip of units hooked together! That will make it easy to remember because the name tells what it looks like to me.*

TEACHER *(Pointing to the overhead.) Look at this larger piece—we'll call it a mat. What do you notice about it?*

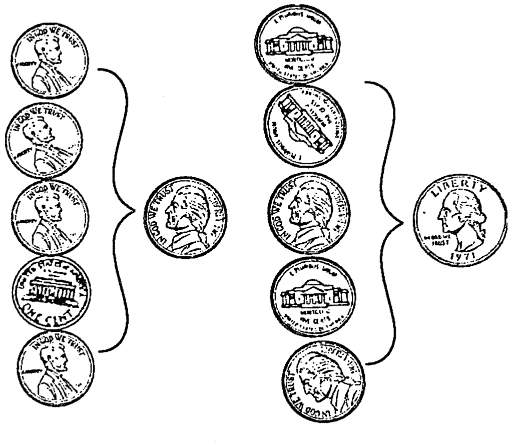


ZACHARY *It's a bigger square.*

BETSY *It looks like 5 strips have been scotch-taped to make that big square.*

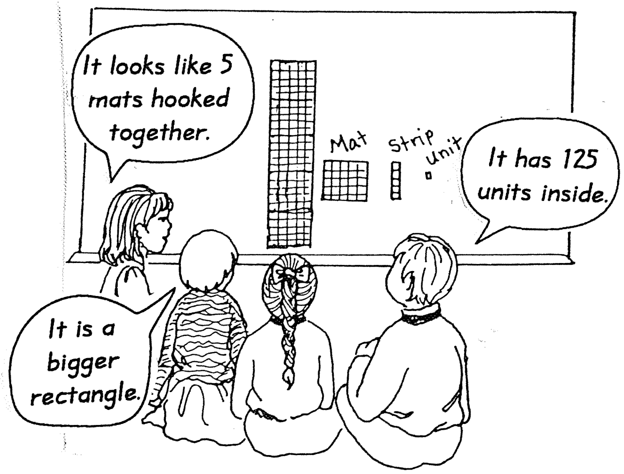
DIRK *I see that, too. That means a mat has 25 units in all.*

LISLE *That reminds me of money, too! A mat is like a quarter. Each mat represents 5 strips, just like each quarter represents 5 nickels! And a quarter equals 25 cents which kind of goes with the mat and its 25 units.*

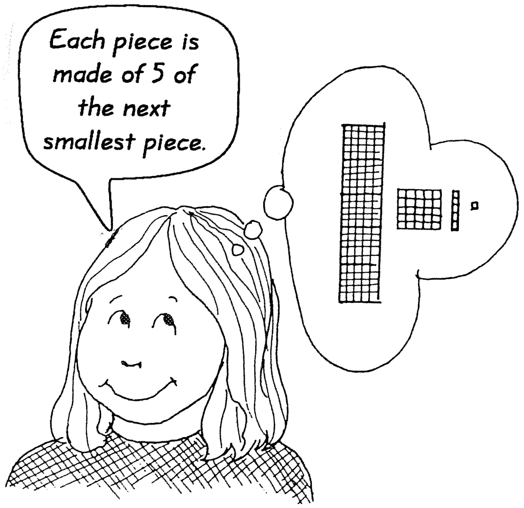


5 pennies = 1 nickel   5 nickels = 1 quarter

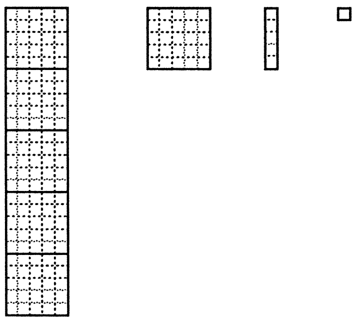
As the discussion continues, the teacher identifies a *strip-mat* and the children describe it.



The children then close their eyes and think about the pieces mentally, describing what they "see".



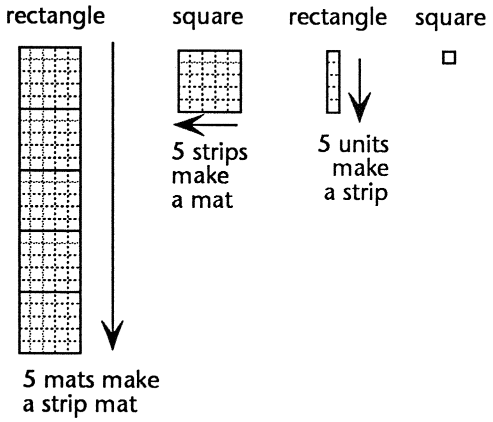
TEACHER *Open your eyes. I am going to place one of each piece on the overhead (see illustration) and I'd like you to do the same at your desks. Please be sure that your pieces are arranged just like mine.*



As you look at the pieces, do you notice any patterns?

LI *(He comes to the overhead.) Well, I see a shape pattern! If I start with the smallest, it goes square,* *rectangle, square, rectangle. Each piece is getting bigger. The next one should be a square.*

RUDY *Yes! The shapes are getting bigger. There is a growing pattern! It takes 5 of one piece to make the next larger one and the shapes take turns between squares and rectangles.*



TEACHER *Yes, you have mentioned two important patterns.*

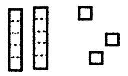
The dialogue can continue in this manner, perhaps with the children predicting the shape and size of the next piece in the collection. In the words of one child, "I think the next larger piece will be a square. You can make it by scotch-taping 5 strip-mats together."

More base five activities

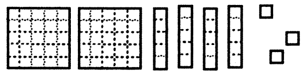
The numeration lessons in this program suggest several types of activities, samples of which follow. Allot enough time for each activity to meet your students' needs. Remember, however, that the major purpose is to model numbers and gain experience with grouping processes with the pieces. No special notation is required. Be informal and have fun!

ACTIVITY 1

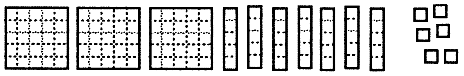
Lay out a collection of pieces and ask your children to determine the total number of unit squares that are present. Note also the number of units represented by the different pieces in the collection.



The 2 strips show 5 units each or 10 units.
The total number of units is 10 + 3 or 13 units.



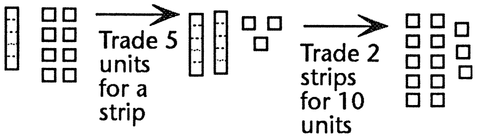
The 2 mats show 50 units and the 4 strips show 20.
The total number of units if 50 + 20 + 3 or 73.



The 3 mats show 75 units and the 7 strips show 35.
The total number of units is 75 + 35 + 6 or 116.

ACTIVITY 2

Ask the children to show 13 (or some other number) units with their pieces. Make a listing of the various collections that are found on a Mats-Strips-Units chart. Include a column for the total number of pieces used in each.



Strips	Units	Number of Pieces
1	8	9
2	3	5 *
0	13	13

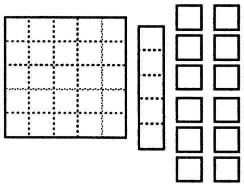
Note how the collections can be formed from one another by making equal exchanges (or trades) of the pieces. The one that uses the fewest number of pieces is marked with an asterisk and is called the minimal collection for 13.

Here are some ways to show 39 units. The minimal collection is identified.

Mats	Strips	Units	Number of Pieces
0	0	39	39
0	1	34	35
0	2	29	31
0	3	24	27
0	4	19	23
0	5	14	19
0	6	9	15
0	7	4	11
1	2	4	7 *

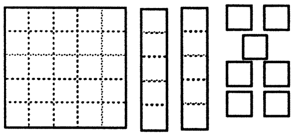
ACTIVITY 3

Show the children an assortment of pieces and have them make some equal exchanges of pieces. Keep a record of the collections that are formed on a Mats-Strips-Units chart. What trades lead to the minimal collection?



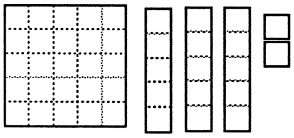
mats	strips	units
1	1	12

trade 5 units for a strip



mats	strips	units
1	1	12
1	2	7

trade 5 units for a strip



mats	strips	units
1	1	12
1	2	7
1	3	2

minimal collection

ACTIVITY 4

Have the children represent an amount of units by its minimal collection.

ACTIVITY 5

Ask the children to model some simple addition and subtraction problems with the pieces. The following dialogue illustrates a way of doing this. Notice that the children work only with the pieces and that no recording takes place at this time.

TEACHER *Will you please place the following collections in front of you: 1 strip and 3 units along with 2 strips and 2 units. What will you have if these pieces are combined?*

OTTO *That's easy! Push them together—you'll have 3 strips and 5 units.*

BETH *Wait a minute! We can make a trade and replace the 5 units by a strip. That will give us 4 strips and no units. That's the minimal collection.*

TEACHER *Thank you. Let's try another.*

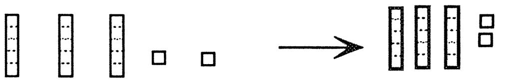
KAREEM *Sure! Give us a harder one this time!*

TEACHER *Okay, harder it is! Why don't you work with a partner on this one? It might be more fun for you. Set up the minimal collection for seventeen units.*

(After a period of time)

Who can show-and-tell how to do this?

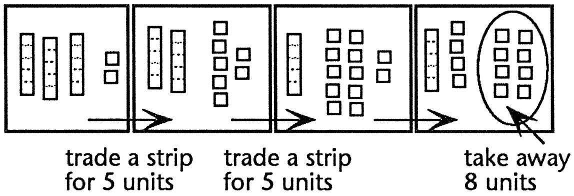
KAREEM *We knew that we can count by fives, just like we do with nickels and pennies. So we started out with strips—5, 10, 15. Then we added 2 more units to get to 17. So we used 3 strips and 2 units.*



TEACHER *I think that's nice. Now, working with your partner, will you please remove 8 units from your collection?*

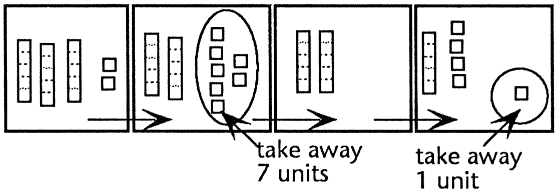
The teams work for a while and then share their thinking about this question. Three possible responses are shown below.

Strategy 1



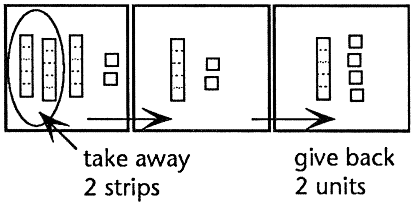
"We traded one of the strips for 5 units. That gave us 7 units. That wasn't enough. So, we traded another strip for 5 more units. That gave us 12 units. Then, we removed 8 units. That left 1 strip and 4 units."

Strategy 2



"We started with the 3 strips and 2 units. Then we traded one of the strips for 5 units. That gave us 2 strips and 7 units. Then we removed the 7 units. We knew we needed to remove one more unit. So we exchanged another strip for 5 units and took 1 away."

Strategy 3



"We started with 3 strips and 2 units. We needed to remove 8 units, so we just removed 2 strips. That was 10 units—2 too many. So to fix it we gave back 2 of the units. That left us with 1 strip and 4 units."

The dialogue shows how the children were given the opportunity to work together and to explain their thinking. It also illustrates how a problem may be solved in several ways, each of which is acceptable. This makes it possible for the children to grow in confidence and to feel good about their work.

Moving on to base ten

Base ten pieces are just like those of base five except each piece contains 10 times as many unit squares as the next smaller one. Children recognize this during an initial period of exploration with the pieces. In fact, as a result of their experiences with base five, the transition to base ten is a very natural and easy one.

The main goal is to help children visualize numbers and think about place value in their "mind's eye". This can be accomplished by a progression of activities in which children model numbers first with base ten pieces and then with diagrams and sketches. This leads to the development of mental images that can be used to give meaning to numerical symbols and to construct numerical relationships.

Activities with base ten pieces

Once children have been introduced to the base ten pieces, they are ready to engage in activities that parallel the ones previously done with base five pieces. They explore the different ways a number can be represented by collections of pieces, with the intent of identifying the minimal collection and gaining familiarity with the regrouping that takes place when pieces are exchanged. Illustrations of this were given earlier in Examples 1–5.

It is important to vary the size of the numbers that are modeled, being sure to include ones that are 3 digit or that involve zero. Here are two more examples—in each, the minimal collection is marked with an asterisk. Please note the comments that follow these examples.

EXAMPLE 6

How can 57 units be shown with the base ten pieces?

The chart below reports the different collections that answer this question, together with the number of pieces used in each.

Strips	Units	Total Number of Pieces
0	57	57
1	47	48
2	37	39
3	27	30
4	17	21
5	7	12*

EXAMPLE 7

What are some ways to show 102 units with base ten pieces?

There are several collections that could be used and they are recorded below.

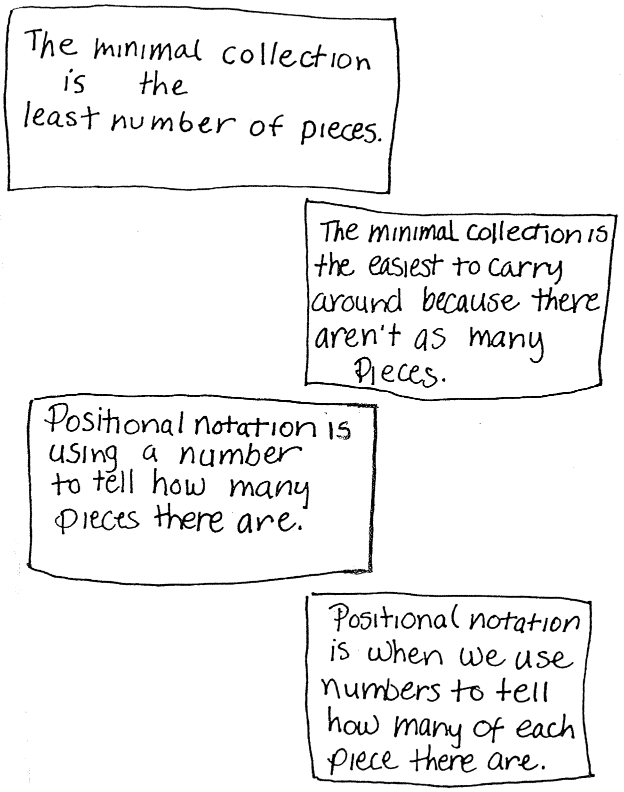
Mats	Strips	Units	Total Number of Pieces
0	0	102	102
0	1	92	93
0	2	82	84
0	3	72	75
0	4	62	66
0	5	52	57
0	6	42	48
0	7	32	39
0	8	22	30
0	9	12	21
1	0	2	3*

As the charts indicate, there are many answers in Examples 6 and 7. Children need not find all of them, though they may enjoy the challenge of doing so. It is important, however, that they determine the minimal collections.

There are enough base ten pieces in a base ten counting piece set for each child to form all the collections for any number up to 60. The children will have to combine materials in order to show larger numbers. We have found that, in general, the children do not feel the need to actually form large collections of units. For example, they will usually realize that a total of 102 units can be shown by a collection of 102 separate unit pieces without actually laying them out.

As usual, conduct these activities in a relaxed, happy way. As your children participate in them, encourage them to work cooperatively and to show-and-tell their results. They will begin to realize that:

- Numbers can be modeled by collections of base ten pieces in various ways, one of which is the minimal collection. They will later learn that a number is most often thought of in terms of its minimal collection. However, the other collections are okay, too, and are sometimes used when working or computing with numbers.
- When forming the minimal collection for a number, it helps to refer to the digits of that number. That is, they understand the use of positional notation. This illustration shows some of the ways children have described these ideas:



The minimal collection is the least number of pieces.

The minimal collection is the easiest to carry around because there aren't as many Pieces.

Positional notation is using a number to tell how many pieces there are.

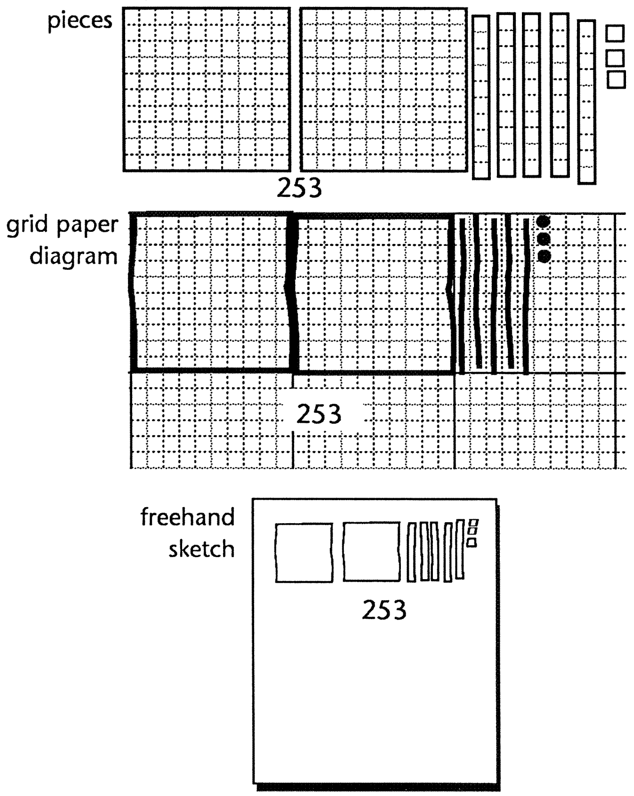
Positional notation is when we use numbers to tell how many of each piece there are.

- For each zero in a number, there will be one type of base ten piece that will not be used in the minimal collection. Children often remember that the base ten pieces form an alternating square-rectangle pattern. If they observe that the pieces in a minimal collection do not follow this pattern, they know that a zero will be needed to record the total number of units.

Making connections: Diagrams, sketches and mental images of numbers

Children gain valuable insights about numbers and place value concepts through their experiences with counting pieces. With this as a basis, it might be tempting to think they are ready to move directly to thinking about these concepts symbolically. We believe, however, that some connecting activities are needed.

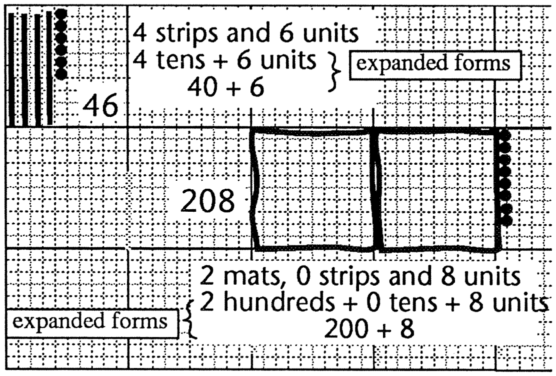
In this program, these connections are made with the help of diagrams (on grid paper) and freehand sketches of base ten pieces. (See illustration on next page.)



Diagrams and sketches are helpful options for picturing numbers of all sizes and for exploring how numbers are related to one another. The following examples illustrate their use.

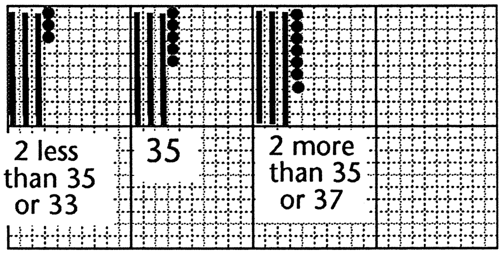
EXAMPLE 8: EXPANDED FORMS OF NUMBERS

Make diagrams of the minimal collections for 46 and 208. Write expanded forms for each number.



EXAMPLE 9: NUMBER PROGRESSIONS

a. Diagram the minimal collections for 35, 2 less than 35 and 2 more than 35.

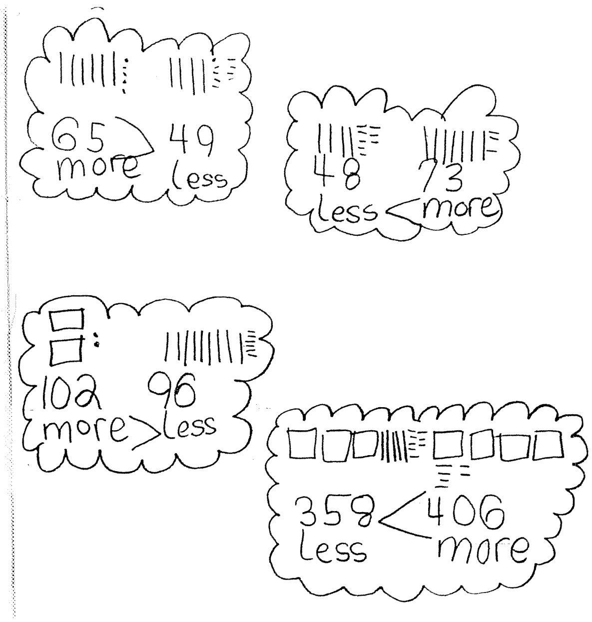


b. Use base ten pieces to form the minimal collections for 107, 19 more than 107 and 19 less than 107. Then make a diagram of each set.

The diagrams on page 29 depict two ways children might trade pieces to form the required sets.

EXAMPLE 10: COMPARING NUMBERS

Ask the children to compare two numbers, deciding which is larger.



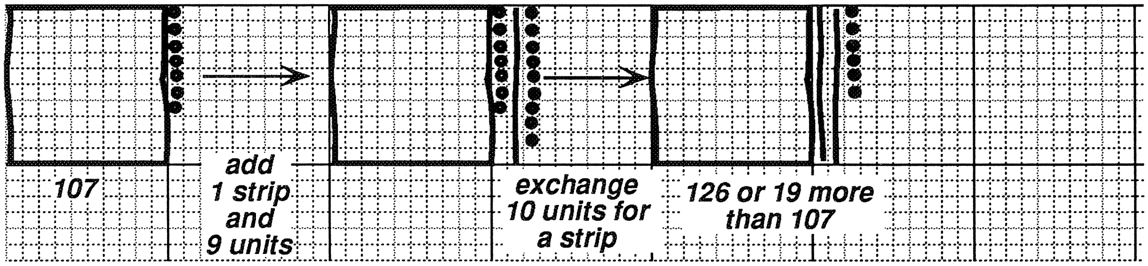
We introduce the symbols for less than (<), greater than (>) and equals (=) in conjunction with examples of this sort. After the numbers have been sketched, the children use the appropriate symbol to compare them.

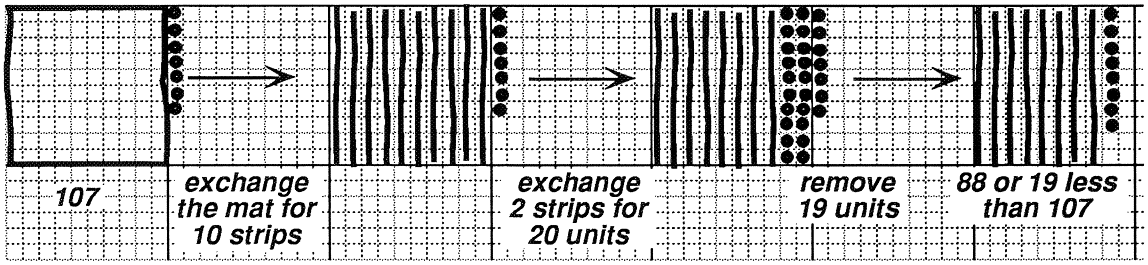
Children quickly recognize that making a diagram or sketch is an alternative to setting out actual pieces. Being able to draw a picture that helps solve a problem makes children feel more comfortable—and powerful.

Diagramming numbers on base ten grid paper is an important feature of this program. While it is always possible to make free-hand sketches (and ultimately more convenient), using grid paper provides a structure that many children need. We help our children become familiar with both options; they can then choose the one that is most helpful in a given situation. There are even times

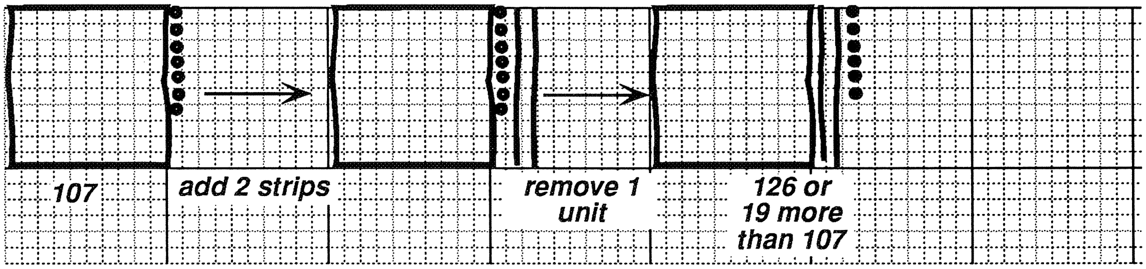
Example 9, b.

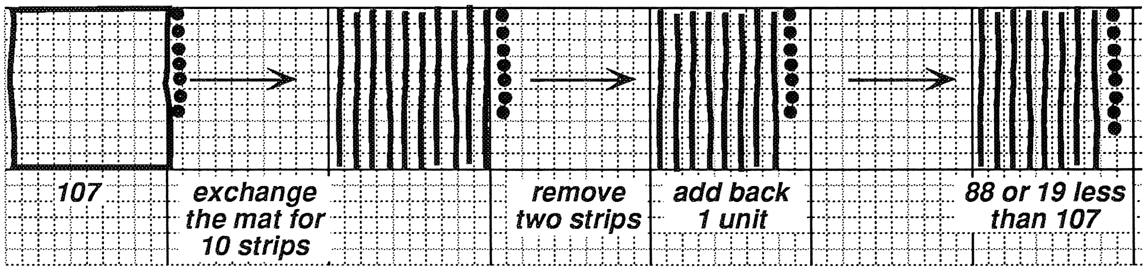
Strategy 1





Strategy 2





helpful in a given situation. There are even times when someone may want to bring out the pieces—that, too, is always an option!

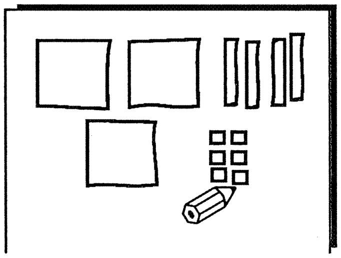
It is not necessary to be overly meticulous when diagramming on grid paper. Our children usually use dots to indicate units and lines for strips. We do recommend using grid paper that has some writing space on it and have included different versions in the blacklines.

All the activities described thus far in this chapter—using counting pieces, diagramming, sketching—will strengthen your children's visual perceptions of number and place value, and aid them in developing and recreating mental images of these concepts. We extend these activities with exercises that promote this imagery. Here are some examples.

EXAMPLE 11: MINIMAL COLLECTIONS

Illustration A

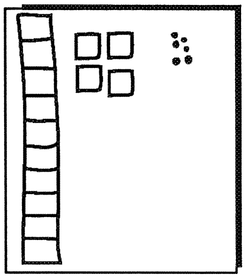
TEACHER *(Holding up a large handful of units.) I have 346 units in my hand. Imagine what the minimal collection would look like. Can you describe or sketch it?*



"I see 3 mats, 4 strips and 6 units."

Illustration B

TEACHER *Make a sketch of 1 strip-mat, 4 mats and 6 units. How many units do these pieces contain altogether?*

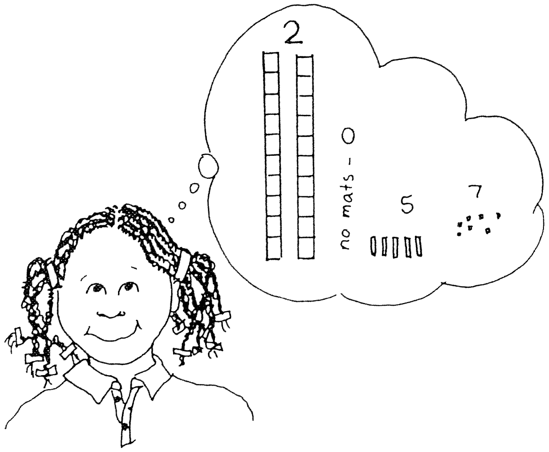


"There will be 1,000 + 400 + 6 or 1,406 units."

Illustration C

TEACHER *Close your eyes. Try to see 2,057. Describe what you see.*

MONIQUE *I remember that the pieces follow a square, rectangle, square, rectangle pattern, starting with the smallest one. So the 2 must refer to a rectangle—there must be 2 strip-mats or 2 thousands. There are no mats. I also see 5 strips and 7 units.*



EXAMPLE 12: PLACE VALUE

TEACHER *Imagine the number 4,503. What can you tell me about the 5 in this number?*

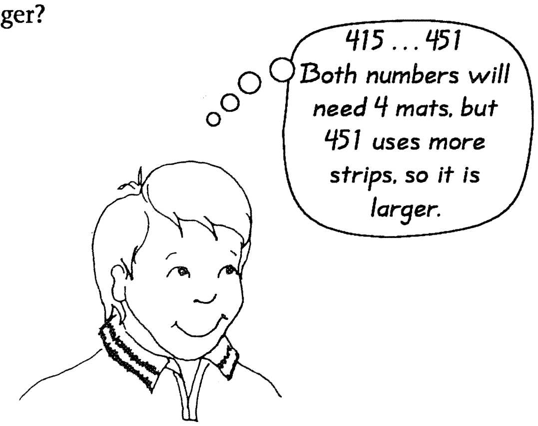
AKEEM *There are 5 mats.*

TEACHER *How many units are represented by the 5 mats?*

AKEEM *Each mat has 100 units. Five of them make 500 units.*

EXAMPLE 13: COMPARING NUMBERS

Think of the numbers 415 and 451. Which is larger?



EXAMPLE 14: ROUNDING NUMBERS

Think of the number 634. What collection of pieces that uses only mats comes closest to showing this number?

Offer your children plenty of opportunities to think about numbers in these ways. Encourage them to draw pictures or set out pieces to assist them in their thinking. You will enjoy seeing how well they do and how much their understanding of number concepts and place value has increased.

Decimals

In *Opening Eyes to Mathematics* children participate in several kinds of decimal activities:

- Throughout the year, many Contact lessons provide experiences that strengthen children's understanding of money. These experiences illustrate an important application of decimals.
- Decimal concepts and notation are introduced during the Insight lessons. Children learn that the models used to visualize place value and whole numbers are also helpful for thinking about decimals.
- The Money Decimal Records component of the Calendar Extravaganza serves to link children's understanding of money with the decimal models developed in the Insight lessons.

Modeling Decimals

In our numeration system, the concepts of place value and positional notation used to describe whole numbers apply also to decimals. For example, in 54.789 the place values of 5, 4, 7, 8 and 9 are ten, one, one-tenth, one-hundredth and one-thousandth, respectively. This number can be written in expanded form in several ways:

54.789 = 5 tens + 4 ones + 7 tenths + 8 hundredths + 9 thousandths

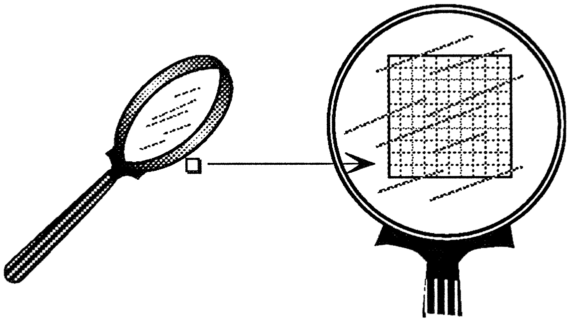
= 5 + 4 + .7 + .08 + .009

= 50 + 4 + $^{7}/_{10}$ + $^{8}/_{100}$ + $^{9}/_{1000}$.

Because of this, the visual models discussed earlier in this chapter can be extended to represent decimals. This is done through the following kinds of activities:

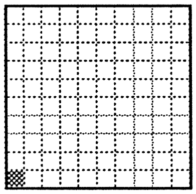
ACTIVITY 6

Show the children a unit square from the base ten pieces. Ask them to close their eyes and imagine looking at this unit through a magnifying glass for "hidden" surprises. While their eyes are closed, replace the unit with a larger one in which tenths and hundredths can be seen.



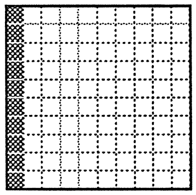
Use this "magnified" unit, which looks like a mat, to motivate a discussion of the following points:

a) The unit square has been subdivided equally into 100 smaller squares. Each smaller square represents one-hundredth of the unit square.



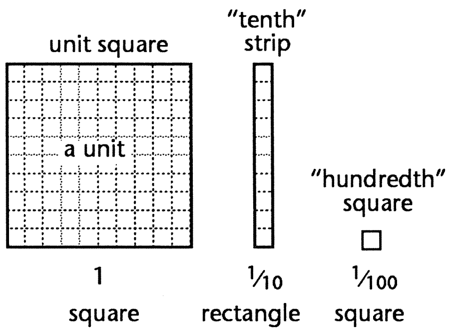
$^{1}/_{100}$ of a unit square

Ten of the smaller squares also form a strip that is one-tenth of the unit square. Ten of these strips make up the unit square.



This strip is $^{1}/_{10}$ of a unit square

b) When a unit square, a "tenth" strip and a "hundredth" square are arranged in the order shown here, their shapes alternate between squares and rectangles.



Each piece also is ten times as large as the piece to its right. These pieces model the numbers 1, 1/10 and 1/100, respectively.

ACTIVITY 7

Give each child a collection of "decimal" pieces (unit squares, "tenth" strips and "hundredth" squares). Children are usually quite willing to pretend and use base ten pieces for this purpose. (They imagine that a mat is a "magnified" unit.) Otherwise, the pieces can be cut from centimeter grid paper.

Ask the children to show 37 hundredths (or some other amount) with their pieces. Make a listing of the various collections that can be formed and identify the minimal collection.

units (1)	"tenth" strips (1/10)	"hundredth" square (1/100)	total number of pieces
0	0	37	37
0	1	27	28
0	2	17	19
0	3	7	10*

Each collection shows 37 hundredths of a unit.
The minimal collection is marked with an asterisk.

This activity strengthens the children's understanding of the regrouping that takes place when pieces are exchanged.

ACTIVITY 8

Form the minimal collections for other sets of pieces:

	1	1/10	1/100
a) 37 hundredths		3	7
b) 2 units, 3 hundredths	2	0	3
c) 37 tenths	3	7	
d) 2 units, 14 hundredths	2	1	4

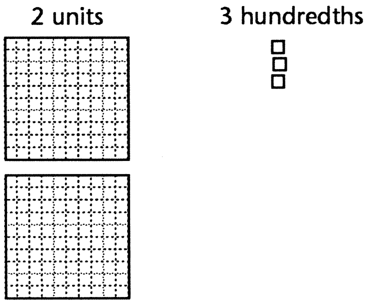
Introduce decimal notation as a means of representing these collections. For example, there needs to be some way of distinguishing the collections reported in entries a) and c) of the above chart. We make the distinction with a decimal point, although historically other ways were also used.

3 units, 7 tenths	vs	3 tenths, 7 hundredths
3⌊7	examples	⌊37
3,7	of historical	0,37
	notation	
3.7	decimal notation	.37

Notice that positional notation is used when describing these minimal collections. The decimal point serves as a punctuation mark that separates the units place from the tenths place. As with whole numbers, the digits tell how many times each different piece is used in the collection.

	1	1/10	1/100	decimal
a) 37 hundredths		3	7	.37
b) 2 units, 3 hundredths	2	0	3	2.03
c) 37 tenths	3	7		3.7
d) 2 units, 14 hundredths	2	1	4	2.14

Sometimes a zero is needed as a placeholder (entry b) above). A zero indicates the absence of a particular piece in a collection.



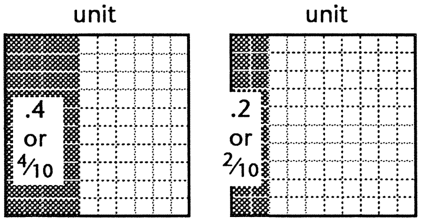
"These pieces are different sized squares. Since there's no rectangle in between, a zero is needed. There are no 'tenth' strips. This means 2.03 represents this collection."

ACTIVITY 9

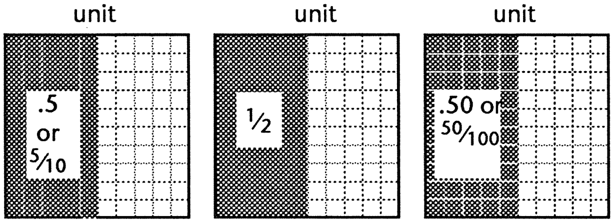
Have the children model various decimals by their minimal collections. In so doing, they develop a visual way for thinking about decimals.

decimal	minimal collection	standardly read as
a) .14	1 tenth, 4 hundredths	fourteen hundredths
b) 2.17	2 units, 1 tenth, 7 hundredths	two and seventeen hundredths
c) 1.05	1 unit, 0 tenths, 5 hundredths	one and five hundredths

With this model, children can compare decimals and observe how they are related to common fractions.

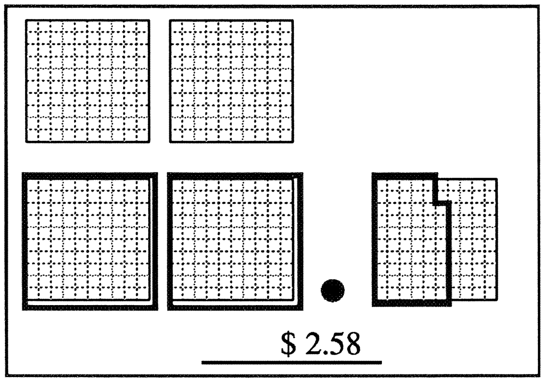


.4 > .2



In all of these, the same portion of the unit is shaded.
So .5 = $^5/_{10}$ = $^1/_2$ = .50 = $^{50}/_{100}$

We also use it in the Calendar Extravaganza to relate decimals to units of money (please refer to the Money Records component).



Related lessons

We have included several lessons in which children play games that use the models discussed in this chapter. Games such as Up-and-Back and Dizzie Lizzie are enjoyable ways to practice representing numbers and regrouping and to learn about the relative sizes of numbers. In addition, there are many Contact lessons where children apply their understanding of numbers to estimation and other problem situations.

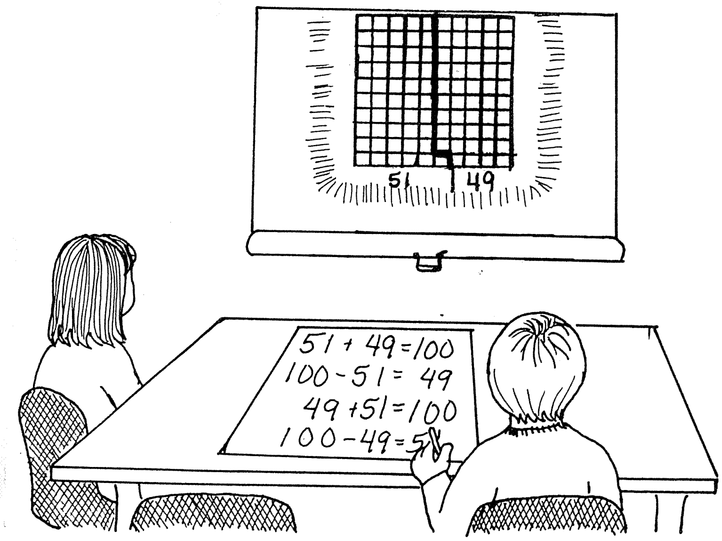
Conclusion

We have seen how the models discussed in this chapter have strengthened our children's perceptions of numbers and place value, regardless of ability level. We are confident that your children, too, will learn from them. They will develop understandings about images of numbers that will be useful now and in the years to come.

5 Numeration:

Addition and Subtraction

> ***Intuition about number relationships helps children make judgments about the reasonableness of computational results and of proposed solutions to numerical problems. Such intuition requires good number sense.*** *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 38*



As teachers, we recognize the importance of helping children acquire the computational facility and problem-solving skills needed to analyze numerical problems. This involves much more than teaching them paper-and-pencil procedures for computing with numbers. Children need to understand the meaning of number operations such as addition and multiplication, how numbers are related and how to estimate and evaluate answers. The visual models presented in Chapter 4 can assist them in these areas. This chapter discusses how to do this when teaching addition and subtraction.

During the Middle Ages an ongoing debate occurred among those who taught and used arithmetic. Some people preferred to work with an abacus and resisted the numerals we use today. Others saw the advantages of these numerals and developed procedures for computing with them. The latter group eventually prevailed and their methods of doing arithmetic evolved into the algorithms that have been taught in schools.

A similar debate is currently taking place, only this time the major issue is related to the role of calculators in the classroom. The widespread use of calculators and computers in the home and workplace stands in contrast to the pressure on many teachers to drill children on paper-and-pencil computation.

The availability of calculators lets you reduce the time spent practicing written arithmetic skills. You can then place greater emphasis on developing number sense and a conceptual understanding of number operations. With these, your children can successfully apply numbers to everyday situations. They will also recognize that they have many options for making needed computations. Some of these are mental arithmetic, estimation, calculators and paper-and-pencil techniques.

By doing the activities described in this chapter and the next, children will strengthen their sense of numbers; learn addition, subtraction, multiplication and division concepts with the help of visual models; and grow in their ability to use different calculating options.

Promoting number sense

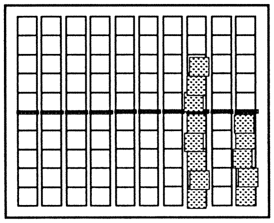
Number sense, like common sense, is hard to define specifically. It might be described as thinking about and working with numbers in an intuitively reasonable manner. People with good number sense confidently analyze numerical situations. They understand number relationships, make helpful estimates and judge the reasonableness of their answers. They are also able to select and use calculating options appropriate to the problem being solved. For example, they will likely recall the sum of 3 + 5, mentally compute 17 + 20 and use a calculator to determine exact answers for cumbersome computations.

We also feel that for young people trying to recall basic facts, number sense includes the ability to think in terms of tens. For example, one can construct an answer for 8 + 5 by making a 10 out of either the 8 or the 5 and thinking of the problem as 10 + 3. We call this *ten-ness* and illustrate it in the following lesson.

LESSON 1

TEACHER *One Halloween, Bill was given 8 pieces of candy at the first house and then 5 more at another house. How could we use the base ten pieces to show the candy collected at the 2 houses?*

MONA *(She uses a transparency of a ten-strip board and base ten pieces for the overhead.) The units of our counting pieces are like candy. Place 8 units in one column of the board and 5 in another like this.*



TEACHER *Now, how many pieces did he receive altogether.*

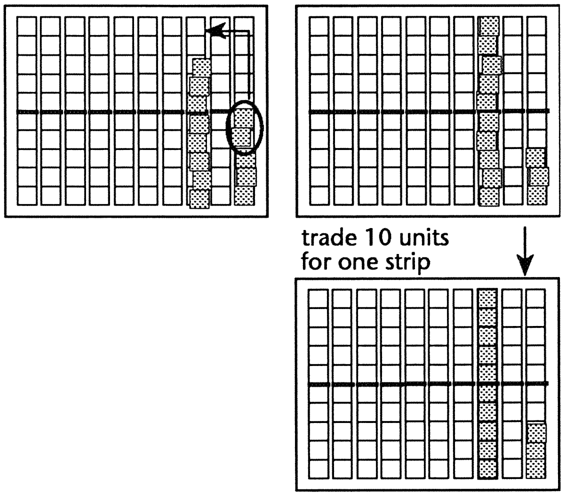
RYAN *You can count the units—8, 9, 10, 11, 12, 13! I wouldn't want to do it that way if the numbers were bigger!*

TEACHER *That's true—I wouldn't want to either! How else can it be done?*

TAD *I have a way.*

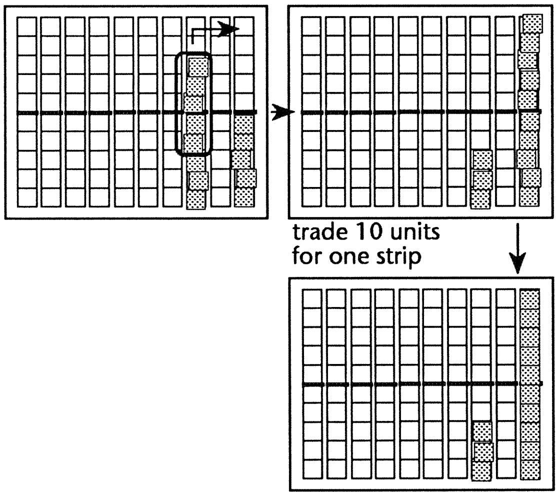
TEACHER *Please demonstrate at the overhead.*

TAD *I see that the 8 just needs 2 more units to become a strip. So I can make a strip of 10 out of the 8 by moving 2 units over. I can make the minimal collection by trading the 10 units for a strip. Now I have 1 strip and 3 units—that shows 13.*

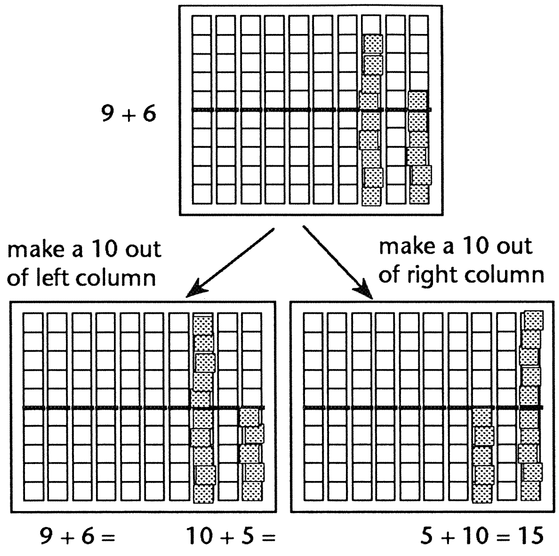


TEACHER *Thank you, Tad. (The stacks of 8 and 5 are recreated.) Does anyone have a different way of thinking about it?*

MONA *(She stands at the overhead.) Well, the 5 needs 5 more units to become a strip of 10. You could make a strip by moving 5 units over from the 8 and then trading. You'd still have 1 strip and 3 units.*



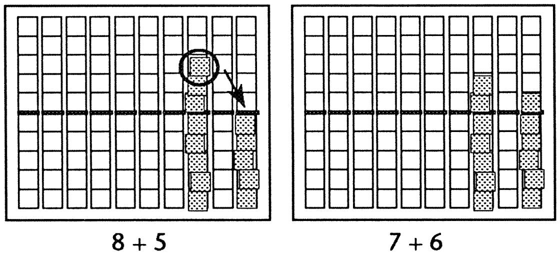
In this lesson, Tad and Mona demonstrated a sense of ten-ness by moving units to create a strip (or 10). This allowed them to form the minimal collection for 13 and thus construct an answer for 8 + 5 by thinking of the problem as 10 + 3. As your children work daily with base ten pieces, they will find thinking about making strips helpful for recalling basic facts. Here is another example of this.



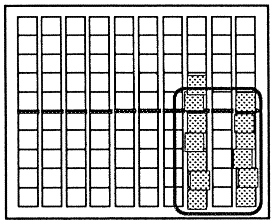
This sense of ten-ness is related to understanding place value and to forming minimal collections. Your children can use it when they compute with larger numbers.

Lesson 1 continues by asking the children to make columns of 8 and 5 units on individual ten-strip boards. The teacher challenges them to make other observations about numbers by looking at the columns of units. The children discuss their thoughts with one another and then demonstrate some of them at the overhead. Here are some possible responses.

a. "Move a unit and make columns of 7 and 6 like this."

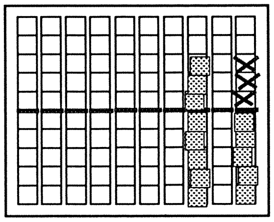


"Now here is a double—2 sixes, that's 12—and an extra unit. That makes 13."



Two sixes are 12, so 8 + 5 = 12 + 1 = 13

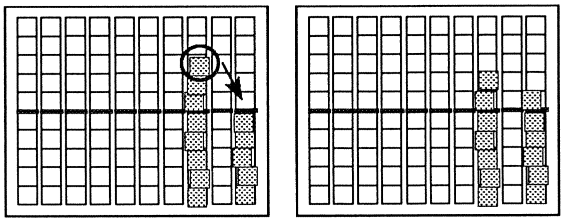
b. "I see the difference between 8 and 5. One column has 3 more than the other. I can show this by placing 3 more units on the one with 5."



The difference between 8 and 5 is 3. 8 − 5 = 3

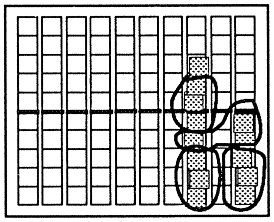
c. "I can make 1 strip and almost half of another. Altogether, 8 + 5 is closer to 1 strip than it is to 2. It's a long way from a mat, however!"

d. "The total number of units is odd. I can match all but 1 of them."



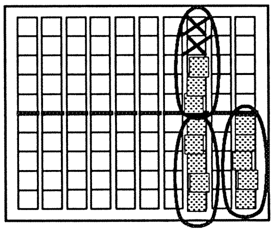
Thirteen is an odd number.

e. "I see groups of 3. Thirteen is 4 groups of 3 and 1 more."



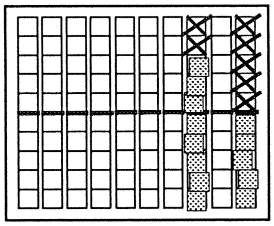
Four groups of 3 plus 1 make 13.

f. "If I put 2 more units in the column with 8, I can see 3 fives. So 13 must be 15 minus 2."



13 is 15 minus 2

g. "I filled up both columns and made 2 strips or 20. It took 7 more units to do this, so 20 is 7 more than 13."



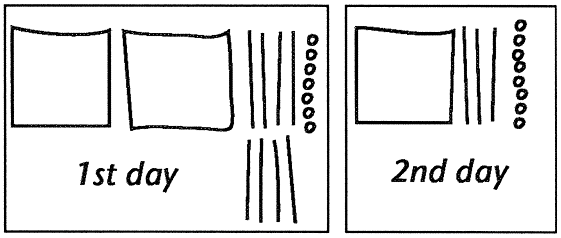
20 = 13 + 7

In this lesson, the children formed their collections on ten-strip boards (see Materials Guide). Our children find them very helpful when considering sums and differences that are less than 100.

The preceding chapter outlined activities that used base ten pieces for understanding place value concepts and forming mental images of numbers. Another way to use these pieces (or sketches of them) to promote number sense within your classroom is shown in this estimation lesson.

LESSON 2

TEACHER *This summer I took a trip. I have used base ten pieces to sketch a picture of the number of miles I drove on the first and second days. (The following illustration is displayed on the overhead.)*



Can you tell how many miles I drove on each day?

ANGELA *I can! You went 287 miles on the first day and 138 on the second.*

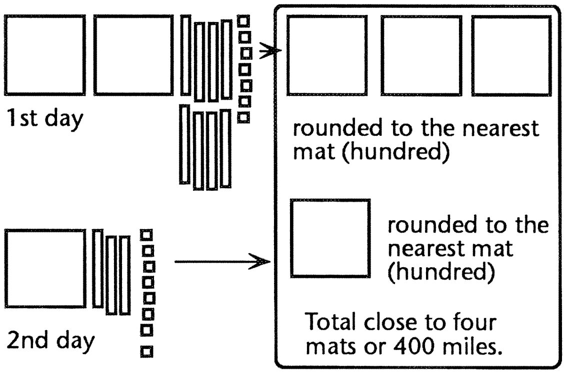
TEACHER *Thank you, Angela. Now I would like each of you to form these collections of pieces at your desks.*

The children set out pieces on their desks that show 287 and 138 units, respectively.

TEACHER *Using only mats, how would you estimate the total number of miles that I drove?*

SHAWN *That's easy! There are 3 mats, so you went around 300 miles.*

RICHARD *Wait a minute! I'd say 4 mats, because the first collection is closer to 3 mats than it is to 2. The second collection is closer to 1 mat. I think you drove closer to 400 miles.*



RUDETTE *I think I'd go with 4 mats, too. I see the 3 that Shawn saw. But if I put the strips together, there will be more than enough to make another mat. I'd say you drove more than 400 miles.*

TEACHER *Those are all useful ways to make estimates. Does anyone want to go for 500 miles?*

RUDETTE *I don't want to. There are enough strips to make 1 more mat, but not enough for 2!*

JIM *Well, 500 is an okay estimate, but the answer is closer to 400.*

TEACHER *How about 200 miles?*

PENNY *No, just like Shawn said, there are already 3 mats and that's 300 miles.*

TEACHER *There are many ways to make estimates. Can you think of a reason why 300 miles might be a useful estimate? What might be a reason for wanting to know that more than 400 miles were driven?*

The discussion continues with the children thinking about times when different types of estimates are appropriate. For example, they might observe that "more than 400" may be more useful if one is concerned about fuel consumption.

The children thought about the problem of Lesson 2 in different ways. Shawn's method of looking only at the largest pieces present is called *front-end* estimation. This is a helpful way of getting a sense of where an answer lies (the term itself needn't be added to the children's vocabulary). Rudette refined Shawn's answer by looking at the strips. Richard rounded each collection to the nearest mat (100) and then combined his results. Each of these procedures is helpful and leads to sound estimates. In a given situation, a person might find one estimate more useful than another.

As a general rule, encourage children to make estimates and to think about their answers. Sometimes, as in the above lesson, these processes may be identified as the main objectives of the activity. Ideally, though, your children will make estimates a natural part of all their work with numbers.

In many of the Contact lessons, children are asked to think about numbers in relation to the theme of the month. They look for and describe patterns, devise estimation strategies and create problems that apply numbers to situations in their world. Here are summaries of two of these lessons.

LESSON 3

Contact Lesson 38
Football—Money

You will need

- different catalogs that advertise footballs
- money feely boxes, record sheets and calculators

Has football mania taken over your classroom yet? Perhaps your children will want to buy a football to donate to the school.

In groups of four, have the children explore the manner in which department store catalogs are organized. In particular, have each group search for a football that suits its needs and lay out (and record) different combinations of money from their feely boxes that could be used to purchase the football

Teacher Tip: We like to have each group report to the class about its work. The report usually includes a description of the ball that has been selected, some of the money combinations that could be used to pay for the ball and some related story problems.

LESSON 4

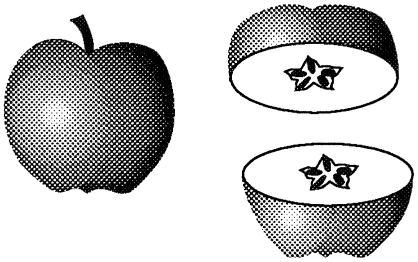
Contact Lesson 19
Apples—Extended Number Patterns

You will need

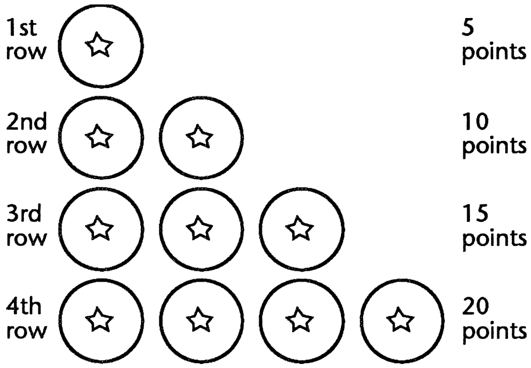
- Chapter 3, Patterns, and Chapter 6, Multiplication and Division
- an apple and a sharp knife
- a 4 × 4 sheet of white paper, crayons, scissors for each child
- a large sheet of bulletin board paper for display, glue

If you cut an apple against the grain and look at the inside of one of the parts, what type of design will you see? How many points will it have? What type of seashell will it look like?

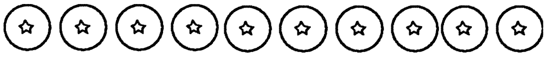
Answer these questions by cutting an apple as shown in the illustration.



Have the children recreate the apple's 5-point design by coloring a circular shape cut from paper. Glue some of these shapes on the bulletin board paper to create the sequence shown below; then identify the multiples of 5 points seen in each row.



Ask the children to visualize what the tenth row in this display will look like. Have volunteers build this row without building those in between and discuss reasons for their answer. The tenth row looks like this:



10th row

Now challenge the children to determine the total number of points that are in the tenth row. Have them share their thinking.

"I see 10 apples with 5 points each. So just add 10 fives (or multiply 5 × 10) to get the total. That's 50 points."

"Two apples have 10 points. I see 5 pairs of apples, so I counted 10, 20, 30, 40, 50. There are 50 points."

"Each row has 5 more points than the one above it. I started with the 25 in the fifth row and counted by fives until I hit the tenth row—25, 30, 35, 40, 45, 50."

This type of imaging may be extended: What does the 20th row look like and how many points does it have? How about the 33rd row? How many apples will be in the row that has 75 points?

Teacher Tip: Encourage the children to look for different ways to total the points in a row. Continue to ask, "Does anyone have another way to share?" or "Can you find another way?"

Activities to enhance understanding addition and subtraction

Most children have kept a running total of their scores in a game and are generally quite aware of who's ahead (and by how much) at the end. This indicates that addition and subtraction are already part of a child's world. By linking these experiences with a knowledge of place value, your children will grow in their understanding of these operations.

Begin by telling a variety of stories that provide a context for addition and subtraction. Invite your children to tell some, too. They will show special interest in stories that are about them—their soccer games, the buses they ride, the lunches they've brought from home, etc. Vary the level of difficulty and size of the numbers.

Have the children model the stories with base ten counting pieces and explain their thinking as they answer questions. As with place value, working with the pieces will promote a visual way of thinking about addition and subtraction. When the time comes, this work will make it easier for the children to understand written procedures for finding sums and differences.

LESSON 5

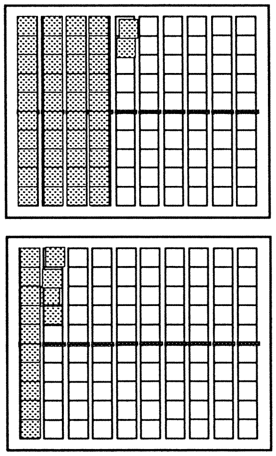
TEACHER *As I was reading the newspaper, I noticed that the Shelby County Rockets football team won the scrimmage last Friday night against the Trinity Bluejays. Yeah, Rockets!*

KRISTEN *I went to that game because my big brother is the quarterback; it was really great! My dad says we might win the state championship this year. He says Coach Pollett is the best coach ever!*

TEACHER *It appears that the Rockets are going to be a strong team this year! The score of the game was Shelby County, 42, and Trinity, 14.*

PAUL *Wow! We scored 42 points! That's great!*

TEACHER *With a partner, place your green counting pieces on your ten-strip boards to show 42 and 14.*



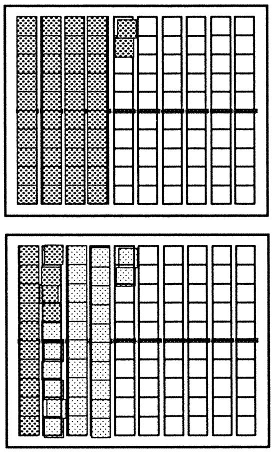
About how many more points did the Rockets score than Trinity?

MARCY *Well the 42 has 3 more strips than the 14 so the Rockets scored around 30 more points.*

TEACHER *I think that's a nice way to describe it, Marcy. Now let's find out an exact answer. How many more points did the Rockets score?*

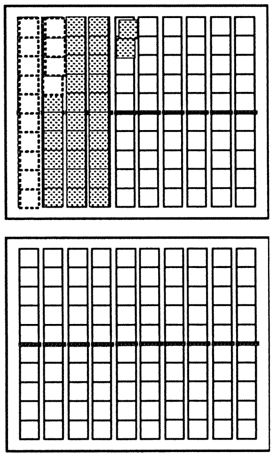
The students work for a time and demonstrate methods such as:

ELVINA *I made the 14 look like the 42 by filling in with yellow pieces until they looked the same.*



Then I counted the yellow strips and units. There were 2 strips and 8 units in the difference, so the Rockets scored 28 more points. That's pretty close to 30.

SETH *The Rockets are my favorite team! I picked up the pieces for the other team's 14 points and laid them on top of the 42. The pieces without any pieces on top of them show the difference.*



28 units are not covered. 28 is the difference between 42 and 14.

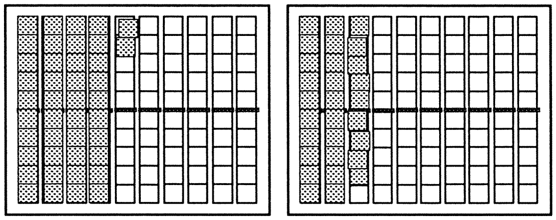
ISIAH *I imagined it in my head. I just started with 14 and counted by tens—14, 24, 34, 44. If the Rockets had 44 points, they'd be 30 ahead. So I counted back 2 to 42. They must be 28 points ahead.*

The teacher now poses a new problem.

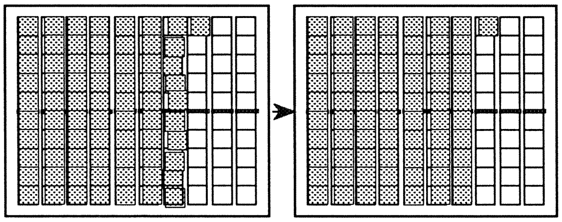
TEACHER *Suppose the Rockets score 29 points in this week's game. Can you use your pieces to show the total number of points they scored in these 2 games?*

The children work together on the problem and share their procedures.

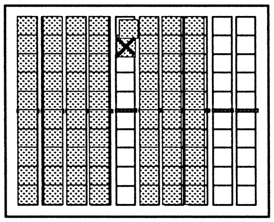
MARTHA *On our boards, we set out 42 points for the first game and 29 for the second.*



Then we put the pieces together and traded 10 units for a strip. That made 71 units in all.



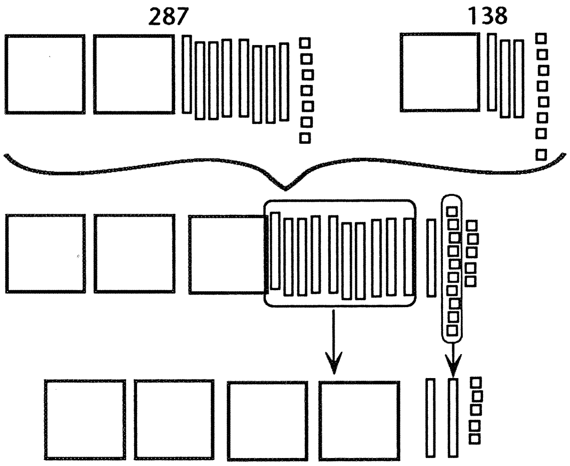
JAMES *We have discovered something really neat! When we went to set up the pieces, we already knew it would be an addition problem. So we looked for a short-cut. We laid down the 42 points for the scrimmage. Then, when we started to lay down the 29 points, we just took 1 away from the 42 and gave it to the 29. We figured that 42 and 29 is the same as 41 and 30. How about that! We still got 71 points!*



Modeling addition

The sum of two numbers can be modeled directly with counting pieces. The children set out collections representing the addends and combine them. Create the minimal collection for the entire set to see the answer. As an example, consider a problem discussed earlier in this chapter: On a two-day trip, the teacher drove 287 miles on the first day and 138 on the second. How many miles did she travel altogether?

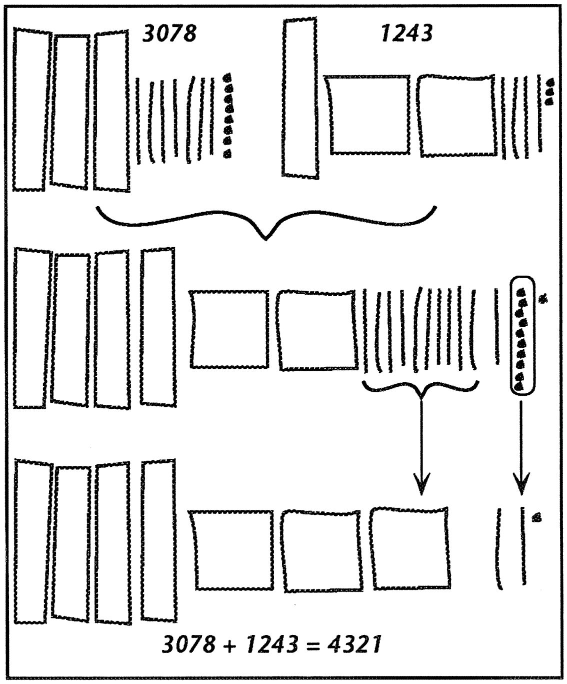
Previously, the children formed collections of 287 and 138 units and estimated the total mileage to be somewhat larger than 400. One way to arrive at an exact answer to the question is pictured here:



287 + 138 = 425
The teacher drove 425 miles.

Notice that the answer compares favorably with the estimate that was made.

As another example, here is a way of modeling 3078 + 1243 with sketches of base ten pieces.



3078 + 1243 = 4321

Modeling subtraction

There are two general kinds of problems that are addressed by subtraction: take-away and comparison (or finding differences). Here is an example of each kind.

Take Away: Angie had 6 pieces of candy and her brother ate 3 of them. How many pieces did she have left?

Comparison: Peter's score in a card game is 23. Colby's score is 18. Who is ahead and by how much?

When asked to model these situations with counting pieces, children will often proceed in different ways. The following discussion summarizes some of the more typical responses.

Take Away

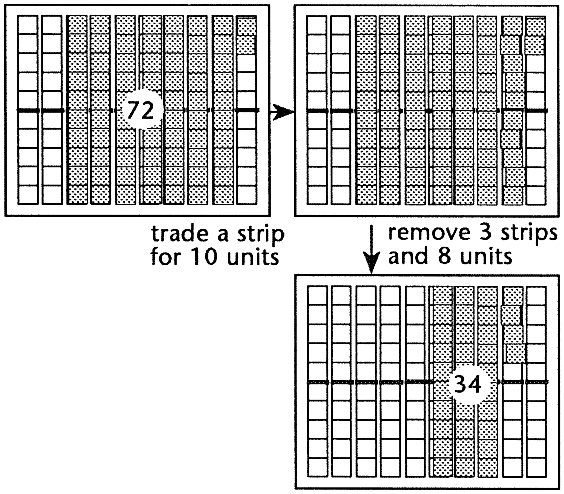
Children have experienced take-away situations many times in their lives.

EXAMPLE 1

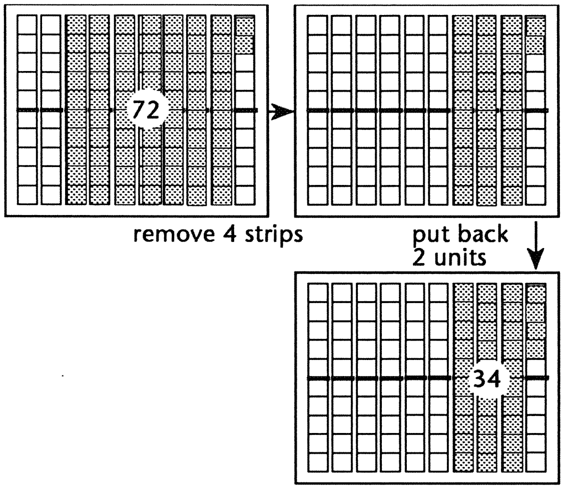
Susan started the day with 72 boxes of Girl Scout cookies and then sold 38 of them. How many did she have left?

This story may be modeled by placing 7 strips and 2 units on a ten-strip board and then removing 38 units.

ANTHONY *I thought there should be around 30 left, since I had 7 strips and about 4 had to be taken away. Before taking anything away, though, I traded a strip for 10 units. Then I removed 3 strips and 8 units. That left 3 strips and 4 units—Susan had 34 boxes left.*



JUANITA *I knew I had to take away about 4 strips, so I did just that! That meant 40 were taken away, so I put 2 units back. That gave me 3 strips and 4 units or 34. That seems reasonable.*



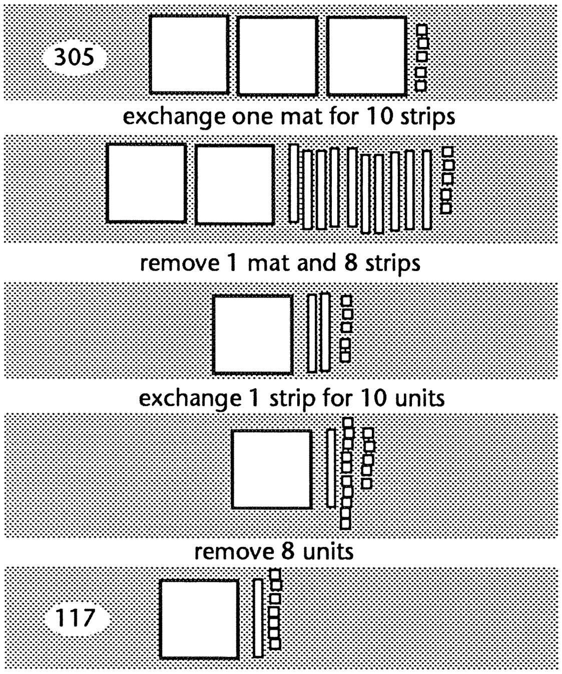
EXAMPLE 2

Jennifer's mom had $305 in her checking account and spent $188 of it. How much money was left in the account?

To model this story, children lay out 3 mats and 5 units to represent $305. They then look for ways to remove 188 units.

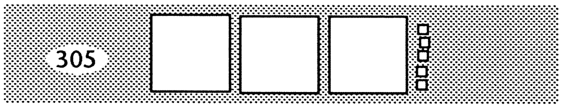
BILL *The answer is around $100 because about 2 mats have to be removed.*

ROSE *I wanted to remove a mat, 8 strips and 8 units. I exchanged pieces to do it.*

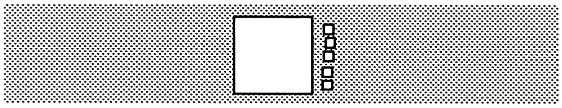


305 – 188 = 117 $117 were left

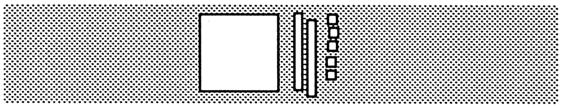
LI *I first removed a mat and 8 strips. I did it by taking away 2 mats and giving back 2 strips. Then I removed 8 units by taking away a strip and giving back 2 units.*



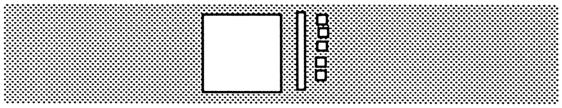
remove 2 mats



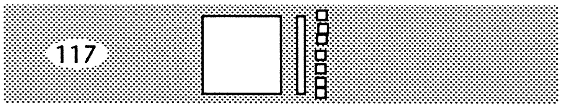
put back 2 strips



remove 1 strip



put back 2 units



The mom had $117 left.

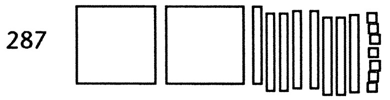
Comparison

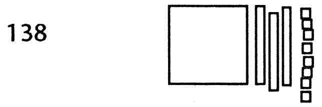
Children are also quite familiar with comparison situations. In Lesson 4, the children found the difference between two football scores in several ways. Another comparison is illustrated in Example 3 below.

EXAMPLE 3

On a two-day trip, the teacher drove 287 miles on the first day and 138 on the second. How many more miles did she drive on the first day?

Each child lays out collections of counting pieces to represent 287 and 138, respectively.

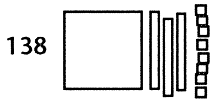




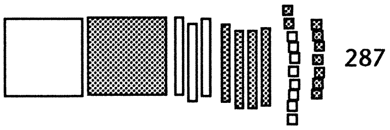
TED *The answer has to be around 200. I know 287 is close to 3 mats and 138 is close to 1 mat, so the difference is about 2 mats. (See illustration at bottom of this page.)*

RHONDA *I compared the mats and the strips; 287 has 1 more mat and 5 more strips than 138, so the answer should be around 150 miles.*

MILES *I added pieces to the 138 to make it look like 287. It took 1 mat, 4 strips and 9 units. So the teacher drove 149 more miles on the first day.*



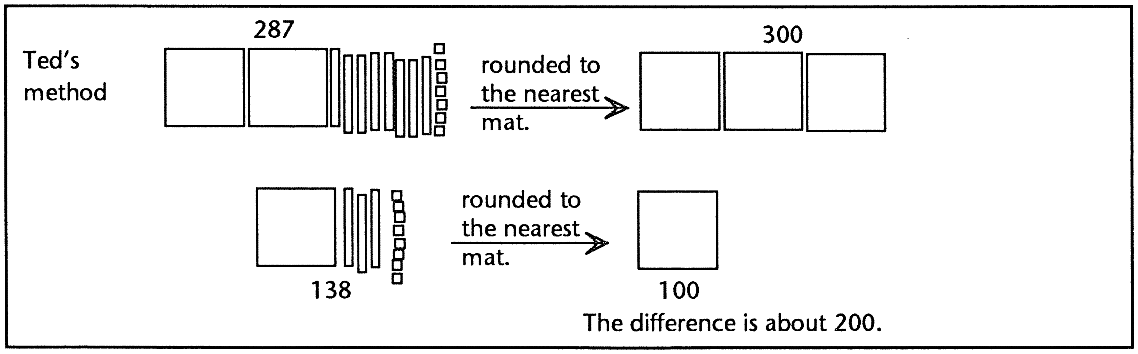
Add enough pieces to make 138 look like 287.



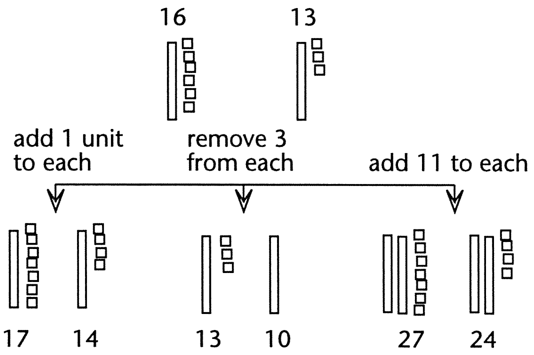
The difference is 149. 287 – 138 = 149 or
138 + 149 = 287

(Note: A variation of Miles' method is to match the pieces of the collections—perhaps by placing those of the 138 on top of those of the 287. In this case, one of the units will have to match with a unit found on a strip. The unmatched pieces will represent the difference between the collections.)

Thinking of subtraction in terms of finding differences is very powerful! Your children will be asked to do this all the way through algebra and beyond. It gives children some helpful options for visualizing and determining answers. Imagine, for example, collections of 16 and 13 units. The difference between these (3 units) is unchanged if you add 1 unit to each. In fact, this difference stays the same if you add (or remove) any equal amount of units from each collection.

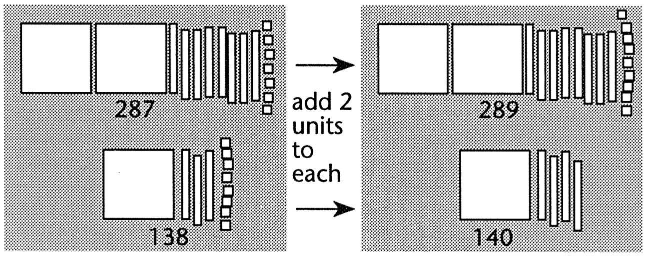


The difference is about 200.



The difference is always 3 units.

This makes it possible to change the form of a subtraction problem. In Example 3, above, the difference between 287 and 138 is not changed by adding 2 units to each collection.



The difference between 287 and 138 = The difference between 289 and 140 = 149

Some children may find it easier to compare the new collections of 289 and 140.

Notice how this idea may be applied to finding the difference between 1000 and 167. If 1 is subtracted from each number, the required difference of 833 can be found, without borrowing, by comparing 999 and 166.

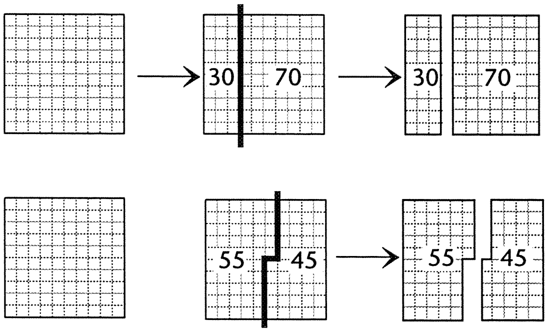
Relating addition and subtraction

In Example 3, Miles found the difference between 287 and 138 by adding 149 units to the collection representing 138. In Lesson 5, Elvina followed a similar procedure when comparing football scores of 42 and 14 points. She found the required difference (28 points) by adding units to 14 until she reached 42.

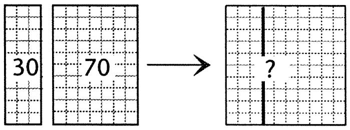
Both children's methods illustrate that addition is very closely related to subtraction. This "fact family" is another example of this:

9 + 5 = 14	14 – 9 = 5
5 + 9 = 14	14 – 5 = 9

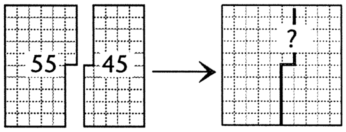
Children can explore this relationship by examining different ways to partition a collection of pieces into two smaller sets. The following illustration shows two ways of partitioning a mat.



Notice that, in addition, one knows the amounts in the smaller sets and seeks the total number in the combined collection.

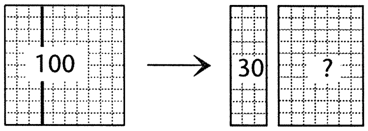


30 + 70 = ?

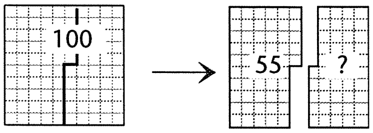


55 + 45 = ?

In subtraction, however, the number of units in the combined collection and the number of units in one of the smaller groups are known. The number of units in the other collection is then sought.



100 – 30 = ?



100 – 55 = ?

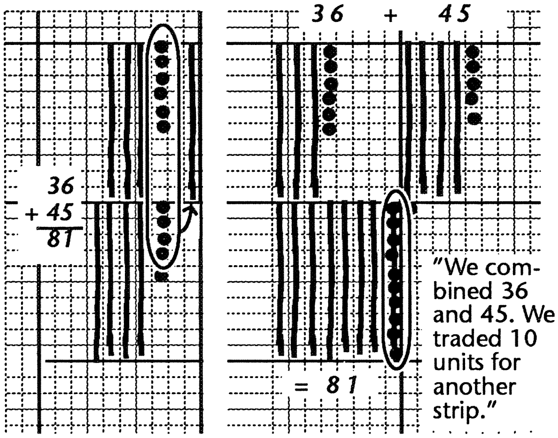
It is this relationship between these operations (often called an inverse relationship) that is the basis for the traditional way of checking subtraction by addition.

$$\begin{array}{r} 100 \\ -30 \\ \hline 70 \end{array} \qquad \begin{array}{r} 70 \\ +30 \\ \hline 100 \end{array}$$

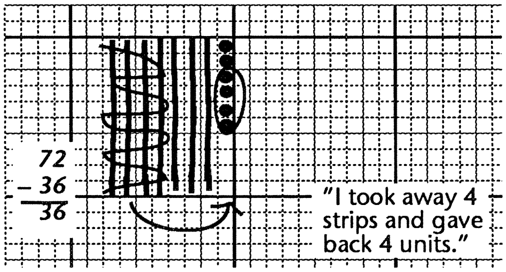
Sketching and symbolizing

All of the examples discussed earlier in this chapter can be modeled on grid paper or with sketches of counting pieces. The children will recognize these as options for representing addition and subtraction problems. They should feel free to choose the option they find most helpful in any given situation.

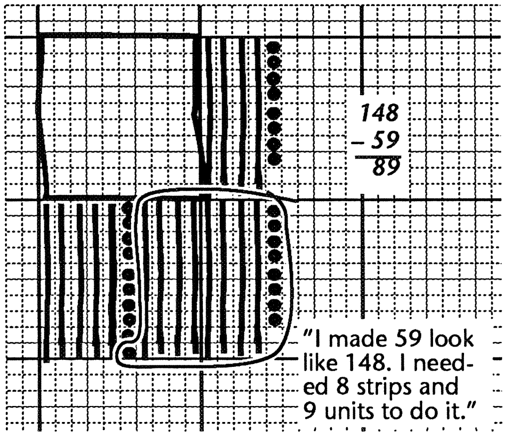
When your children are ready to make written records of how they add and subtract, let them do it in their own style. Below are some sketches our children made on base ten grid paper.



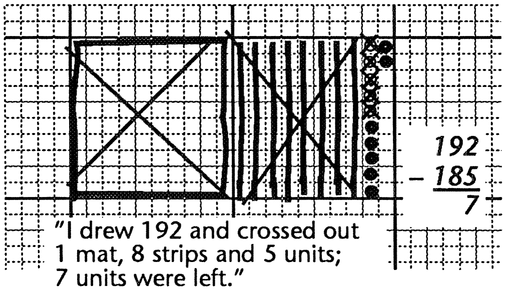
"We combined 36 and 45. We traded 10 units for another strip."



"I took away 4 strips and gave back 4 units."

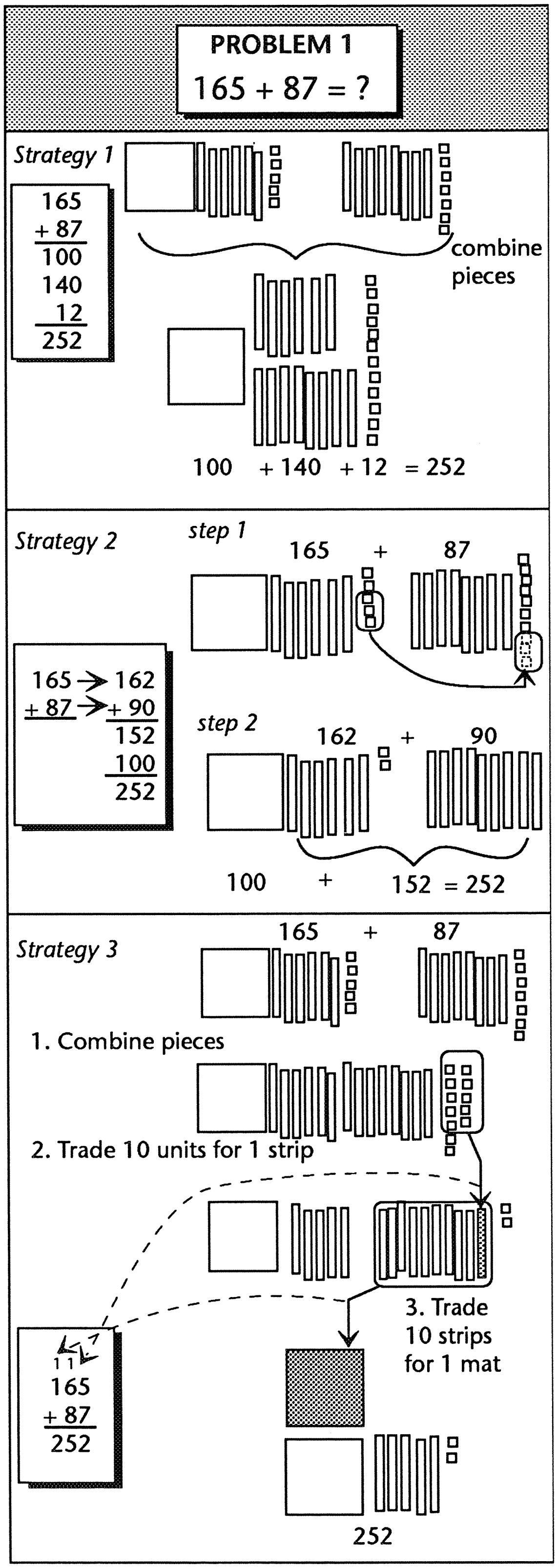


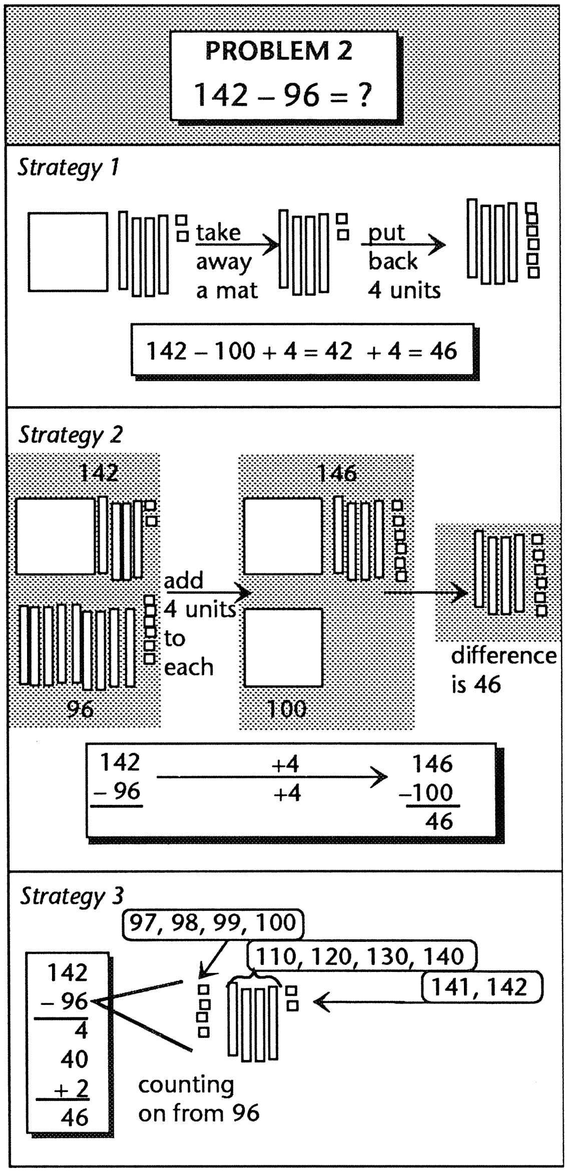
"I made 59 look like 148. I needed 8 strips and 9 units to do it."



"I drew 192 and crossed out 1 mat, 8 strips and 5 units; 7 units were left."

The children will eventually become comfortable with describing their work symbolically and will enjoy the challenge of inventing written procedures for this purpose. We are amazed at some of the ways our children record their results.





This is not to suggest that the traditional algorithms that we learned are inappropriate. But, we need to remember that using the traditional algorithms is just one of several effective methods for working with numbers. In fact, if the children don't bring up these algorithms, you may wish to do so.

Practice occurs naturally

In general, practice occurs as a natural part of working with counting pieces (or diagrams) and answering story questions. As children develop number sense, they will devise their own effective and meaningful methods for computing and solving problems.

Suggestions for added independent practice are included in many of the lessons. We also recommend using discussion cards that are designed to help children strengthen their sense of ten-ness. These cards are listed in the Materials Guide and strategies for using them are given in the lessons.

Finally, a good way to motivate practice is to have your children write and illustrate mathematics story books. Instructions for making these Tell It All books are included in the Materials Guide and a number of the lessons focus on this activity.

Using various calculating options

People usually perform calculations with some purpose in mind. The context of the problem being solved, together with the nature of the numbers used, often determine how a calculation is made. In some cases, some mental arithmetic that leads to a quick estimate might be sufficient (e.g., deciding if one has enough money to buy items at a store or enough gas to reach a destination). For computations that are more complex, a calculator is most commonly used. Even in the latter instances, people find it helpful to make estimates that permit them to judge the reasonableness of answers.

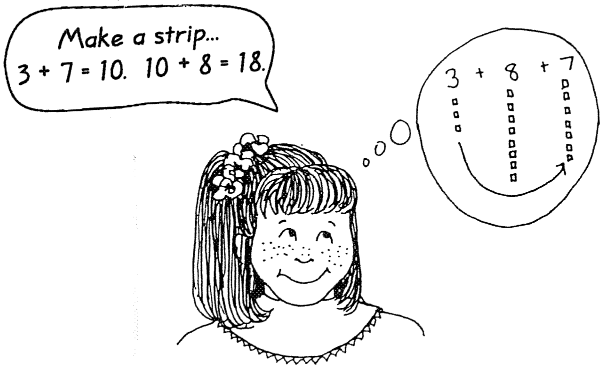
Traditional paper-and-pencil algorithms are still valid ways to determine answers. However, the degree to which they need to be emphasized in the classroom has lessened considerably. Certainly, speed is no longer emphasized as it once was. Also, as has been discussed, there are other written procedures that can be used and that children might prefer.

In general, try to intersperse mental arithmetic and estimation activities throughout your numeration lessons. Children will become more adept in mental arithmetic and estimation and at recalling basic facts as they learn more about place value, develop a sense of ten-ness, increase their awareness of number relationships and make use of visual models.

While there are no best methods or hard-and-fast rules for making estimates or mental calculations, children often discover ways to use any special number combinations or relationships that might be present in a problem. This was demonstrated in the estimation activity described in Lesson 2 of this chapter and is illustrated in the following examples. Children enjoy sharing their discoveries and procedures and, in so doing, learn more about these calculating options. In fact, our children often say, "The best way is the way that is best for ME!"

EXAMPLES OF MENTAL CALCULATIONS

a. 3 + 8 + 7



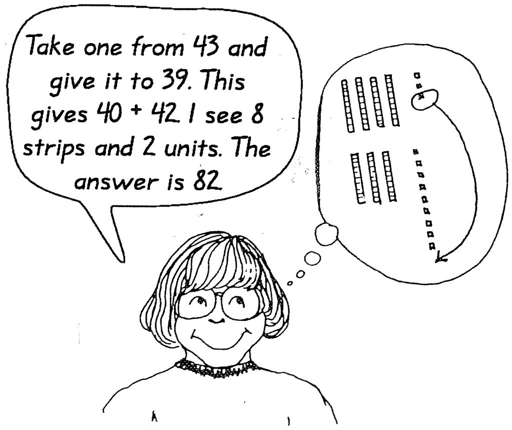
b. 18 + 8

"Take 2 from the 8 and give it to the 18. That makes 20 and 6, or 26."

"Add 10 (or a strip) to 18. That makes 28. Now take 2 away to get 26."

"Make a strip out of the 8 by taking 2 from 18. Now you have 16 + 10 or 26."

c. 39 + 43



"Think of adding 4 strips to 39 to make 79. Now 3 more make 82."

"Imagine adding 4 strips to 43. Now that makes 83, but that is 1 too many. So take 1 away and you have 82."

d. 400 + 400

"I see 8 mats—the answer is 800."

e. 60 + 50

"I see 11 strips. They make a mat and a strip, or 110."

"I know 50 + 50 is 100. Ten more makes 110."

"Take 40 from 50 and give it to the 60. You'll get 100 with 10 extra. The answer is 110."

"Six and 5 is 11. But they are strips so you have 11 tens. Ten of these make 100. You have 100 + 10 or 110."

f. 16 – 9

"16 – 10 is 6. Give back 1 to make 7."

"This difference is the same as 17 minus 10. Seventeen is 7 more than 10, so the answer is 7."

g. 82 – 68

"Count up from 68. You'll need 2 to make 70, then 10 more to get to 80 and 2 to reach 82. The answer is 2 + 10 + 2 or 14."

"Take 7 strips (or 70) away from 82. That gives 12. But you've taken away 2 extra units. So add 2 on to the 12. That makes 14."

h. 1100 – 700

"11 – 7 is 4. But they are hundreds. The answer is 400."

"I imagine adding 300 to 700. That makes 1000. Then another 100 is needed to reach 1100. So the difference is 400."

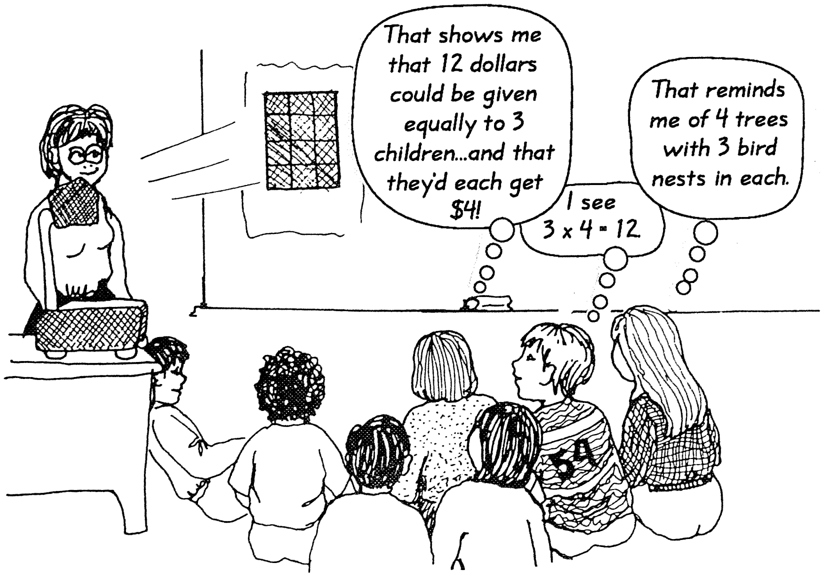
"I see 11 mats; if I remove 7 of them, 4 are left. That's 400."

Allow your children access to calculators whenever needed. Also, encourage them to think about the use of calculating options as they solve problems. How can the answer be estimated? Is an estimate sufficient for answering the question or is an exact answer needed? What seems to be an appropriate way to perform the required calculations? Should they be done mentally, with paper-and-pencil, or with a calculator? By considering questions such as these, your children will become confident problem solvers.

6 Numeration:

Multiplication and Division

Understanding the fundamental operations of addition, subtraction, multiplication and division is central to knowing mathematics. ... Children with good operation sense are able to apply operations meaningfully and with flexibility. Operation sense interacts with number sense and enables students to make thoughtful decisions about the reasonableness of results. *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 41*



Chapter 5 described activities that motivate a conceptual understanding of addition and subtraction. In those activities, children model and apply these operations, become aware of the relationship between them and learn about their properties. In the words of the National Council of Teachers of Mathematics, the children are gaining "operation sense".

To help children develop operation sense relative to multiplication and division, the lessons of *Opening Eyes to Mathematics* provide many hands-on experiences and use visual models that help children understand and apply these operations. This chapter focuses on the general principles behind these lessons.

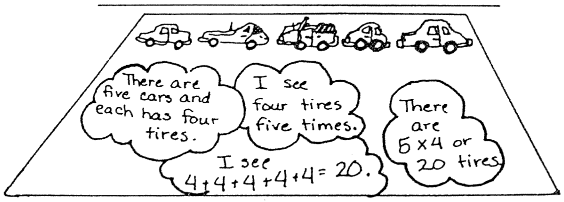
Introducing multiplication and division

Multiplication and division are two mathematical operations that often cause difficulty for students. Even after their school career is far behind them, people often recall struggling to learn and use times tables. The problem situations that require one to multiply or divide, however, are encountered frequently in everyday life. For example, one might estimate or calculate the area of a rectangle, how far they could travel on 18 gallons of gas when averaging 23 mpg, the cost of 6 half-gallons of milk priced at $1.37 each, the miles per gallon statistics for an auto trip, or the amount of paint needed to paint a room.

In all likelihood, your children have had previous experience with multiplication and division concepts. They may have counted by twos or threes to find the total points made in a game or to determine the number of cookies on a tray. They've had plenty of opportunities to share items and usually know when they have—or have not—received their fair share. These experiences provide a natural context for introducing multiplication and division.

Begin by engaging your children in a "multiples" search. Have them look for sets that illustrate multiples of a number. Begin the search in the classroom and later extend it to the whole school building, to magazines and newspapers, and to their homes. Here are some things that might be noticed.

Have children make displays of the objects brought in from home and describe the multiples they see. Encourage the children to describe their discoveries in their own words and do not be concerned with "correct" language and symbols. Their descriptions will often be in terms of repeated additions, with familiar symbols used occasionally. The following illustration shows possible ways to describe groups of tires.



Modeling multiplication

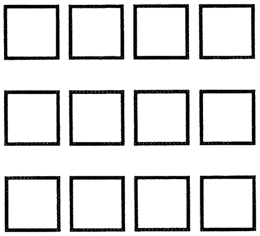
The results of the "multiples" search offer a natural setting for modeling multiplication. Square tile (we use the units of the base ten pieces) are especially useful for this purpose, as illustrated in the following lesson.

LESSON 1

TEACHER *Larry's display shows 3 cars, each with 4 tires. I'd like you to represent these sets of tires with your tile. Imagine that each tile represents 1 tire. How many tires are there altogether?*

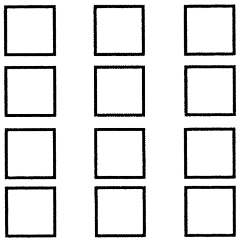
TEACHER *(After a short time has elapsed.) Would someone like to show their model at the overhead?*

ISIAH *(He comes to the overhead.) Sure! I see 3 groups of tile with 4 in each. Each tile represents a tire. There is a group for each car. There are 12 tires altogether.*



TEACHER *Thank you, Isiah. Does anyone have another arrangement to share at the overhead?*

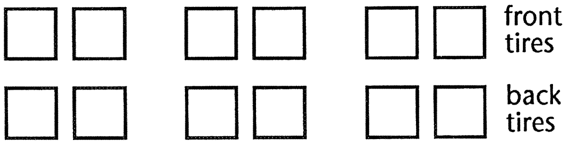
MONICA *I saw the same groups but I arranged them differently.*



I also see 4 groups of 3 in this arrangement—that's still 12 tires.

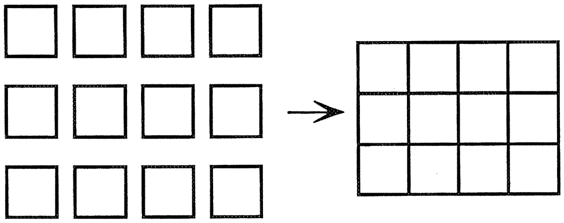
ISIAH *Mine and Monica's are alike—turn mine and you have hers!*

JOSH *I have another way. I placed the tile like this. I thought of the tires like they are on a car. See these are the front tires and these are the back ones.*



I still see 3 fours or 12 tires altogether.

TEACHER *That's nice, Josh. Now, I'm going to build Isiah's way again at the overhead and would like you to do the same at your desks. Look at what happens when we move the tile together like this.*

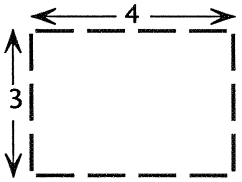


MORRIS *We get a rectangular array. It looks like one that was made when we counted by fours.*

TEACHER *Yes. It's also like the ones made in Today's Array. Can you still see the 3 groups of 4?*

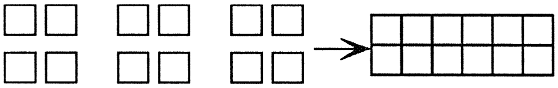
ALICE *Yes! And if I turn my head sideways, I see 4 groups of 3 as well. Its area is 12 square units.*

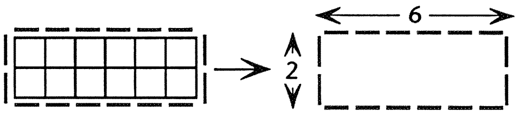
NATALIE *(She uses linear units to show the dimensions.) The dimensions are 3 and 4. It's a 3 by 4 array.*



TEACHER *We can also call it a 4 by 3 array.*

JOSH *Look, you can make a rectangular array out of my way, too! (He lays out his model again at the overhead.) I can push the tiles into a rectangle like this.*

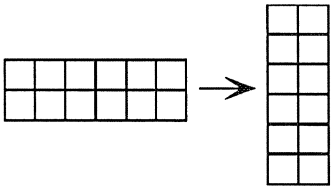




TEACHER *Yes, you have just made a 2 by 6 rectangular array with your 12 tile. What can be seen in Josh's array?*

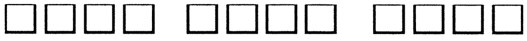
JOSH *Well, I think of the top row as the 6 front tires and the bottom as the 6 back ones. I see 2 rows with 6 in each.*

ANDY *Turn Josh's array like Monica did earlier. (He does so at the overhead.) Now I see 6 groups of 2.*

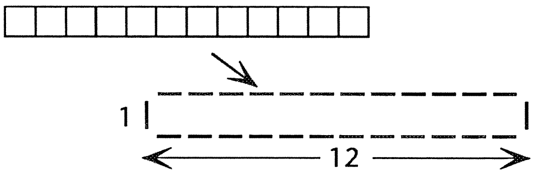


I guess that's like pairing a front and a back tire together in each group.

JUDY *Hey! I made another one! May I show? (She comes to the overhead) I arranged my tile like this at first. I thought of each of these groups as a car with 4 tires.*



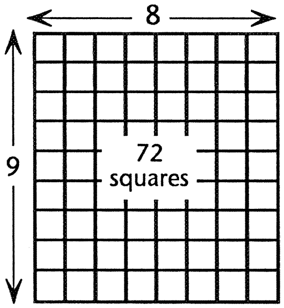
Then I pushed them together and I got a long, thin rectangle.



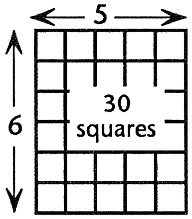
I guess this could be called a 1 by 12 array! That's like laying all the tires out in a line.

The children discuss further Judy's array and then model other multiplication and division stories with their tile.

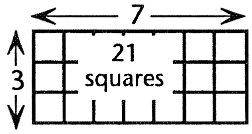
In Lesson 1, the children formed rectangular arrays using 12 square tile. In this program, arrays of this sort are used to model multiplication and division concepts (see Insight Lessons 78 and 79 and the Today's Array component of the Calendar). As examples, the arrays in the following illustration demonstrate that 9 x 8 = 72, 6 x 5 = 30, 7 x 3 = 21 and 17 x 8 = 136, respectively.



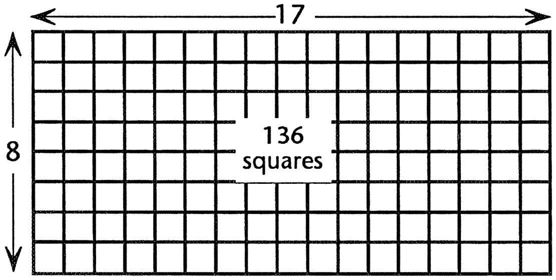
9 by 8 array (or 8 by 9)



6 by 5 array (or 5 by 6)

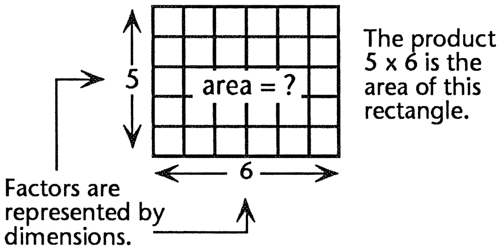


3 by 7 array (or 7 by 3)



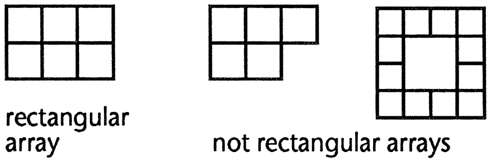
8 by 17 array (or 17 by 8)

This model is sometimes called the *area model of multiplication* because products can be represented by areas of rectangles and factors by the corresponding dimensions of the rectangles.

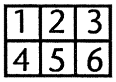


In this model of multiplication, it is important to distinguish the area of a rectangular array from its dimensions. Here is a summary of these concepts:

- **Rectangular array:** an arrangement of square tile that form a rectangle.

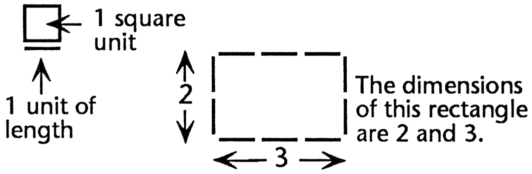


- **Area of a rectangular array:** the number of square units used to build the array. In the earlier examples, each tile represents a square unit.

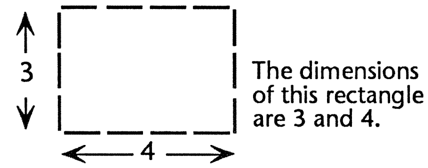


The area of this rectangle is 6 square units.

- **Dimensions of a rectangular array:** the length of adjacent sides of the array. Dimensions are measured in terms of units of length (linear or line units). In the picture below, the unit of length is the edge of a square tile.

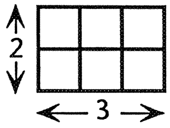


The dimensions of this rectangle are 2 and 3.

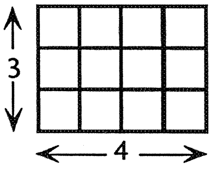


The dimensions of this rectangle are 3 and 4.

- **Identifying a rectangular array:** an array is often referred to in terms of its dimensions.



A 2 by 3 (or 3 by 2) array.

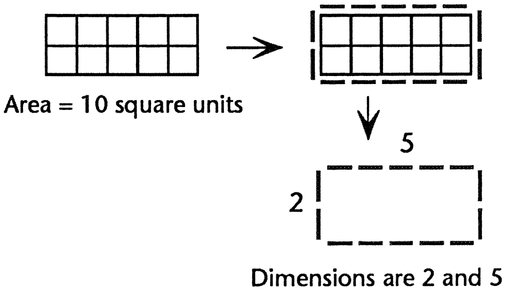


A 3 by 4 (or 4 by 3) array.

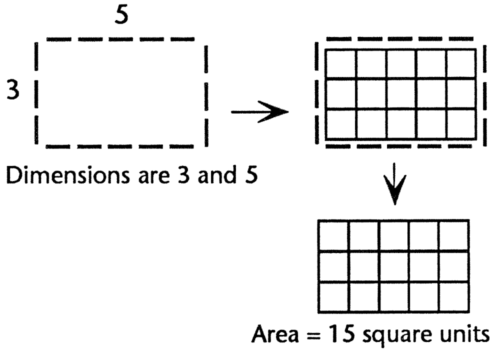
Note: In reporting a numerical value for an area or dimension, it is important that the unit of measure is known. This will generally be clear from the context. For example, if it is reported that the area of a rectangular array is 12, it is understood that the unit of measure is a unit square.

You can help your children clarify their understanding of area and dimension with exercises such as:

- Form an array with 10 tile on a piece of paper or on the overhead and make an outline of it that shows the number of linear units along each edge. Remove the tile and identify the dimensions of the array.



- Form the outline of 3 by 5 rectangular array, using linear units that have been cut to a length equal to the side of a square tile (see Materials Guide). What are the dimensions of this array? Now use tile to form the complete array and identify the area. What is the area of the array?



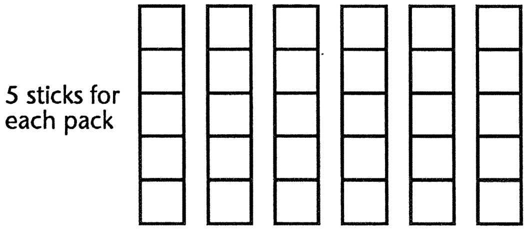
When discussing exercises of this sort, try to gain insight into how your children are thinking by asking them to describe exactly what they are counting when reporting an area or dimension.

Modeling products of whole numbers with rectangular arrays is closely tied with the traditional view of multiplication as repeated addition. In Lesson 1, for example, the children were able to see both 3 sets of 4 and 4 sets of 3 when looking at a 3 by 4 rectangle. This is also a visual way of observing the commutative nature of multiplication.

Modeling division

The "multiples" search can also motivate an awareness of division situations. While discussing the children's displays, pose questions that illustrate both the sharing and the grouping methods of division. Ask them to model these problems with square tile.

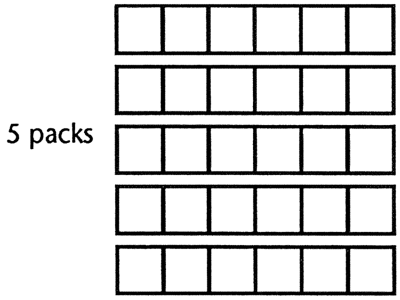
Question 1 (grouping): Joan's display shows packs of gum that have 5 sticks each. Suppose she has 30 sticks. How many packs does she have?



30 ÷ 5: grouping method

"There are 30 sticks of gum. Each tile represents one stick. If each pack has 5 sticks, there will be 6 packs."

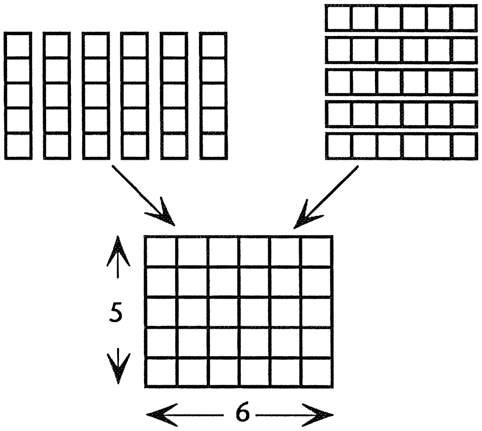
Question 2 (sharing): Suppose Joan wishes to divide her 30 sticks of gum evenly into 5 packs. How many sticks would go into each pack?



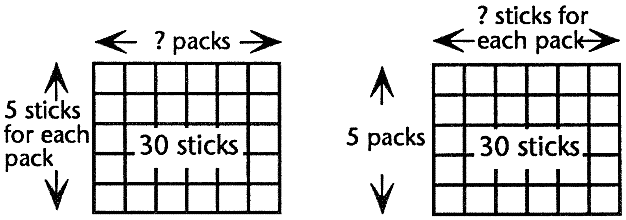
30 ÷ 5: sharing method

"There are 30 sticks of gum. Each tile represents one stick. If I divide them evenly into 5 packs, each pack will have 6 sticks of gum."

Like multiplication, both types of division situations can be represented by a rectangular array. The tile used in the two illustrations above can be moved together to form a 5 by 6 array.



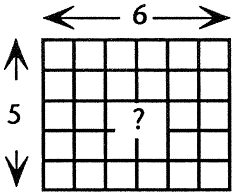
In terms of arrays, the division problems of Questions 1 and 2 can be modeled by a rectangle that has area 30 and one dimension 5. The required quotient is the other dimension of this rectangle.



For each question, 30 ÷ 5 is the missing dimension.

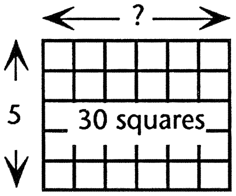
A single rectangular array therefore models the following types of questions:

Multiplication: What is 5 times 6?—or—What is the area of a 5 by 6 rectangle?



5 x 6 is the number of squares

Division: What is 30 divided by 5?—or—A rectangle has an area of 30 and one dimension of 5. What is the other dimension?



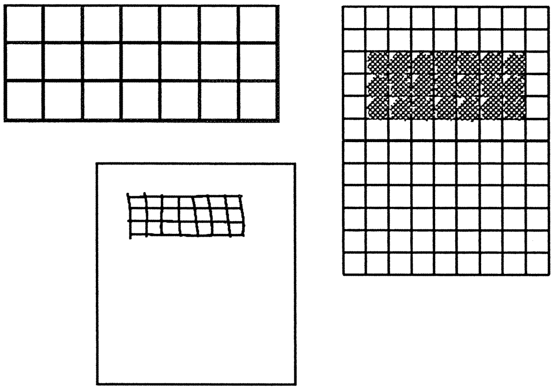
30 ÷ 5 is the missing dimension

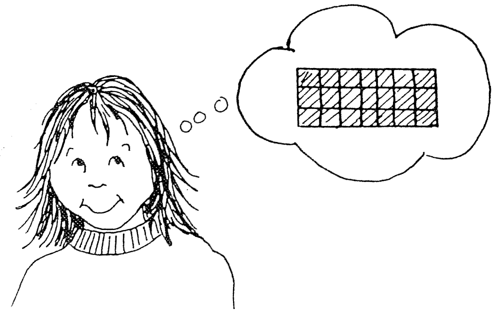
This is a useful way to think about the inverse relationship between multiplication and division. It is this relationship that makes it possible to use multiplication to check a division problem.

$$\begin{array}{r} 6 \\ 5\overline{)30} \end{array} \quad \text{check: } 5 \times 6 = 30$$

Using rectangular arrays in the lessons

Rectangular arrays can be built directly from base ten counting pieces, drawn on grid paper, sketched freehand or pictured mentally.





Give children experiences of representing arrays in these different ways. They can then best judge which is most appropriate for a particular situation.

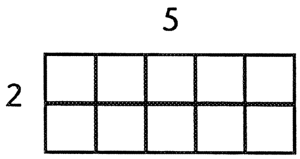
In the Insight lessons, we have suggested a progression of activities that use arrays to develop multiplication and division concepts. Pace the lessons according to the needs of your children. Here is a summary of these activities.

A. Stories and applications

Lesson 1 and the previous discussion illustrated how arrays can be connected with problem situations that are familiar to children. Emphasize this association as much as possible. For example, after the multiples search has been completed, ask the children to make up multiplication and division stories that are modeled by arrays. A favorite activity for doing this is "Rolling for Rectangular Arrays".

EXAMPLE 1

The children roll a 2 and a 5 with number cubes. They then form a 2 by 5 array with tile and create a story related to it.



"There were 2 boys with 5 pennies each. How many pennies did they have altogether?"

"Two frogs ate 5 flies each. How many flies were gobbled by the hungry frogs?"

"There were 10 boys and they wanted to form 2 teams that have the same number of players. How many were on each team?"

"There were 10 cookies and Sue wanted to put 2 on each plate. How many plates does she need?"

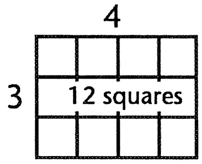
After having more experience with multiplication and division patterns and concepts, the children can analyze problems you pose or they suggest (see Lessons 3 and 4 of this chapter). They will also enjoy creating books of stories that are bound together for all to enjoy. (Directions for making these books are included in the Materials Guide.)

B. Number patterns and basic facts

Once your children have been introduced to modeling products and quotients by building rectangular arrays, they will be ready to explore the patterns formed by multiples of numbers. Prepare displays of arrays that show the first 10 multiples of 2 through 9, respectively. Have the children shade each of these sets of multiples on a hundred's matrix and discuss the patterns that are created. (See illustration of number patterns on following page.)

Using the foregoing diagram of a hundred's matrix, pose questions such as: What patterns do you notice? If the patterns are extended, would 80 be shaded? How about 69? or 132? How can you tell?

This is an appropriate time to introduce the language and symbols associated with multiplication and division.



Multiplication

$$\underbrace{3 \times 4}_{\text{factors}} = \underbrace{12}_{\text{product}}$$

Division

$$\underbrace{12}_{\text{dividend}} \div \underbrace{3}_{\text{divisor}} = \underbrace{4}_{\text{quotient}}$$

$(3)(4) = 12$

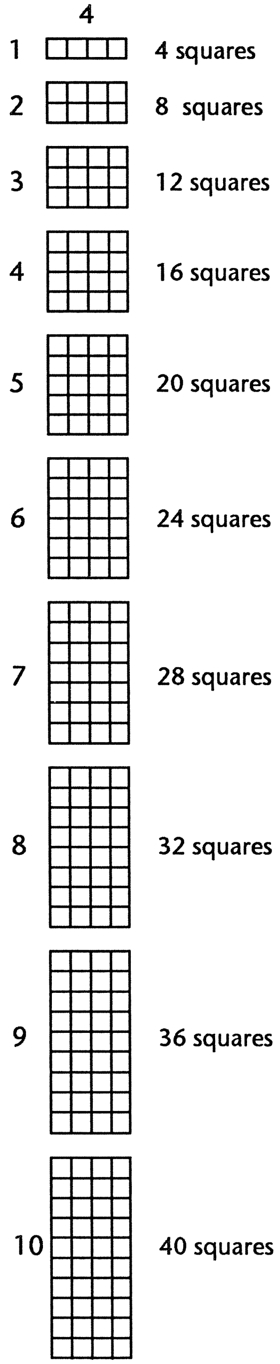
$3 \bullet 4 = 12$

$12 = 4 \times 3$

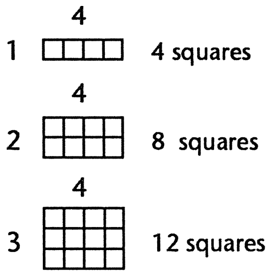
$$\begin{array}{r} 4 \\ 3\overline{)12} \end{array}$$

By attaching number statements to their arrays, your children will have pictures of the basic facts of both operations. (See illustration in the second column of the next page.)

Number patterns illustrated



1	2	3	**4**	5	6	7	**8**	9	10
11	**12**	13	14	15	**16**	17	18	19	**20**
21	22	23	**24**	25	26	27	**28**	29	30
31	**32**	33	34	35	**36**	37	38	39	**40**
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



$1 \times 4 = 4 \quad 4 \div 1 = 4$
$4 \times 1 = 4 \quad 4 \div 4 = 1$

$2 \times 4 = 8 \quad 8 \div 2 = 4$
$4 \times 2 = 8 \quad 8 \div 4 = 2$

(continued through 4 by 10 array)

Encourage the children to organize this work into booklets. These will then be available for reference purposes in later lessons. We like to have them make individual books of matrix patterns and a classroom book of array displays of multiples of 2 through 10. The children use these booklets, as well as calculators, to help them predict how these patterns can be extended.

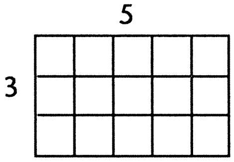
The children will also enjoy singing number pattern songs, such as *Two's Shoes* and *Slicing by Fives,* to remind them of arrays and think about multiples. These are on the cassette tape that accompanies the *Musical Array-ngements* booklet of this program. (Tape and booklet are available from MLC.)

c. Visually determining products and quotients

In the area model for multiplication, the number of square units (or area) in a rectangular array represents the product of two numbers. Products can be determined by using various visual techniques which children will enjoy discovering. This is exemplified in Lesson 2 in which children search for "hidden" arrays contained within a 3 by 5 rectangle.

LESSON 2

TEACHER *Please build this array with the units from your counting pieces. (She builds a 3 by 5 array at the overhead while children build their arrays.)*



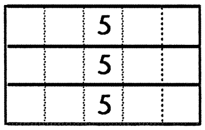
TEACHER *What multiplication problem is represented by this array?*

MIA *The dimensions are 3 and 5, so the array shows 3 times 5.*

CURTIS *It takes 15 units to make up the array.*

TEACHER *How did you decide that there are 15 units, Curtis?*

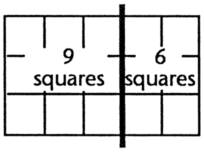
CURTIS *(He stands at the overhead.) I can see 3 sets of 5, so I counted by fives—5, 10, 15. So 3 × 5 = 15.*



There are 15 squares, so 3 x 5 = 15.

TEACHER *I see. Can anyone see the 15 in another way?*

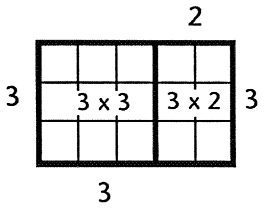
LORNA *If you split the array like this, you can see 9 and 6. (She demonstrates at the overhead.) So, 9 plus 6 equals 15.*



9 + 6 = 15 squares

TEACHER *Thank you, Lorna. You and Curtis both found some arrays that are hidden in the 3 by 5 rectangle. What are the dimensions of Lorna's hidden arrays?*

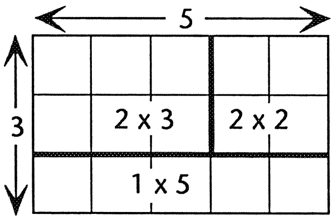
TRICIA *She made a 3 by 3 and a 3 by 2. That's 3 × 3 = 9 and 3 × 2 = 6.*



3 x 5 = 3 x 3 + 3 x 2
= 9 + 6
= 15

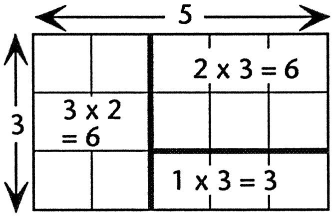
TEACHER *(Arranges the pieces into a 3 by 5 rectangle once more.) Yes, she did. Can anyone find any other hidden arrays?*

The lesson continues with the children describing other hidden arrays. Here are some other possibilities.

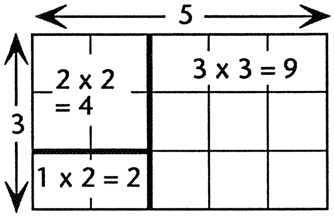


2 x 3 = 6
2 x 2 = 4
1 x 5 = 5

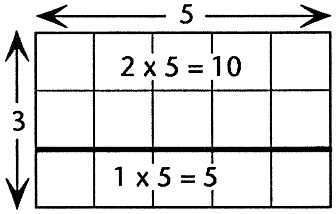
3 x 5 = 6 + 4 + 5 = 15



3 x 5 = 6 + 6 + 3 = 15



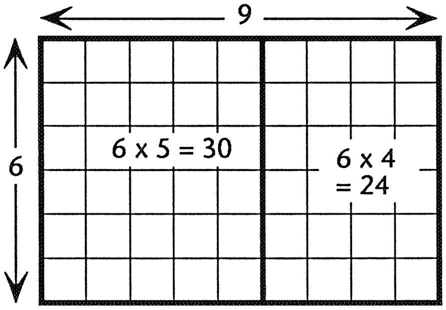
3 x 5 = 4 + 2 + 9 = 15



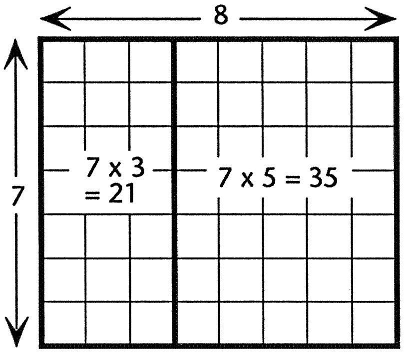
3 x 5 = 10 + 5 = 15

Notice that, in each case, the 3 by 5 array has been subdivided into smaller rectangles. The product of 3 × 5 is then found by summing the areas of the smaller rectangles.

This process for determining products is very important in mathematics—it demonstrates the *distributive property of multiplication.* It provides your children with a visual way of learning and recalling basic multiplication facts. Here are two examples of this:



6 x 9 = 6 x 5 + 6 x 4
= 30 + 24
= 54



7 x 8 = 7 x 3 + 7 x 5
= 21 + 35
= 56

It also lays the foundation for helping them find larger products. Challenge them, for example, to find collections of base ten pieces that can be arranged into rectangles. Have them total the areas of these rectangles and describe the products that are represented. (See illustration at the bottom of this page.)

In the following lesson, the children combine the use of arrays with some mental arithmetic to estimate and compute the cost of a field trip.

LESSON 3

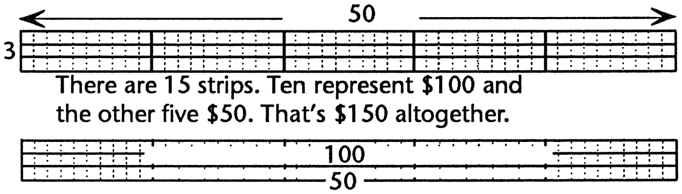
TEACHER *Our grade is going to have to raise enough money to go on a field trip at the end of the year. It's going to cost each child $3 dollars and there are 54 children. About how much money will we need to collect?*

The children work in groups for a short time.

TEACHER *Could we please have some group reports?*

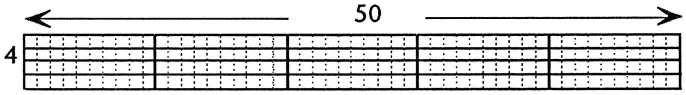
GROUP 1 *We decided that more than $150 are needed. There are about 50 children. Instead of paying the $3 all at once, each child could pay $1 at a time. Every time they do this, it gives $50 They'd do this three times. That gives 50 + 50 + 50 = 150.*

GROUP 2 *We thought of a 3 by 50 rectangle. There will be 3 groups of 5 strips. That's 3 times 5 or 15 strips. Each strip is like $10. Ten of them will be $100. Add in the other 5 tens and there will be $150. This will not be enough money since there are 54 children, but it will be close!*



There are 15 strips. Ten represent $100 and the other five $50. That's $150 altogether.

GROUP 3 *We estimated that about $200 would be collected. There are about 50 kids and if each pays about $4, that would give $200. We chose 4 because we could make imagine 4 × 50 or 20 strips in a rectangle. Ten strips make 100, so 20 of them will make 200.*



20 strips or $200 in all

We aren't sure if that is more or less money than is needed. We used fewer kids but a larger price!

TEACHER *These are all interesting methods. Would we need to know exactly how much money is to be collected?*

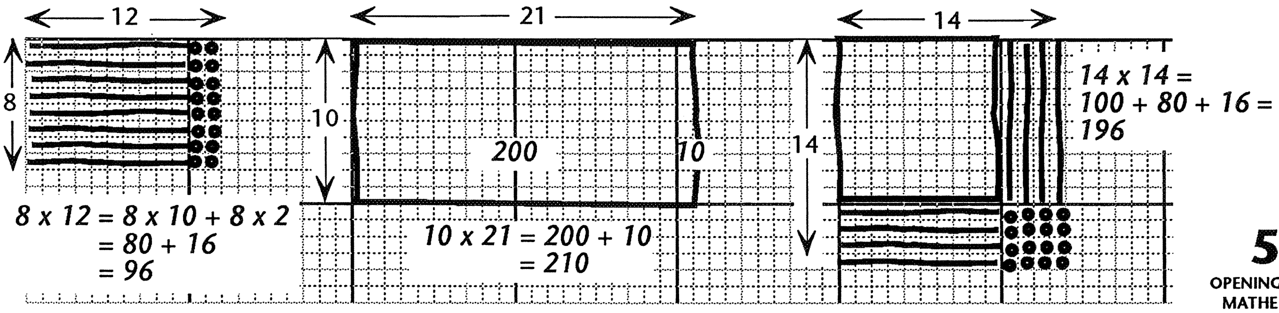
JUANITA *Well, I think so. That way you'd be sure each person will be able to go.*

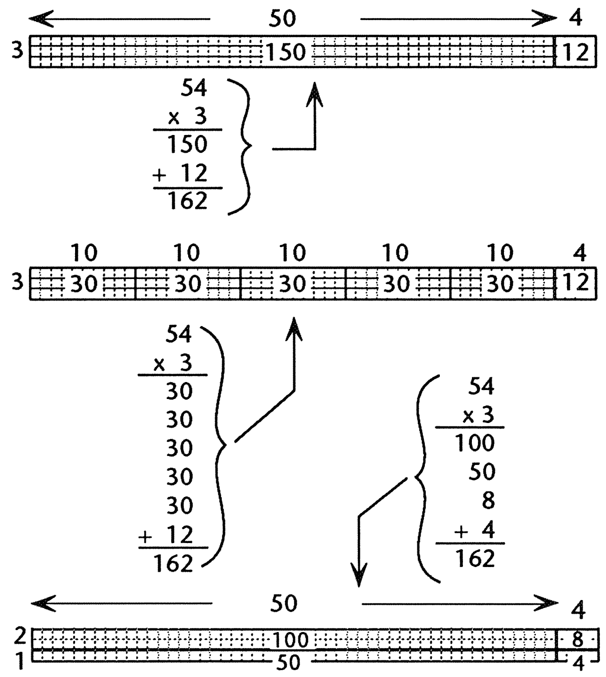
MYRNA *Couldn't you just check each person's name off as they paid? That way, you'd know that everyone's paid, but you wouldn't need to count all the money.*

LING *I think the people who are getting the money would want to know the exact amount. That way they'd be sure they got the right amount.*

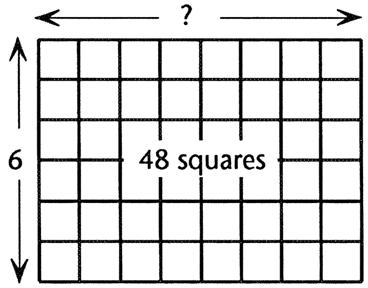
TEACHER *What is the exact amount? See if you can find some ways to calculate it.*

The lesson continues with the teacher encouraging the children to find as many ways as they can for answering the question. As time permits, these ways are shared and the exact answer is compared with the previous estimates. Some possible methods (see next page):





Rectangular arrays can also be used to find quotients in a visual manner. We saw earlier how arrays such as the one shown below are helpful for learning basic facts.



The missing dimension is 8, so 48 ÷ 6 = 8 $\begin{array}{r}8\\6\overline{)48}\end{array}$

Larger quotients, including those with remainders, can be found in a similar way.

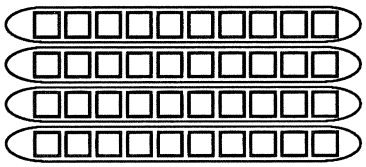
LESSON 4

TEACHER *Jennifer's mom has 68 cookies that she wants to give to her 4 neighbors. If she wants each of her friends to receive the same number of cookies, about how many cookies should each person get?*

ANDREA *That's a lot of cookies! I'd like to be one of her friends! You could do it by fours. Take 4 cookies at a time and give 1 to each friend. Do this until all the cookies are gone. I know you could do it more than 10 times because that would only use 40 cookies. Each person will get more than 10 cookies!*

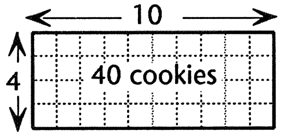
TEACHER *Could you show that with tile?*

ANDREA *(She stands at the overhead.) Each tile is a cookie; take 4 at a time and imagine giving each neighbor 1 of them. You can do this 10 times. (Andrea begins to lay out tile.)*



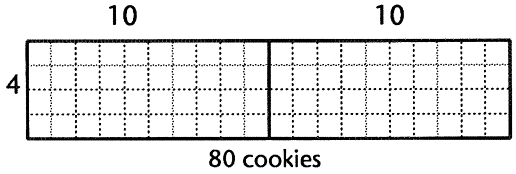
"Each neighbor gets at least 10 cookies. That uses 40 cookies."

JOAN *You can use strips to show what Andrea's done. You'll get a 4 by 10 array. (Joan makes and describes the following array.)*



"The 4 reminds me of the 4 neighbors. Each gets 10 cookies."

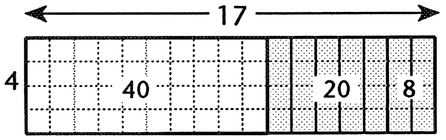
MILLIE *Yes, but there aren't enough to give each person 20. You would need 80 cookies to do that! (She comes to the overhead.) If you make Joan's array longer with 4 more strips, you'll get 80. That's too many.*



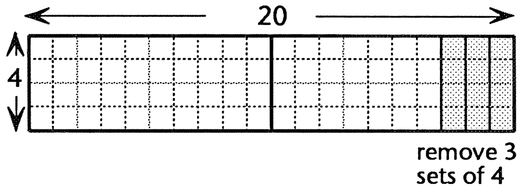
TEACHER *So each person should receive between 10 and 20 cookies.*

The teacher now asks the children to determine the exact number of cookies each person receives. Here are two methods that could be used.

JEREMIAH *Each person gets 10 cookies for sure. That uses 40 of them. Begin with a 4 by 10 array and add more fours until all 68 cookies have been used. Five more fours will use 20 more—that uses 60 of the cookies—now 2 more fours uses the other 8. Looks like each person gets 17 cookies. That makes sense and agrees with our estimates.*

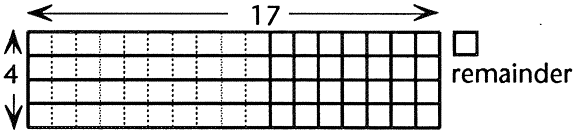


DIANE: *Start with a 4 by 20 array. That's 80 cookies. Now remove sets of 4 until 68 cookies are left. That will make 4 rows of 17 cookies.*



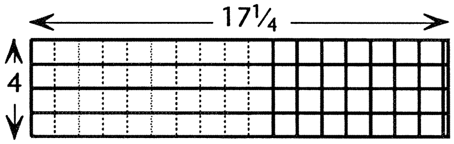
Both of these methods lead to an array that contains 68 square units and has one dimension of 4. The other dimension is 17 and this represents the quotient 68 ÷ 4.

Had there been 69 cookies instead of 68, the same model could be used. There would be 1 extra cookie as shown here.



69 cookies

The extra cookie could also be split evenly among the 4 friends so that each receives $17^{1}/4$ cookies.



69 cookies

As with addition and subtraction, when your children are ready to make written records of how they find products and quotients, encourage them to do it in their own style, using their visual models as guides. It will be fun for them to invent procedures for describing their work symbolically; they may surprise you with some of their methods. Here are samples of what we have observed. (See illustration A.)

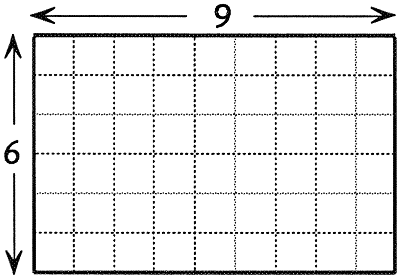
The traditional paper-and-pencil procedures for multiplying and dividing also take on meaning when developed from visual models. (See illustration B on the following page.) Some of the children's methods may suggest these procedures. If not, you may wish to present them as other alternatives.

Area models for multiplication and division appear throughout mathematics, so the children will encounter them again in higher grades. Children soon learn that the same kinds of pictures can be used to describe problems involving large numbers. (See illustration C on the following page.) They will find themselves solving both basic and difficult problems with understanding and confidence.

Illustration A

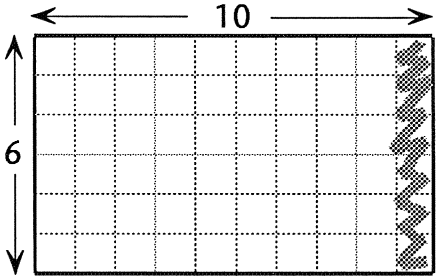
Problem: 9 x 6 = ?

Strategy 1



"I see 6 rows of 9 each." 6 x 9 = 54

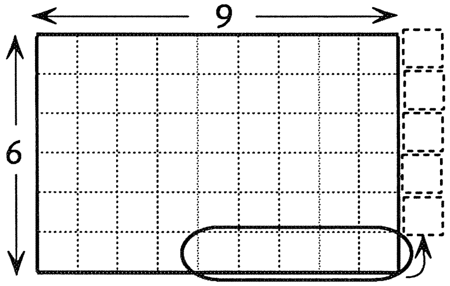
Strategy 2



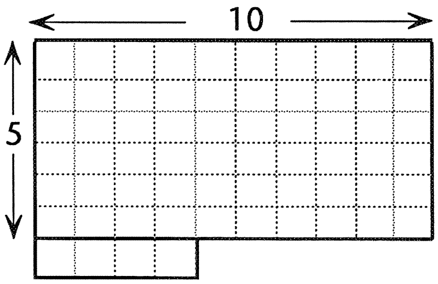
"I thought of 6 times 10 and took away 6."

$$\begin{array}{r} 6 \times 10 = 60 \\ -6 \\ \hline 54 \end{array}$$

Strategy 3



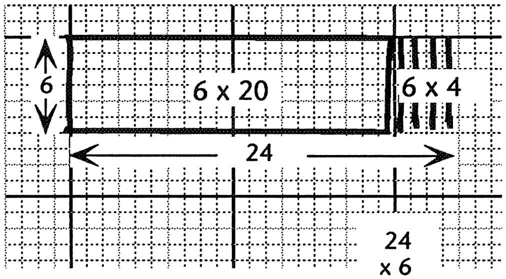
"I imagined making rows of 10. I could make 5 of these. The bottom row then is left with 4."



9 x 6 = 5•10 + 4 = 54

Illustration B

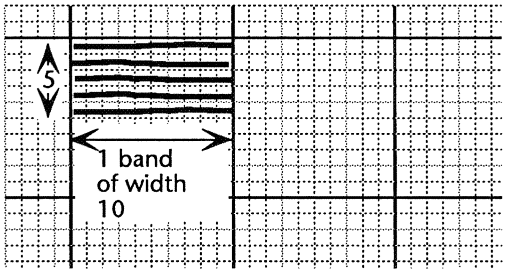
Problem: 24 x 6 = ?



24
x 6
24 (6 x 4)
120 (6 x 20)
144

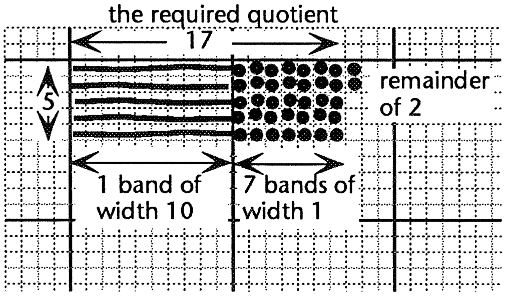
Problem: 87÷ 5 = ?

Step 1



5)87
50 (1 band of width 10 uses 50 units)
37 37 units are left

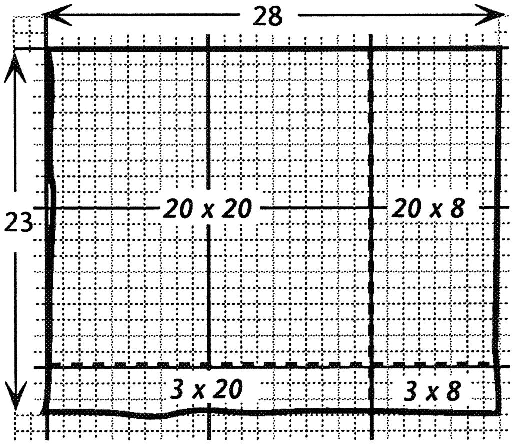
Step 2



17 (The required quotient is a band of width 17)
5)87
50
37
35 (7 bands of width 1 uses 35 units)
2 (2 units are left)

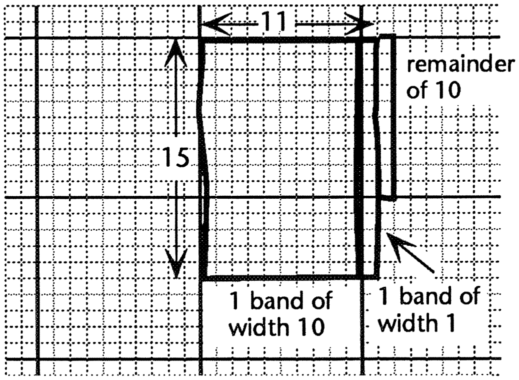
Illustration C

Problem: 23 x 28 = ?



23
x 28
24 (3 x 8)
160 (20 x 8)
60 (3 x 20)
400 (20 x 20)
644

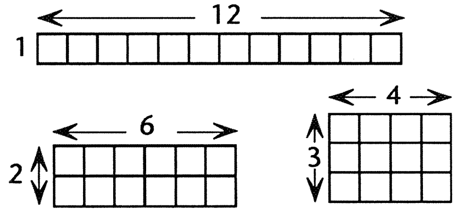
Problem: 175 ÷ 15 = ?



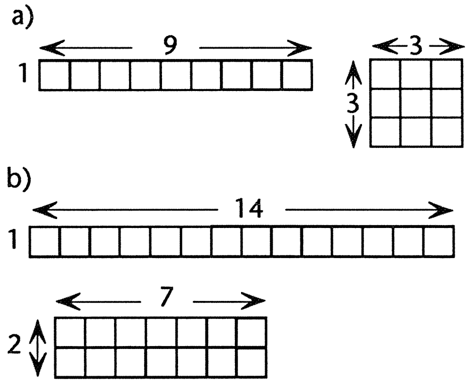
11 (The required quotient)
15)175
150 (1 band of width 10 uses 150 units
25 leaving 25 units to be distributed)
15 (1 band of width 1 uses 15 units)
10 (10 units remain)

d. Special numbers and factors: primes and composites; odd and even numbers; 0 and 1

In Lesson 1 the children discovered that it was possible to form several rectangular arrays with 12 tile.



Nine and 14 are examples of other amounts of tile that can be arranged into an array in more than one way.



9 and 14 are composite numbers.
They each have more than two factors.

The above arrays show that 12, 9 and 14 each have more than two factors.

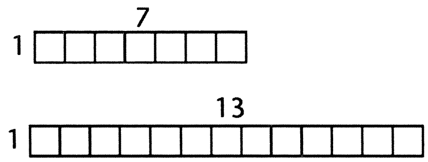
1 x 12 = 12
3 x 4 = 12
2 x 6 = 12

1 x 9 = 9
3 x 3 = 9

1 x 14 = 14
2 x 7 = 14

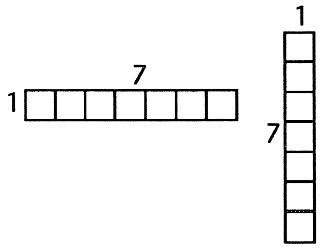
Numbers that have more than two factors are called composite numbers. Other examples of composite numbers are 4, 21 and 100. These numbers can be represented by rectangles in more than one way.

Some numbers have exactly 2 factors and can be represented by a rectangle (with dimensions that are whole numbers) in only one way. These are called prime numbers.



7 and 13 are examples of prime numbers.
They each have *exactly* two factors.

In the above description of a prime number, the orientation of a rectangle is disregarded. For example, the following rectangles are considered to be the same because one can be made to fit exactly on top of the other.

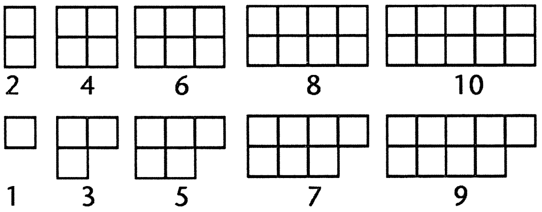


The number 1 is special. It has only one factor so it is considered neither prime nor composite.

You can initiate a discussion of prime and composite numbers by having children construct the different rectangles that can be formed with a given number of tile. We usually do this by forming arrays of tile that correspond to the date of the month or the number of days school has been in session (See illustration on top on next page from "Today's Array" in the Calendar Extravaganza.)

As children model numbers in this fashion, they notice that 1 and 0 are very special factors. The product of any number and 1 is that number, whereas 0 times any number is 0.

They also notice that if they try to form arrays that have a dimension of 2, some amounts of units have 1 left over while other amounts don't.



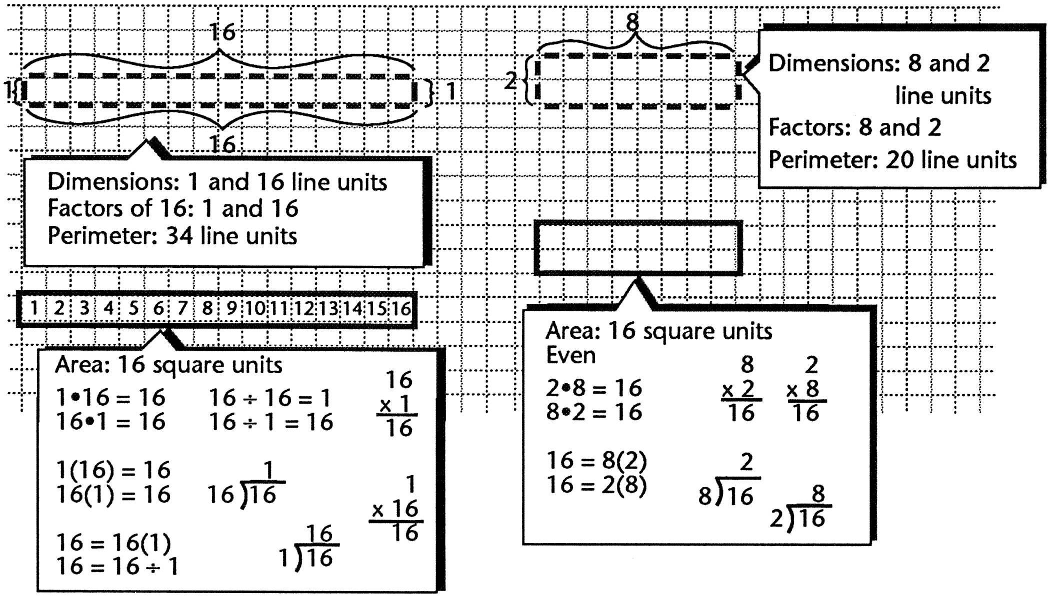
This is a visual way of thinking about odd and even numbers.

Calculating options

Children will be able to analyze multiplication and division problems more confidently and successfully if they can make sound estimates of the answers. We usually include estimating as a natural part of any discussion on problem situations (refer to Lessons 3 and 4 of this chapter).

Children find it helpful to think about multiples of 10 when estimating products and quotients. They often do this by combining some mental arithmetic with a picture of an array. Examples 2 and 3 illustrate this.

Today is the 16th day of March



Dimensions: 1 and 16 line units
Factors of 16: 1 and 16
Perimeter: 34 line units

Dimensions: 8 and 2 line units
Factors: 8 and 2
Perimeter: 20 line units

Area: 16 square units

$1 \bullet 16 = 16$
$16 \bullet 1 = 16$

$16 \div 16 = 1$
$16 \div 1 = 16$

$$\begin{array}{r}16\\ \times 1\\ \hline 16\end{array}$$

$1(16) = 16$
$16(1) = 16$

$$\begin{array}{r}1\\ 16\overline{)16}\end{array}$$

$$\begin{array}{r}1\\ \times 16\\ \hline 16\end{array}$$

$16 = 16(1)$
$16 = 16 \div 1$

$$\begin{array}{r}16\\ 1\overline{)16}\end{array}$$

Area: 16 square units
Even

$2 \bullet 8 = 16$
$8 \bullet 2 = 16$

$$\begin{array}{r}8\\ \times 2\\ \hline 16\end{array} \qquad \begin{array}{r}2\\ \times 8\\ \hline 16\end{array}$$

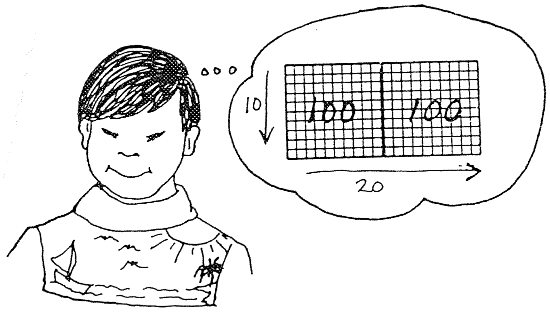
$16 = 8(2)$
$16 = 2(8)$

$$\begin{array}{r}2\\ 8\overline{)16}\end{array} \qquad \begin{array}{r}8\\ 2\overline{)16}\end{array}$$

EXAMPLE 2

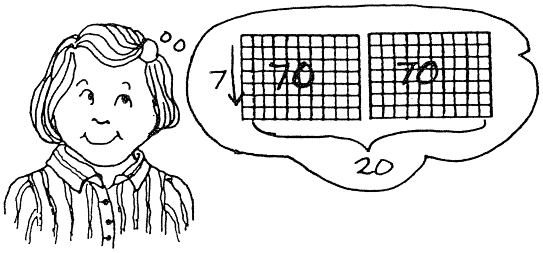
Estimate 17 × 7.

Strategy 1



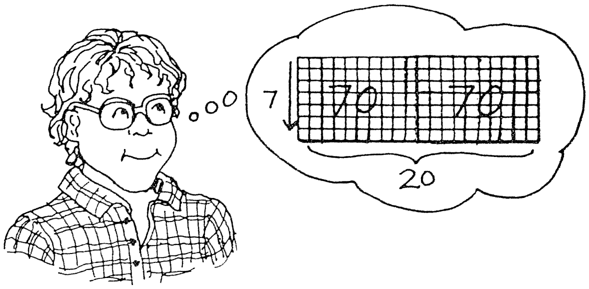
"The problem is close to 20 × 10 or 20 strips. This makes 200 since 10 strips are 100. Seventeen times 7 will be less than 200."

Strategy 2



"The answer should be about 20 × 7. That's 14 tens. Ten of these make 100. The other 4 give 40 more. So the answer is close to 140."

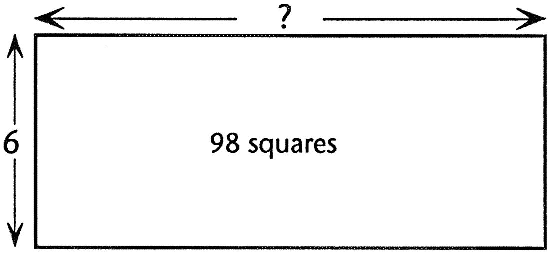
Strategy 3



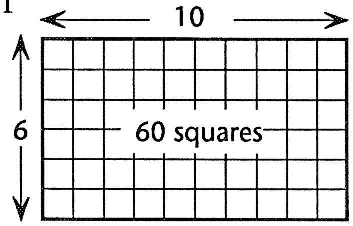
"The answer should be close to 20 × 7. I see a 7 by 20 rectangle. That will be 70 + 70 or 140."

EXAMPLE 3

Estimate the answer to 98 divided by 6.

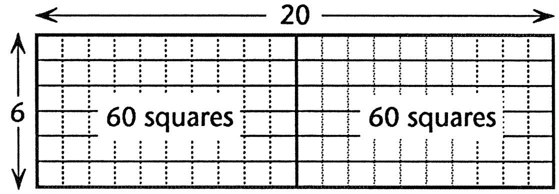


Strategy 1



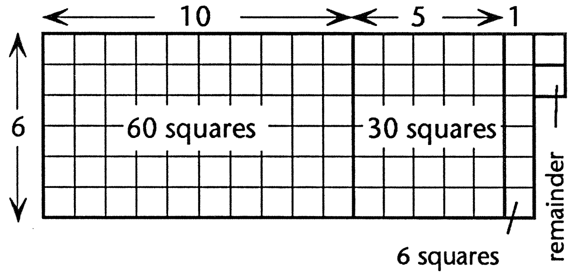
"The answer is more than 10 since 10 × 6 is 60."

Strategy 2



"We want to make a rectangle that has 98 square units. One side must be 6. If the other side is 10, that would make 60 squares. If it's 20, that would make 60 plus 60 or 120 squares and that's too much. The answer is between 10 and 20."

Strategy 3

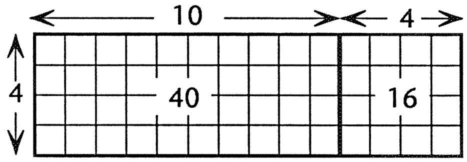


"Six times 10 is 60 and 6 times 5 is 30. So 15 sixes make 90. The answer is more than 90. There's also room for one more 6 with 2 left over. That makes an answer of 16 with 2 remaining."

Children also enjoy discussing different ways of mentally computing simple products.

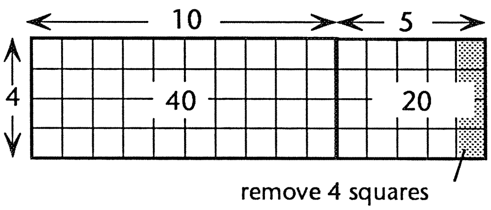
EXAMPLE 4

Multiply 14 × 4.
Strategy 1



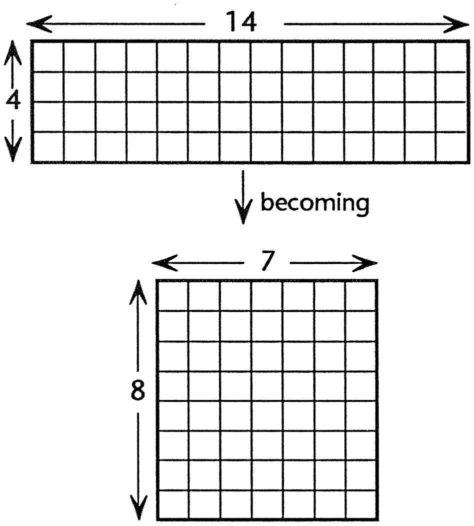
"I see 10 fours—that's 40. Four more fours is 16, so 40 + 16 = 56."

Strategy 2



"Ten fours are 40. Five more are 20. So you get 40 + 20 = 60. Now take a 4 away and you have 56."

Strategy 3



"Think of a 14 by 4 rectangle. You can make a 7 by 8 rectangle out of it. I know 7 × 8 = 56, so 14 × 4 = 56, too."

Remember there is no one best way to make estimates or calculations. Please refer back to Chapter 5 for the discussion of this. Of course, paper-and-pencil procedures and calculators are there to be used as needed!

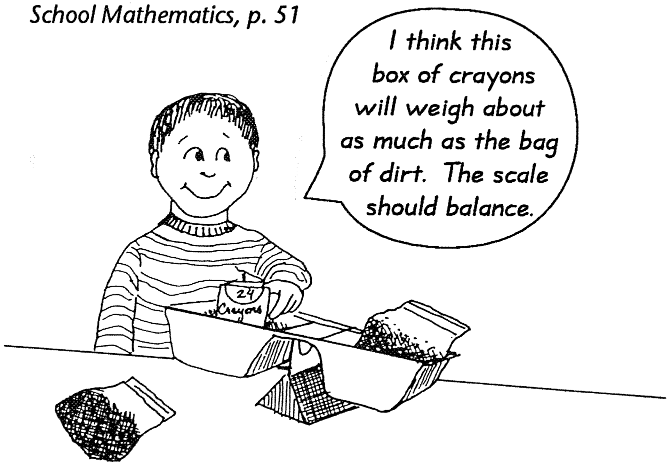
Providing practice

As with addition and subtraction, practice with multiplication and division occurs naturally as children solve problems. We have included suggestions for independent practice throughout the program. We also recommend the use of discussion cards that are designed to help children visualize and use rectangular arrays. These cards are listed in the Materials Guide and strategies for using them are given in the lessons.

You will find that, as your children develop their sense of number and of operations, they will feel comfortable with multiplication and division problems and devise meaningful ways of solving them. As adults, we hope they will remember their school experiences with multiplication and division as challenging and fun.

7 Measurement

> ***Measurement is of central importance to the curriculum because of its power to help children see that mathematics is useful in everyday life and to help them develop many mathematical concepts and skills.*** *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 51*



From the time when people first kept track of sunrises and seasons to today when a child wants to know their height or a person wishes to carpet a floor, measurement has been an important application of mathematics.

Accurate methods for measuring were developed only after many centuries. At first units varied as people typically used body parts or familiar items as measuring units. (Some of these are listed in Table 1 below.) Your children might enjoy learning more about the history of these and other units, many of which are still in use.

Table 1

span
cubit
hand
foot
1 jigger = 2 mouthfuls
1 jack (or jackpot) = 2 jiggers
1 jill = 2 jacks
1 cup = 2 jills
1 pint = 2 cups
1 quart = 2 pints
1 pottle = 2 quarts
1 gallon = 2 pottles
1 yard = the distance from the nose to the thumb of England's King Henry I

Today our children need to learn about and use standard units such as inches, meters, pounds and liters. To facilitate this, *Opening Eyes to Mathematics* works to develop a sound "measurement sense" in children.

Measurement sense has many of the same qualities as number sense. People with good measurement sense understand the process of measuring. They recognize the kinds of measurements required in problem situations and select appropriate measuring units and tools for obtaining them. They can also make helpful estimates and judge the reasonableness of their answers.

It is important to engage children in many hands-on activities that help them experience and understand the process of measuring. The following discussion summarizes this process.

Length, area, volume and weight

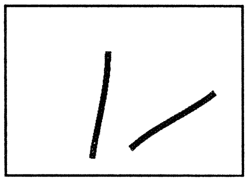
There are several attributes of an object that are commonly measured, such as length, area, volume and weight. Children need to distinguish among these and recognize the questions related to each.

Table 2 below contains information about each of these attributes.

Help your children learn about these attributes by having them observe or directly compare objects to determine their relative size. Problems such as the following are useful for this purpose.

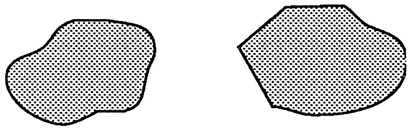
Take out the pencils in your school boxes. Are they the same length? Please arrange them in order from shortest to longest.

Which of these two lines is longer? If they were each divided in half, which half would be longer?



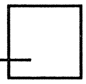
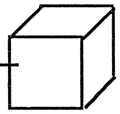
Which of these glasses holds more? How can you decide?

Look at these two regions. What are some ways to figure out which has the larger area?



Encourage the use of different strategies for making these comparisons. As an illustration, in the last problem above, some children may lay one region over the other to see if any information presents itself. Others may cover each region with their hands, cards or tile and then compare. Still others might cut up one of the regions, in order to

Table 2

Attribute	Description	Some Standard Units
length	measures distances	inch foot (12 inches) centimeter meter (100 centimeters)
area	measures in square units the amount of surface covered by a flat region	square inch square centimeter 
volume	the amount of space occupied by a solid or the capacity of an object (i.e., the amount an object can hold)	cubic centimeter  cubic inch cubic foot cubic meter milliliters (1 cubic centimeter equals 1 milliliter) liter (1 liter = 1000 milliliters) pint, quart, gallon
weight	measures how heavy an object is	pound ounce gram, kilogram

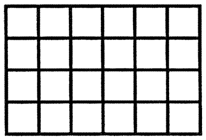
rearrange the pieces on top of the other in a helpful manner.

Using standard and non-standard units

All measurements are obtained in the same general way. An appropriate unit of measure must first be chosen. The attribute being measured can then be described in terms of this unit.

Area

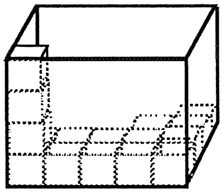
Unit □ 1 square centimeter



The area is 24 square centimeters.

Volume

Unit □ 1 cubic centimeter



"I would guess that it will take about 80 units to fill the box. The volume is about 80 cubic centimeters."

Units such as those listed earlier in Table 2 are examples of standard units. They can be used to communicate measurements accurately. An area of "12 square centimeters" can be understood by all because a square centimeter is fixed in size. Two regions with this area are pictured below. Notice that the areas are the same even though the 12 squares are arranged in two different ways.

Unit □ 1 square centimeter

The area of each rectangle is 12 square centimeters.

It is possible to measure with non-standard units—ones that do not have a fixed size or that have different meanings for people. However, these can lead to conflicting answers and to miscommunication. Consider the problem of measuring an area with beans. Because the size and shape of beans vary, more than one answer can be reported.

Despite the ambiguity that accompanies non-standard units, there are good reasons for asking children to measure with them. Children find them very tangible and meaningful.

Work with non-standard units also establishes a need for standard ones. If children experience the difficulty in communicating or agreeing with answers expressed in non-standard units, they then begin to recognize the usefulness of standard units. Here are some problems that cause children to reflect upon these difficulties.

- Divide the class into small groups and ask each group to fill several plastic bags with a prescribed amount of dirt (say, an amount they would guess to be about the weight of a bag of 35 hex-a-links). Give the bags a name, perhaps "Trid" (that's dirt spelled backwards). Ask each group to determine the Trid weight of several designated items or find an object that weighs exactly 5 Trids. Have the groups compare results. Ask your children to think about bringing an item from home that weighs 4 Trids. Would they be able to do this easily? Would they face any obstacles?
- How big is the top of your desk? Use your hand to find out. How many hands did it take to cover it? Will everyone's answer be the same?

- Ask each child to draw a line on their chalkboard and investigate ways for determining its length without using a ruler. Is your line longer or shorter than your partner's?

As children work these problems, they can discuss such things as:

- the difficulties encountered in using Trids, especially if each child's Trid weighs a different amount.
- the lack of agreement that comes from measuring areas with hands
- the problems of comparing lengths that have been measured with different units.

Their ideas for overcoming these difficulties can prompt a discussion that suggests the use of standard units.

- Find out how many pounds equal a Trid and then use a scale.
- Use just one person's hand to cover and measure the desks or, better yet, use some tile that are the same size.
- Have everyone measure their lines in the same way, perhaps with a ruler.

The measurement lessons of this program provide activities with both non-standard and standard units. Depending on the children's previous experiences, you can adjust the amount of time spent on each type of unit.

You can use many classroom items to make the transfer to standard units easier. For example, the strips of the base ten counting pieces are 20 cms long and can be notched on the back sides to be used as centimeter rulers. Similarly, tape measures can be created from strips of grid paper. Using such close-at-hand materials will prepare your children for work with various rulers and measuring tapes. In the same way, your children's experiences with calibrated jars, wooden cubes and rectangular arrays will prepare them to use standard units of volume and area.

Choosing appropriate units

In any measurement situation, one must select a suitable unit of measure. Children need to learn about the general nature and use of units to enable them to make such selections. They will need to keep in mind two things:

- They may arbitrarily choose a unit of measure, as long as it is consistent with what is being measured. For example, they must use a unit of length to measure lengths.
- The size of the object should be considered when selecting a unit of measure.

PROBLEM 1

How would you measure the length of this line? Would you measure the longest wall in the room in the same way? Why?

In this problem any units of length could be used to obtain the required measurements. Children might use inches, centimeters, feet, or the length of a paper clip or popsicle stick, to name a few. However, depending on the lengths of the line and wall, they may choose a different-sized unit for measuring each one.

PROBLEM 2

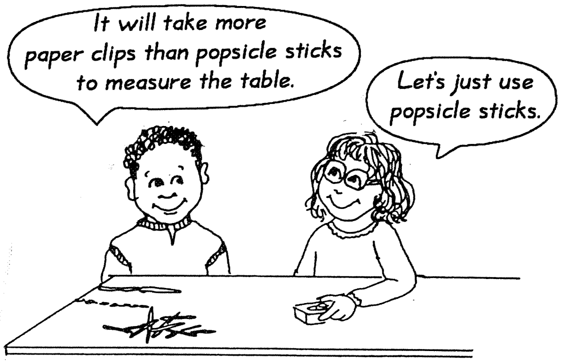
Would you measure the distance between two cities in feet or in miles? Why?

Similarly, in this case, one can measure the distance between two cities in terms of feet, but how much more convenient to use miles. (Of course, if the distance is being determined from a map, then one might have to work with a scale that uses inches.)

Children will also learn to use units appropriately if they have a sense for how the units are related and if they can form helpful pictures of them.

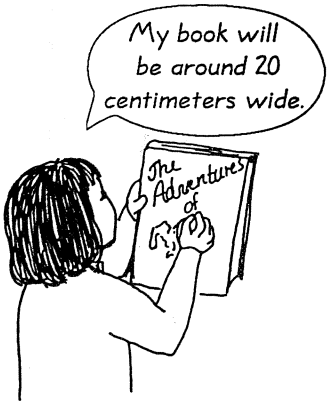
PROBLEM 3

Will you need more paper clips or popsicle sticks to measure the length of this table? Will you need more inches or feet?



PROBLEM 4

Make a space between your thumb and forefinger that is approximately 1 cm. Now look at this book. About how many centimeters wide do you think the book is?



Once our class was determining the length of a row of potatoes. The children first made estimates in terms of the edge of a hex-a-link cube. When we gathered around the floor to measure the length more exactly, we paused after 10 to 12 hex-a-links were in place to ask if anyone wished to change an estimate. Sean called out, "I can tell right now that we are going to need 56 to 58 cubes for sure!" Why was he so confident? He had seen that 12 cubes could be laid out along a floor tile and that the entire "potato train" was a bit shorter than the length of five tile!

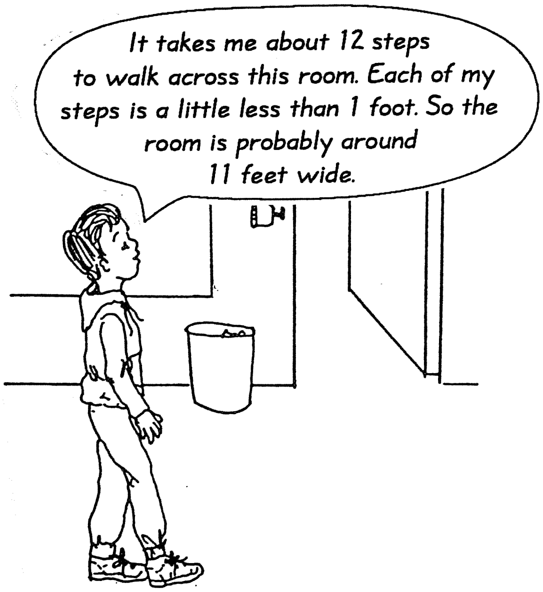
If children can carry an image of an inch or a square yard in their minds or can sense what twenty pounds feels like, then they will have established useful references for future estimation and measuring.

Estimation and measurement

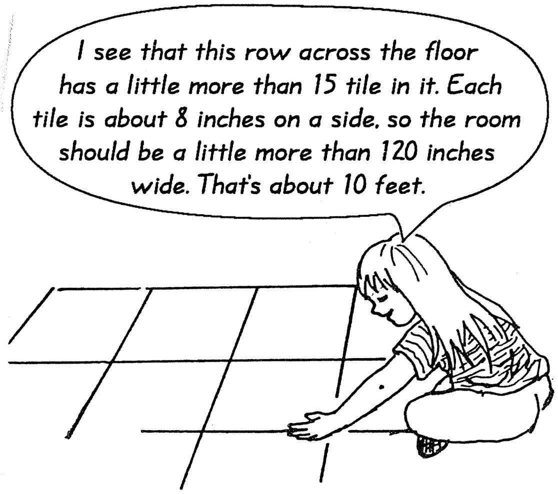
We don't realize how often we make estimates and then act upon them. Is this parking space big enough for my car? Do I have enough time to go to the grocery store before my meeting? Is there enough paint to finish this job? Can I make it home and back on this tank of gasoline?

We've become experts at "eyeballing it" and need to provide experiences for our children to practice this skill as well. Give your children lots of practice with predicting and comparing. They will enjoy developing strategies for making sound estimates and deciding how to use approximations.

EXAMPLE 1



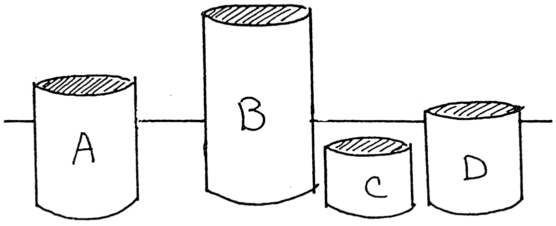
EXAMPLE 2



Children who routinely estimate before measuring are likely to think more carefully about the task at hand. This gives them a greater sense of ownership in their work and a means for judging their answer. Problems like the ones below can be addressed more confidently with the help of appropriate estimations.

PROBLEM 5

Look at these containers. How many of container A will fill container B? Which container will hold about one-half of container A? Which holds about two-thirds as much as A?

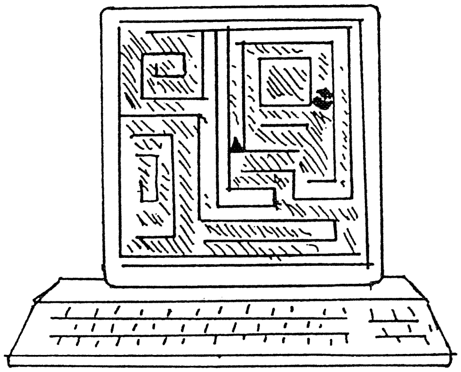


PROBLEM 6

Can you find an object that weighs about 5 pounds? Are there any items in the classroom that weigh approximately 20 grams?

PROBLEM 7

(With Logo) Try to get the turtle through this maze in the fewest number of steps.



Time and temperature

A child's sense of time and temperature can vary widely. How do your children feel when they're told it's 10 minutes to bedtime; when they are told there are 5 hours to go on a trip; when they jump into a swimming pool?

Because units of time and temperature cannot be counted as concretely as those of physical lengths or areas, we approach these topics in a different way. We do this as part of the Calendar Extravaganza.

For developing time sense, the Clock Reading Component of the Calendar Extravaganza has proven to be very successful. It contains experiences designed to increase children's awareness of when events occur and how long it takes to complete various tasks.

- It is 9:30 in the morning. How much longer until lunch time? What will you be doing at 9:30 tonight?
- Do you think you can make a 14 × 17 rectangular array with tile in 5 minutes?
- How long do you think it will take you to put everyone's name in alphabetical order?

We also feel that it is important to talk with children about time, teaching them to respect it and use it wisely.

During the second semester of the year, the Calendar Numberline can be adjusted to look like that of a thermometer (including negative numbers). The children can post daily temperatures during the calendar lesson and discuss the variation that occurs. Do they feel the changes in any special way? How accurate were the weather forecaster's predictions for the week?

Temperature patterns that affect us in more general ways can also be discussed. These include body temperatures and seasonal temperatures found in nature. Why does a frog bury itself in the mud each winter? He notices quite a difference between the temperature of the air and that of the mud!

Planning

As you plan your instruction, keep in mind the following considerations.

Provide your children with measuring experiences that use a variety of materials. Many of the materials suggested in the lessons can be brought in from home; others may have to be purchased. See the Materials Guide for a listing of the main items needed.

Consider integrating this unit into other areas of study. For example, science and measurement are so interrelated that they are hard to separate. Where would science be without calculations, comparisons, estimations, etc.?

Measurement also plays an important role in social studies. What percentage of the world's supply of oil is produced in Iraq? How does a timeline make a sequence of events easier to remember?

Special care must be taken when using certain items for measuring. For example, if lengths are being measured with the help of square tile or hex-a-link cubes, it is the *edge* of each of these objects and not the object that is the unit of measure. If children are not aware of this, they are likely to inappropriately identify lengths with measures of area or volume.

Both metric and non-metric units are used in the world, and our children should grow in their understanding of each. Treat each system in a natural manner and do not emphasize exact conversions between them. Provide opportunities for your children to develop a sense of the size of the different units in each system and to learn when and how to apply them appropriately.

Once your children have developed a sense of "ten-ness" from the numeration lessons, they will find the metric system a very natural one to use.

Conclusion

People use measuring processes in many daily activities such as cooking, cutting and sewing clothes, hanging wallpaper and reading a map or globe. In fact, their sense of measurement often permits them to correctly do this in ways that are quite different than the formal techniques generally taught in schools. Help your children develop a comfortableness with measurement that will serve them well throughout life.

8 Fractions

An understanding of fractions and decimals broadens students' awareness of the usefulness and power of numbers and extends their knowledge of the number system. It is critical in grades K–4 to develop concepts and relationships that will serve as a foundation for more advanced concepts and skills. NCTM's *Curriculum and Evaluation Standards for School Mathematics*, p. 57

People have used fractions for centuries, principally for measuring. The ancient Egyptians solved the problem of sharing 6 loaves evenly among 10 people by giving each person $6/10$ of a loaf. However, because they only understood fractions that had a numerator of one like $1/2$ or $1/5$, they reported the answer of $6/10$ as $1/2 + 1/10$.

Our units for measuring time were conceived of as fractions and come from ancient Babylonia: 1 minute was defined as $1/60$ of an hour and 1 second as $1/60$ of a minute.

Today, fractions are used extensively in many applications of mathematics such as measurement, probability and statistics. Children need to work comfortably with fractions since they are such an important part of our number system. Our goal is to help your children extend their sense of numbers to include fractions. We want to open their eyes to fractions through activities that provide visual models for thinking about them.

Beginning experiences with fractions

Children generally have had some experience with fractions in the course of sharing things. This might be cutting an apple in half, spending a quarter of a dollar or giving a friend a third of a pizza. Find out about their previous experience and add to it by working with fractions in physical situations. Here are two lessons that do this. Adjust them to fit the needs of your class. You will also find other ideas in the Calendar Extravaganza (see People Fractions).

LESSON 1

TEACHER *I'd like half of you to stand over here and the other half to remain seated. How should we do this?*

MARTHA *We could all stand and then pair up. Then one person from each pair can sit down.*

BRIAN *How about counting the number of children? We could split that number in two. That will tell us how many should stand.*

JENNIFER *Perhaps we could do it by rows. Start at the front and then have every other person stand up.*

SUSAN *I have an idea! Just have half of the people in each row stand.*

TEACHER *These are all suggestions worth trying.*

Each method is tested.

TEACHER *How do you feel about these procedures?*

MARIO *Well, when we stood up and paired off, it seemed to be OK.*

JUANITA *That's because we have 26 children and half of 26 is 13, so 13 had to sit down.*

SUSAN *There's a bit of a problem with my idea. If a row has 5 people, then there will be a person left over. Either 2 will stand and 3 will sit or the other way around.*

JOHN *That will happen whenever there is an odd number of children in a row.*

TEACHER *Can anything be done with the left-over people?*

SUSAN *Sure! Half of them could stand!*

JENNIFER *Yes! I see another possible problem. If the number of children in the class is odd, we won't be able to have exactly half of them stand. That didn't happen today.*

LIBBY *(with a smile) Well, if we have an odd number, maybe we could split the extra person in half!*

TEACHER *You each have some nice thoughts. To have exactly half the children stand, we need to divide the class into two equal groups. We can do this if the number of children is even. We can come pretty close if our class has an odd number of people in it.*

The lesson continues, perhaps by investigating the problem of splitting the class into thirds. The teacher encourages further observation by the children with questions such as:

If we can divide the class into 3 equal parts, what fraction of the class is formed by each part? (⅓)

Is it always possible to split the class exactly into thirds? (No. For an exact split, the number of children must be a multiple of 3.)

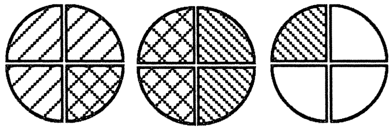
Are there more people in ½ the class or in ⅓ of the class? (½)

When we split into thirds, did the total number of children change? (No)

LESSON 2

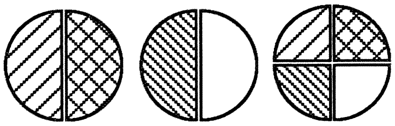
TEACHER *Here are 3 cookies. How can they be shared equally by 4 children?*

RANDY: *I think I'd try to split each cookie into 4 equal parts and then share the parts. Each person would get 3 of the parts that way.*

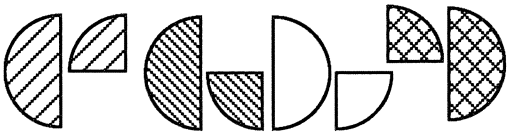


Each persons gets 3/4 of a cookie

AMY *Here's another way! You could split the first 2 cookies in half and give each person half a cookie. Then split the third cookie into 4 equal parts and give each person one of the parts.*



Each persons gets half a cookie plus one-fourth of a cookie.



WAYNE *If you do that, each person would get half a cookie and a fourth of a cookie.*

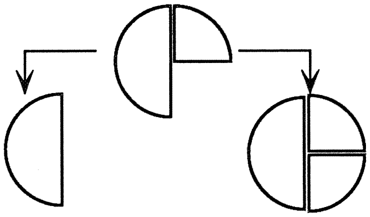
RUBY *It looks like each person gets more than half a cookie, but there isn't enough for one cookie each.*

TEACHER *Is each person's amount closer to half a cookie or one cookie?*

RUBY *It's exactly between!*

TEACHER *Come show us how you decided that.*

RUBY *(Standing at the overhead.) Each person gets half a cookie and a fourth of a cookie. If you put another fourth with it you make a whole cookie and if you take a fourth away you get half a cookie! So it's right in the middle!*



Take 1/4 cookie away and 1/2 a cookie is left.

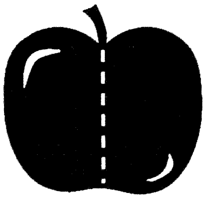
Add 1/4 cookie and 1 whole cookie is formed.

Lessons of this sort will give children a rudimentary insight into how fractions are created and used. In particular, they will see how a unit (or whole) can be subdivided into equal parts. In Lesson 1 the unit is the group of children that is being evenly split. In Lesson 2 Randy treats a cookie as a whole, dividing each into 4 equal parts. Amy also sees each cookie as a whole unit, but makes 2 different divisions. Also during the lessons, the children use appropriate language to describe other observations about numbers and the relative size of fractions.

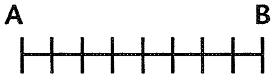
The role of the whole or unit

The concept of a whole establishes a context for thinking about fractions in a meaningful way. For example, when people are asked to imagine "one-half", some may think of half an apple, while others might visualize half a dollar or 1/3 as a point on a number line.

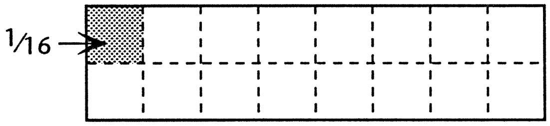
In working with fractions, your children must first identify the whole that is being used. This whole might be anything that can be subdivided into equal parts. Some possibilities are individual objects (such as an apple), collections of objects (like a group of children), lengths or regions.



The apple is divided into two equal parts. Each part is one-half of the apple.



The length AB is divided into eight equal parts. The length of each part is one-eighth that of AB.



The rectangle is divided into 16 equal parts. The area of each part is 1/16 that of the entire rectangle.

If children overlook or don't recognize the whole that is being used in a problem they may have difficulty solving the problem. Provide experiences for the children in recognizing the whole and encourage them to verbalize their understanding with phrases like "one-third of the whole apple" or "one-third of the whole basket of apples."

Sometimes, too, children may be confused by the physical size of a unit. One-third of Sherry's whole apple may be much smaller than 1/3 of Rob's. Are they really each 1/3, even though they are not equal in size? Yes! But they are 1/3 of different wholes!

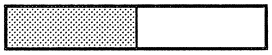
Understanding fractions

In your fraction lessons, use appropriate language and emphasize the development of concepts. Help children establish visual models for thinking about fractions; introduce symbols as they become meaningful.

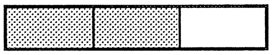
The more concrete experiences children have with forming fractions and identifying the corresponding wholes, the more they will be able to visualize fractions and use them in their world. We create these experiences in the school room using Fraction Bars, pattern blocks and other materials.

Fraction Bars®

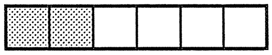
Fraction Bars® are a wonderful tool for helping children picture fractions clearly.*



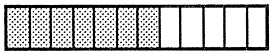
1/2 bar



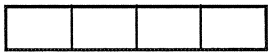
2/3 bar



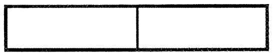
2/6 bar



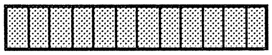
7/12 bar



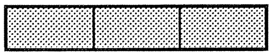
0/4 bar



0/2 bar

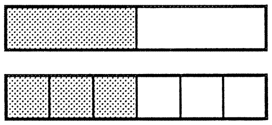


12/12 or whole bar

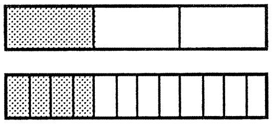


3/3 bar

Fraction Bars are particularly useful for comparing fractions. We see the 1/2 and 3/6 bars have the same amount of shading. The same is true for the 1/3 and 4/12 bars.

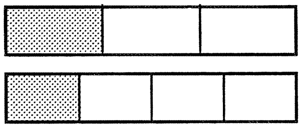


1/2 is equivalent to 3/6



1/3 is equivalent to 4/12

We can show that the 1/3 bar has more shading that the 1/4 bar.



1/3 > 1/4

Our fraction activities begin with People Fraction Bars. (These bars are made from different colors of poster board, with one side shaded with slash marks—see Materials Guide). The following dialogue illustrates the use of these People Fraction Bars.

Four children come to the front of the room, each holding 1 of the 4 parts that have been cut from a blue bar. They show the unshaded sides to the class.

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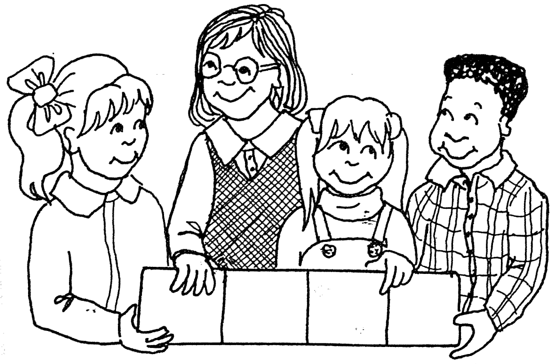
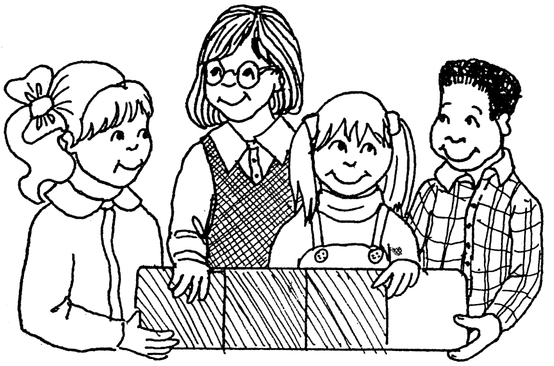
*Fraction Bars® is a registered trademark of Scott Resources, Inc.



TEACHER *What do you notice about these children?*

JOSH *Well, 3 are girls.*

The three girls turn their bars over so that the shaded sides are exposed.

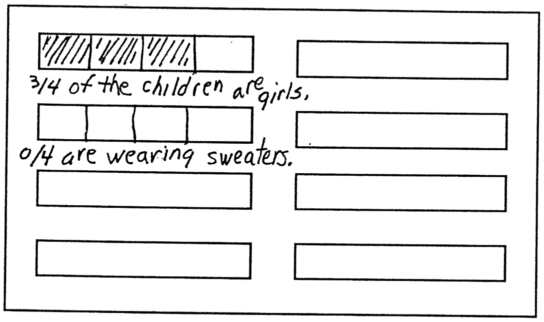


"This shows that ¾ of the children are girls and that ¼ are boys."

MARIE *I see something else. None of them are wearing sweaters.*

All four children show the unslashed sides once more.

The children at their desks make a graphic record of each of these People Fractions as shown here:



This activity is replicated later in a small group setting using similar bars cut from colored construction paper. These, too, have slashed lines on one side (see Materials Guide). We call these Group Fraction Bars as each group shows fractions that convey information about itself.

"In our group, 2⁄4 of us wear glasses."

After these activities, have children examine a complete set of individual Fraction Bars (see Materials Guide). Distribute sets and invite the children to share their observations. Some things they might observe are:

The bars are all the same length.

There are different colors.

Some are shaded, though the amount of shading is not always the same.

If more of a bar is shaded, then less of it will not be shaded.

All the bars of the same color have been divided into the same number of parts. The number of parts that are shaded are different.

Some bars have no parts shaded. (These are called zero bars.)

Some bars are completely shaded. (These are whole bars.)

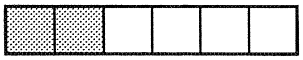
The parts of the red bars are smaller than the ones of the green bars. The more parts there are, the smaller each part will be.

Sometimes the shaded part on one bar matches the shaded part on another bar.

The children are then able to refer to the bars as they describe and compare fractions.

TEACHER *Please take out the green, yellow and red bars from your set.*

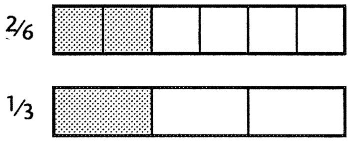
TEACHER *(She holds up a 2/6 bar.) Will someone describe this bar?*



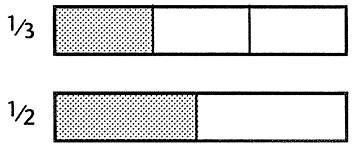
RONDA *I will! It has 6 parts that are all the same, except that 2 have been shaded. Two-sixths of the bar has been shaded and 4/6 is not shaded.*

TEACHER *Yes, Ronda. Now, are there any other bars that have the same amount of shading as this 2/6 bar?*

JOSH *(Demonstrates at the overhead.) I have one. The yellow bar with 1 part shaded will work. This is the fraction 1/3. The 1/3 and 2/6 bars have the same amount of area shaded.*

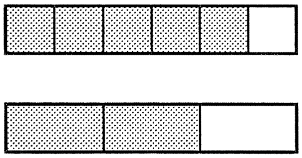


MELISSA *There is no green bar that has the same amount shaded as the 2/6 one. It looks like the yellow 1/3 bar has less shaded area than the green one that shows 1/2.*

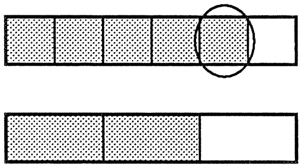


STU *That's like before! We found out that 1/3 of our class is less than 1/2 of it.*

The children explore further. As an example, they notice that the 1/2, 2/4 and 3/6 bars all have the same amount of shading. The children thus have a model for thinking about equivalent fractions, with an entire bar being a unit. They can also compare bars that do not have the same amount shaded and determine which of the corresponding fractions is larger.



5/6 > 2/3



The 5/6 bar has more shaded area than the 2/3. The extra amount of shaded area is 1/6 of a bar.

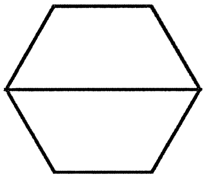
The difference between 5/6 and 2/3 is 1/6.

Pattern blocks

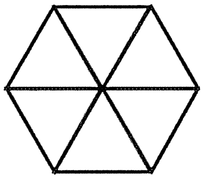
Children enjoy using pattern blocks to make designs and observe how they fit together. The blocks are especially helpful for thinking about geometric concepts such as properties of shapes, angles and symmetry. Their regularity makes it possible to model fractions in a hands-on manner. Here are some ways to do this.

- Ask the children to find different ways for covering a yellow hexagon with other pattern blocks. Some typical solutions are:

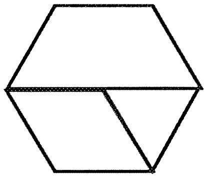
a. two red blocks (trapezoids)



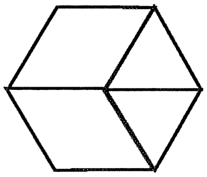
b. six green triangles



c. a red trapezoid, a blue diamond and a green triangle

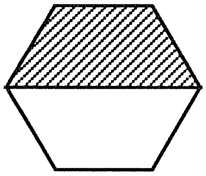


d. two blue diamonds and two green triangles.

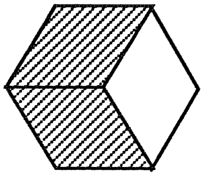


- Ask the children to consider the yellow hexagon as a whole and to then discuss what parts of the hexagon are shown by other blocks.

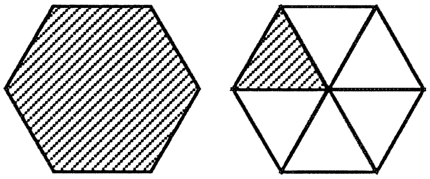
MYRNA *The red trapezoid shows $\frac{1}{2}$ of the hexagon.*



OTIS *Three diamonds placed on a hexagon show $\frac{3}{3}$. Two-thirds of a hexagon is represented by 2 of the diamonds.*

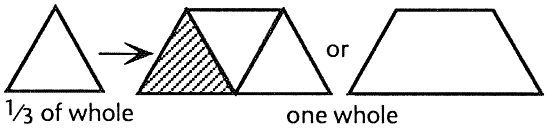


PETER *A hexagon and a green triangle shows $1\frac{1}{6}$ hexagons.*

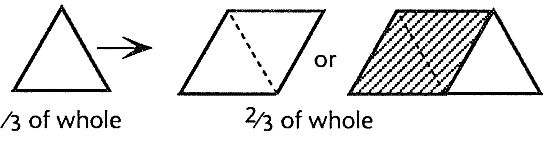


- Tell the children to suppose that $\frac{1}{3}$ of a whole is represented by a green triangle. What else can be observed?

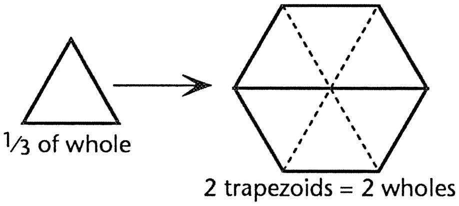
JENNY *Three triangles will be a whole. This means the red trapezoid will be a whole, too.*



MELINDA *The blue diamond will show $\frac{2}{3}$ of a whole.*



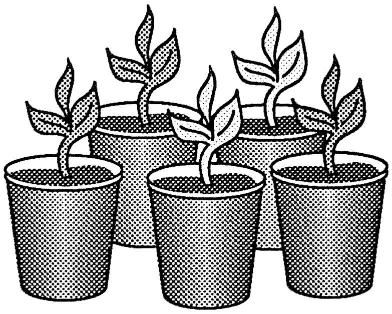
JAMES *The hexagon will show 2 wholes. It is made up of 2 trapezoids and each of them is a whole.*



More fraction activities

The Contact lessons contain many fraction activities that use materials related to things in the child's world (football, peanuts, stamps, etc.). Also, lessons related to other areas of the curriculum can provide experiences with fractions.

- Science: Suppose you have planted 1 bean in each of 5 cups. Stop each day to ask what fractional part of the beans have sprouted or have leaves? If 2 of them have yellow leaves, you might have your children make a Fraction Bar that shows this by folding a strip of paper into 5 equal parts and then coloring 2 of the parts yellow.



$\frac{2}{5}$ of the beans have yellow leaves

$\frac{2}{5}$ is shaded yellow

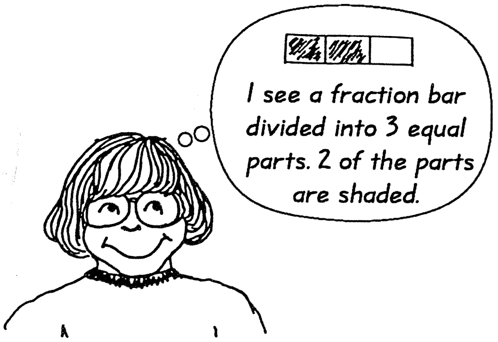
- Social studies: After studying the seven continents, look for places to use fractions as part of a review. For example, what fraction of the continents have parts that are in the southern hemisphere? Create a Fraction Bar or pie chart that shows this.
- Health: Perhaps you have a picture of 6 food products. Sort them using fractions, such as, "$\frac{5}{6}$ of these foods provide calcium for the body."

It's important for your children to think about fractions in many different contexts. Their understanding of a whole and of the meaning of fractions will be reinforced. Also, you can discover and discuss any misconceptions they may have.

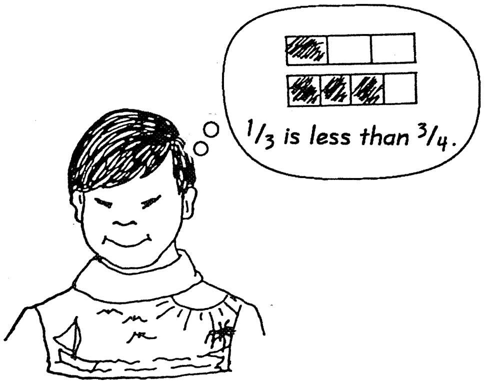
Building mental images

Children soon form useful mental images of fractions as they work with manipulatives such as Fraction Bars and pattern blocks. They will find it helpful to refer to these images when thinking about relationships among fractions. The following exercises from the lessons illustrate this.

Close your eyes and try to imagine ⅔. Describe what you see.

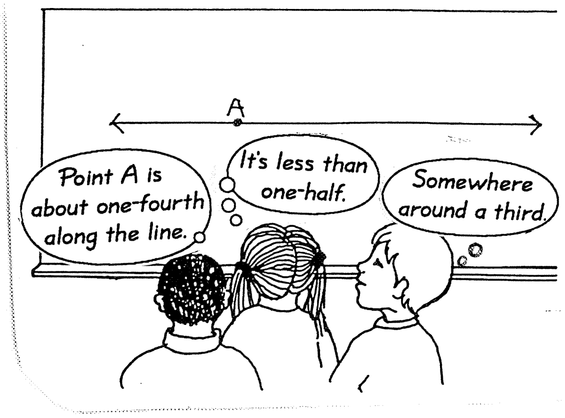


Which is the larger fraction, ⅓ or ¾?

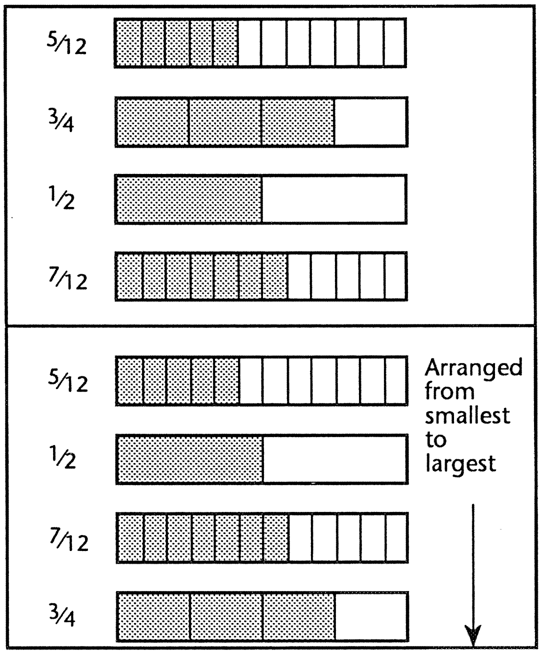


What do you think a Fraction Bar that shows ⅞ looks like?

About how much of the line is to the left of point A in this picture?



Arrange these Fraction Bars in order from least amount shaded to greatest amount shaded.

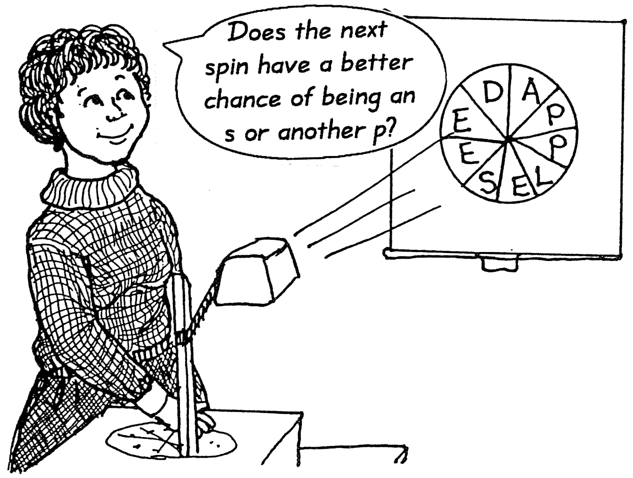


Conclusion

The fraction models discussed in this chapter, along with the mental images that grow from them, are very powerful. Your children will use these many times in years to come as their experience with fractions grows.

9 Probability

Statistics and probability are important links to other content areas, such as social studies and science. They also can reinforce communication skills as children discuss and write about their activities and their conclusions. Within mathematics, these topics regularly involve the uses of number, measurement, estimation, and problem solving. *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 54*



People have to make decisions throughout their lives. When these decisions require a prediction about the future, often the element of chance must be dealt with. When this is the case, a knowledge of probability offers the option of deciding on the basis of theoretical laws of chance.

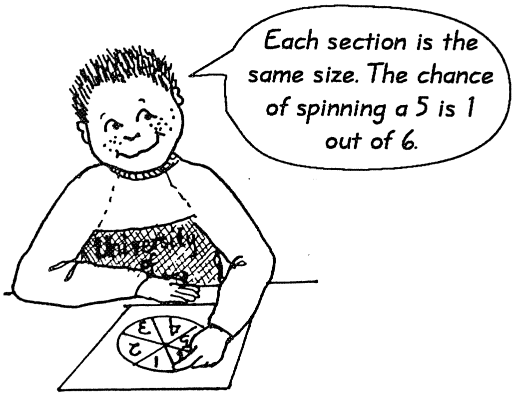
Historically, probability was first studied because of people's interest in gambling. Now, however, decisions and forecasts in fields such as business, weather, sports, medicine and insurance rely on probability theory. Dealing with probability in mathematics requires the use of fractions, decimals, percentages and graphing.

Probability is fun! Children regularly encounter chance in the games they play with one another. They think about the moves that are available to them, trying to decide which is best, and who got more "breaks" in the game. This awareness of chance can be broadened by informal explorations of probability concepts in the classroom. We have included many such explorations in the Contact Lessons of this program. This chapter describes how they might be conducted.

Determining probabilities

A probability is an estimate that an event will or will not occur. For example, the weather forecaster predicts a 20% chance of rain for tomorrow or a child thinks there is a 50–50 chance of winning a coin toss at the start of a game.

Probabilities may be determined in different ways. One method is to make an estimate on a theoretical basis as shown in the following illustrations.



The children in the illustration have theorized that any one number on the cube or the spinner is as likely to turn up as any other number.

A second way to estimate probabilities is to use data from an experiment or simulation as a predictor. For example, if a basketball player has made 60 out of 80 free throws, it may seem reasonable to say that the player's chances of making a free throw are $^{60}/_{80}$ or 75%. Insurance companies use this approach to probability to measure risks when setting premiums.

People sometimes make estimates on a purely subjective basis. They may have no previous experience to guide them. Perhaps they wish to follow their intuition regardless of other information; they might place a bet that feels "right" to them, even though it means going against the odds.

In practice, estimates can change as new information about a situation is uncovered. For example, a person might feel that a coin is "fair" (evenly weighted) and estimate the chances of tossing heads to be 50%. Suppose, however, the coin is flipped 100 times and heads comes up on only 20 tosses. How can these results be explained? Perhaps the coin is actually fair and the number of heads and tails will balance out if more tosses are made; perhaps it is fair, but the tosses have not been random. Or maybe the coin is not fair after all and there is actually a greater chance of tossing tails. The person must judge which explanation seems most reasonable in a given situation.

Probability experiments

Children enjoy conducting experiments where the outcomes are determined by chance. In so doing, they strengthen their intuition about what might be expected to occur and they gain experience with gathering and analyzing data. The following Contact lesson illustrates this.

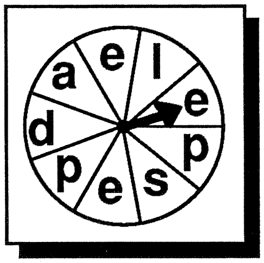
Contact Lesson 14: Apples—Probability and Statistics

You will need
- Chapter 9, Probability
- a transparency of Blackline 101 (9-Section Spinner)

for each group of four children
- a spinner prepared from Blackline 101
- a copy of Blackline 58 (Apple Probability)
- (optional) Blackline 81 (Observation Record Sheet)

Your lesson

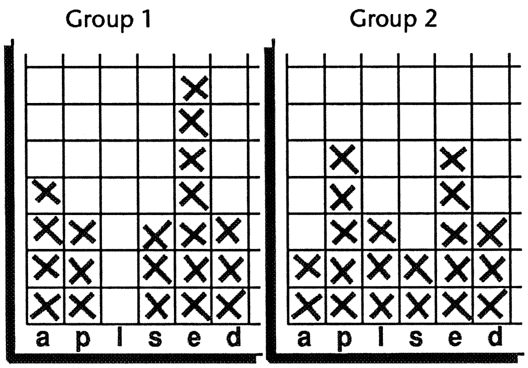
Display your 9 section spinner and explain to your children that the letters ***a-p-p-l-e-s-e-e-d*** are to be written in its sections. Let them determine their placement.



Discuss the following questions:

If you make 20 spins, which letter do you think will be spun most often? Which letter do you think will come up the fewest number of times?

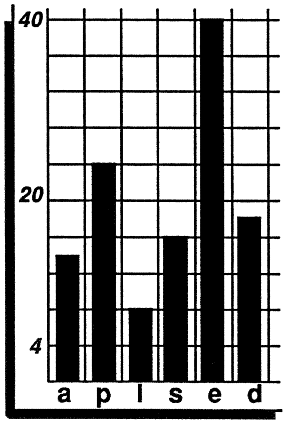
Once their predictions have been shared, have the groups make 20 spins and construct a bar graph of their results. Ask the groups to present their results to the class.



Group 1: We spun ***e*** the most, just as we guessed. We didn't think ***a*** would come up more than ***p***, since ***p*** was on two sections, but it did. We didn't get any ***l***'s which was a surprise.

Group 2: We spun ***p*** and ***e*** the same number of times. We thought we'd get more ***e***'s. The other letters came up about the same amount.

When everyone is ready, pool the results as shown here, and then address the questions listed below.



Letter	Number of Spins
a	14
p	24
l	8
s	16
e	40
d	18

Questions for discussion

How closely does the data compare with what was predicted? What might explain any differences?

What might be expected to occur if the experiment were repeated or if more spins were made?

Is it possible that one of the single letters could turn up most often? Is it probable that this will happen?

What fraction of the results were ***a***'s? What fraction were ***e***'s? What fraction of the results were ***p***'s? Why?

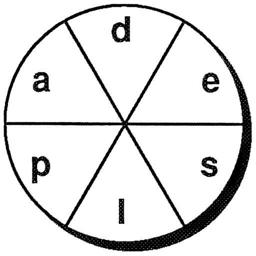
How do the results from each group compare to the pooled results of the entire class?

Does each letter on the spinner have an equal chance of being spun? Why or why not? If not, how could the experiment be changed to give each letter an equal chance of being spun?

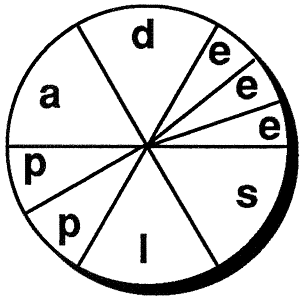
Teacher Tips

It is natural to expect that ***e*** and ***p*** should turn up most frequently. Theoretically, if the spins are random, the chances of landing on ***e*** and ***p*** are $\frac{3}{9}$ and $\frac{2}{9}$, respectively. If this doesn't happen, it may be that not enough spins were made or that the manner of spinning was not random. Also, because 20 is a fairly small number of spins, the pooled results may differ from the results of individual groups. Theoretically, the combined totals of all the groups should be in closer agreement with what might be predicted in this experiment.

The children may have various ideas for making the outcomes of this experiment equally likely. One might be to use a spinner that has six sections of the same size and to place one letter—*a, p, l, e, s, d*—in each section.



Another is to adjust the original spinner so that the amount of area for each letter is the same.



Lesson 1 illustrates several important points about probability experiments:

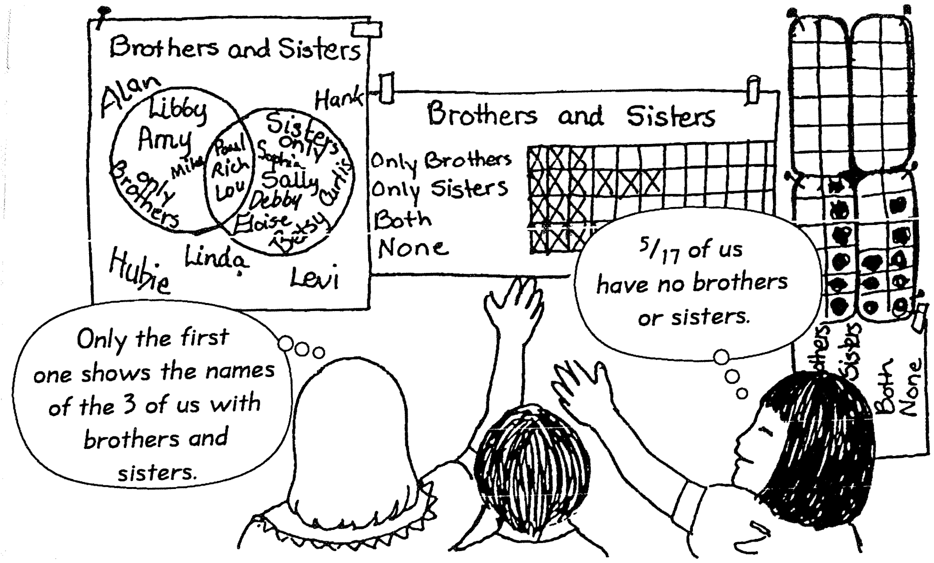
- The children first made predictions about the outcomes of the spins. This developed their awareness of probability and gave them some ownership in the experiment.
- Each group generated data for the experiment by making 20 spins. This information was then organized into a graph and results were pooled. Tossing number cubes and drawing lots are two other ways for gathering data in experiments of this sort. A computer simulation could generate a large quantity of statistics to analyze.
- In planning the experiment, the teacher considered the following questions: What information is needed? How will this information be gathered? How many times should the experiment be repeated? How should the data be organized and shared with others?

  It is also helpful for children to take part in the planning process. In Lesson 1, for example, they could be asked to think of other ways to conduct the experiment. One possibility would be to place in a bag nine identical slips of paper, each marked with one of these letters ***a, p, p, l, e, s, e, e*** and ***d***. Then do the following 20 times: randomly draw a slip from the bag, record the letter and return the slip to the bag.
- The data was analyzed after the experiment was completed. Generally, this includes discussing questions such as: Did the results agree with what was predicted? What might explain any similarities or differences? What conclusions might be drawn from the data? If the experiment were repeated, what might be expected to happen?

All of the probability activities in this book revolve around the above points. The activities will help children develop a sense for what might reasonably be expected to happen in a chance situation. We keep the instruction informal and don't focus on assigning specific percentage probabilities to the outcomes of an experiment.

10 Data Analysis and Graphing

Collecting, organizing, describing, displaying, and interpreting data, as well as making decisions and predictions on the basis of that information, are skills that are increasingly important in a society based on technology and communication. *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 54*



In today's world, statistics crop up everywhere and people must be able to use and interpret them in a meaningful way. Understanding basic statistical concepts enables one to make informed decisions in the workplace and as a consumer. For children, this means providing experiences that require them to order, compare and analyze data. This includes working with graphs, since graphs are commonly used to summarize and display statistical information.

Many lessons in this program ask students to examine and graph sets of data and to draw conclusions about them. This chapter describes the general nature of these activities and the types of graphs that are frequently used.

Collecting and Analyzing Data

Children find many kinds of data interesting. Our children enjoy learning about themselves, looking at results from sports or probability experiments and examining information related to the monthly Contact themes. In the following lesson, the children use people and bar graphs to describe where they live.

LESSON 1

TEACHER *I'm just so curious about where you live! I'd like to know if you come from the city or the country. How could I find out the answers and remember who lives where?*

MYRA *Just ask!*

TEACHER *I'd get 25 answers at once!*

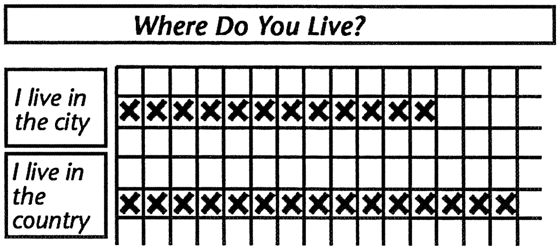
MARIO *Ask us one at a time.*

TEACHER *If I ask each of you individually, I'll forget before everyone has a turn. You must think I have an elephant's memory!*

CARLOS *Perhaps we could form two groups—one for the country and one for the city.*

TEACHER *Okay, let's do that right now.*

After the children have made this "live" graph, ask them to make a bar graph to record where they live.

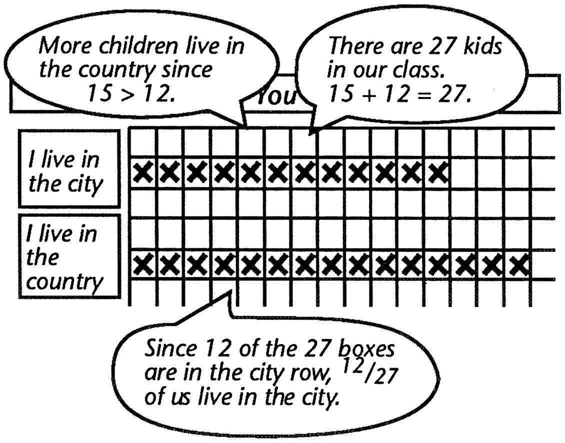


The purpose of a graph is to visually display information so it can be readily understood. Once a graph has been constructed, always take time to see what it tells.

TEACHER *We need to make our graph talk to us now. How can we make it tell us some information about where you live?*

JENNIFER *Newspaper cartoons show people talking with little bubbles of conversation. Each of us could cut a bubble from paper and write on it what the graph says to us —cartoon style.*

OTHER CHILDREN *Yeah! We like the idea of a talking graph.*



Conversation bubbles are just one way of recording observations about graphs. Here are some other methods that can also be used.

- In a large group setting, record the students' observations on chart paper posted near the graph.
- Have students work in small groups and challenge them to record as many statements as possible on paper. When they finish, post the papers near the graph and discuss the findings of each group.
- Divide the students into teams of two and ask each team to write a question about the graph on a sentence strip. Post the questions and discuss them.

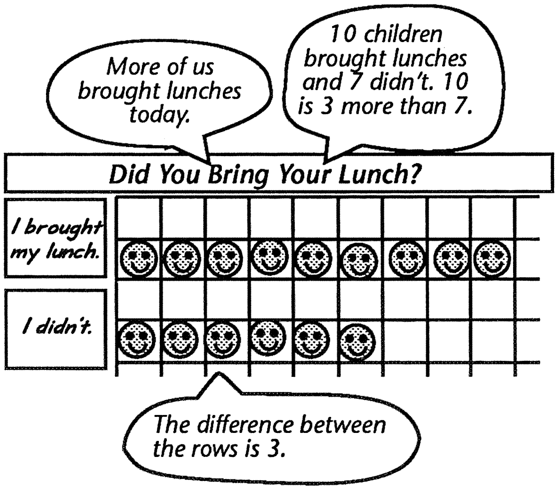
As the year proceeds, encourage the students to incorporate mathematical vocabulary and symbols (such as more than, <, fractions, =) in their findings. Eventually, they will be able to accompany their written statements with supporting number sentences or sketches.

If your children are at a loss for words, you might stimulate their thinking by asking such questions as: Did more of us live in the country or in the city? Was the number of those living in the city an odd or even number? How many children are present today? Do more of our boys live in the country or the city?

When finished, post your graph and the accompanying analysis in the classroom or hall for everyone to enjoy! This will remind your children of the fun they shared when making the graph. Also, other teachers might see new ways to incorporate graphing into their curriculum.

More ways to graph

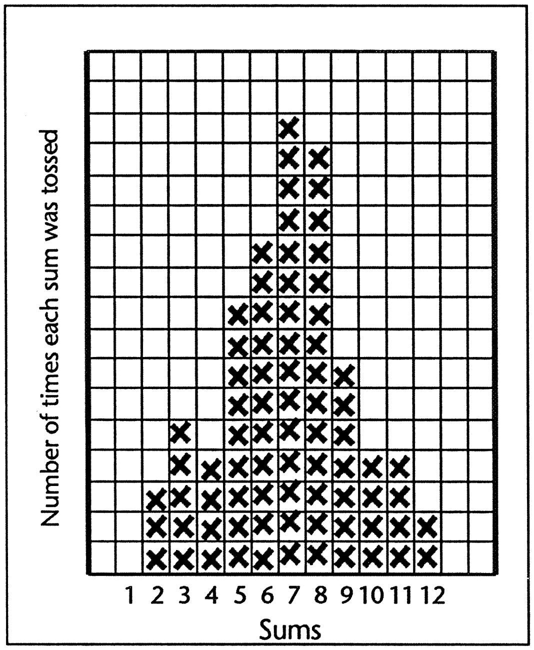
The entire graphing process provides many opportunities for students to make decisions. The children will enjoy deciding how to record data when making a bar graph. Instead of shading the frames, they may suggest filling them with initials, checks or stickers.



They can also share their thoughts about such questions as:

What kind of data is needed? How shall it be collected and displayed? What inferences can be drawn from the information?

A graph can add meaning to related mathematical topics. For example, the chart below summarizes the results of tossing two number cubes in Lessons 160–161, Crossing the Mississippi.



The results of a Crossing the Mississippi game. Each block represents one toss.

Based on these outcomes, one might predict that sums close to (or equal to) 7 are highly likely to turn up when the cubes are tossed. Why is this? Why did a sum of 1 never turn up? Sums of 3 and 11 came up about the same number of times, will this always happen?

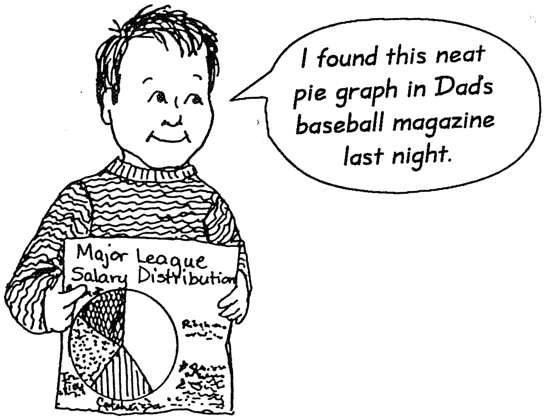
Used in these ways, graphing can help young people explore mathematics, observe relationships and become better problem-solvers.

Using a variety of graphs

There are many types of graphs that can be used in addition to the bar graph illustrated above. Some suggestions are:

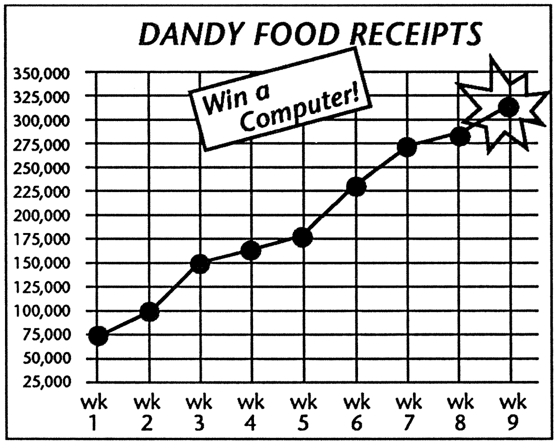
Egg carton and pom-pom graphs

Pie or circle graphs

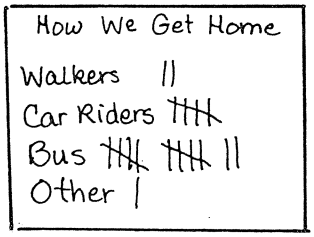


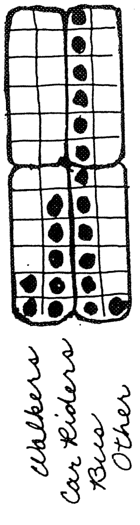
We have included blacklines of pie graphs for classes ranging from 18–35 children.

Line graphs

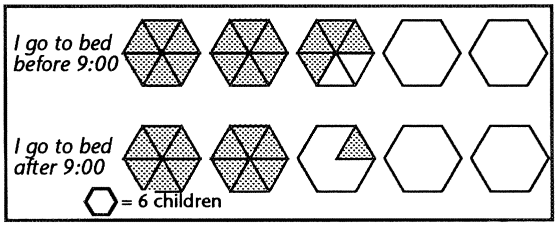


Line graphs are especially useful for displaying information that has been gathered over a period of time. An alternative to drawing dots and connecting lines is to use push pins and join them with yarn or string.

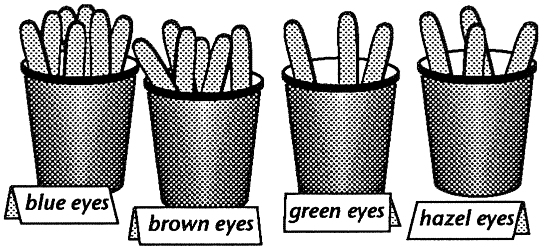


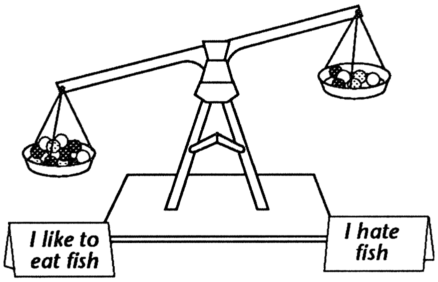


Other types of displays

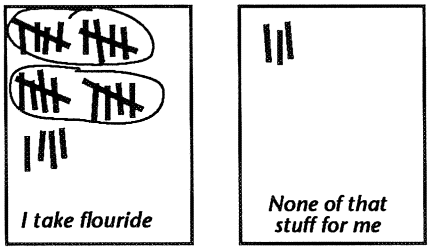


pattern blocks

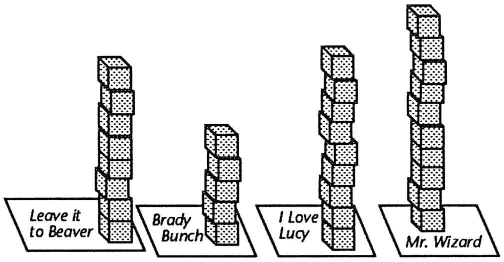




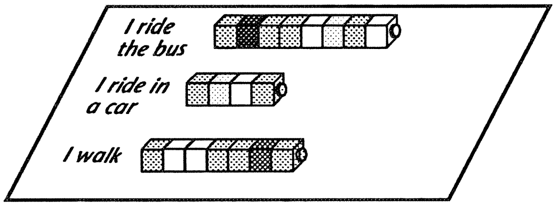
balance scale



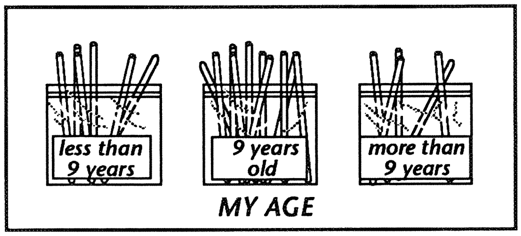
tally graph



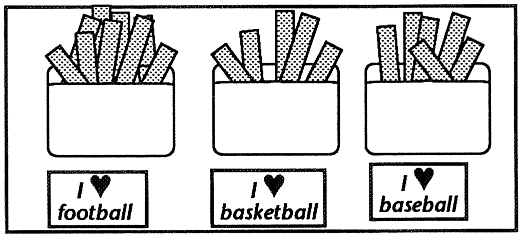
wooden cubes



hex-a-link



Ziplock bags and straws



library pockets

Whenever possible, make your graphs reusable by laminating them before labeling. After each use, the labels can be erased and the graph stored. Permanent markers write well on lamination. The marks can be erased with fingernail polish remover, hair spray or duplicating fluid. Vis-a-vis markers also work and can be erased with a tissue or paper towel.

For more graphing ideas, use the many weather related graphs in the Calendar Extravaganza.

You may wish to use different graphing strategies or topics. The lessons give some examples which can be changed to meet the needs of your classroom. As you develop this personal touch, you will find that your children will respond more positively to the activities.

To help children see that different types of graphs can show the same information, first gather data on a piece of chart paper. Challenge the students to display the information in different ways. For example, one group of four can create a bar graph, another can make a pie chart and a third a pom-pom graph. The children will enjoy comparing the various displays, noting similarities and differences, and discussing which display seems to convey the information most effectively.

Keeping all the children involved as a large group graph is being created can be a problem. You can have each child make an individual graph of the information being discussed. Keep a supply of appropriate blacklines for this purpose. Thus, all children can remain on task as the information is being recorded. This also provides each child with the opportunity to become physically involved in the whole graphing process. Our children are tickled to have a personal copy to take home. Before the graph goes home, we make sure that each child has labeled the graph and included at least one analytical statement.

You can help children connect these graphing experiences to the graphs found in environmental literature. Newspapers, magazines and textbooks are full of many types of graphs. Encourage your children to share the examples they find. When possible, recreate the graphing style that is being displayed. For example, we like to cut a weather prediction graph from the newspaper and post it in the room. We then make a blank replica of this graph and record the actual weather as it occurs. We can then observe how well the forecaster has done for the week.

Finding averages or means

The word "average" is very familiar. We often speak of such things as an average family, batting averages, test averages or average temperatures. In statistics, the word refers to a number that can be used to summarize a set of data as shown in Example 1.

EXAMPLE 1

In her first 7 basketball games, Lisa scored 4, 7, 13, 13, 11, 9 and 13 points, respectively. She averaged 10 points per game.

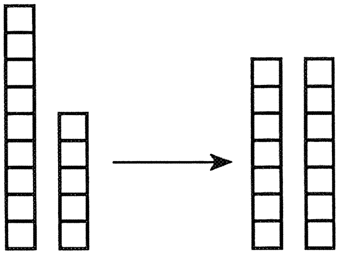
The average reported in this example is often computed as follows: (4 + 7 + 13 + 13 + 11 + 9 + 13)/ 7 = 70/7 = 10. It is also known as the *mean* of this set of data.

There are two other averages that are used in statistics: the *median* and the *mode.* The median refers to the middle value in a set of data. If the point totals of Example 1 are listed in order from smallest to largest, we have 4, 7, 9, 11, 13, 13 and 13. The median is 11 since there are 3 values below it and 3 above. The mode is the most frequently occurring value and in Example 1 it is 13.

The averaging lessons of this book refer to the mean—the median and mode will be studied in later grades. This type of average is related to the process of making things even and can be thought of visually as shown in Examples 2 and 3.

EXAMPLE 2

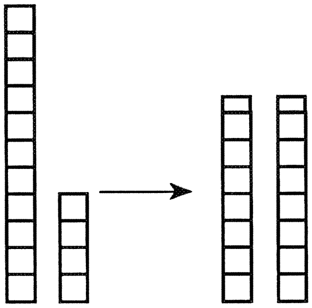
Form 2 columns of tile, one of height 9 and the other of height 5 and then even them off so that both columns have the same height.



The average of 5 and 9 is 7.

EXAMPLE 3

Find the average of 4 and 11. This may be done by evening off columns of 4 and 11 tile so that both have the same height.



The average of 4 and 11 is $7^1/_2$

Children find this process a very natural one since they make things even all of the time. They divide cookies equally, make relay teams even and make sure they get their fair share of the time in the front of the car on a trip. Stories related to these experiences can be used to introduce averaging.

LESSON 2

TEACHER *Rob and Abby took pennies from their piggy banks to purchase candy from the convenience store on the corner. Abby began to feel badly because she had more pennies than Rob. So guess what she decided to do.*

LUCAS *She decided to share with her friend so they would have the same number of pennies.*

TEACHER *That's right. Will you lay our two columns of tile to represent Rob's 7 pennies and Abby's 11 pennies? I'll also do so at the overhead.*

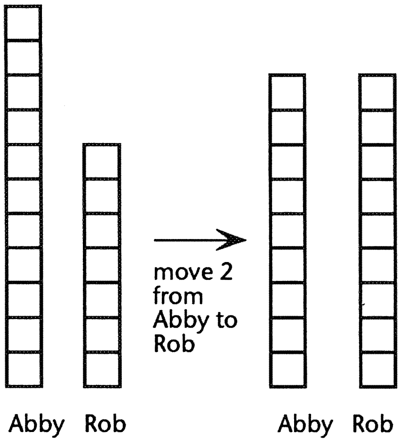
Wait until all are ready.

TEACHER *Will you now move the tiles to make their pennies even?*

CLARK *I have a way!*

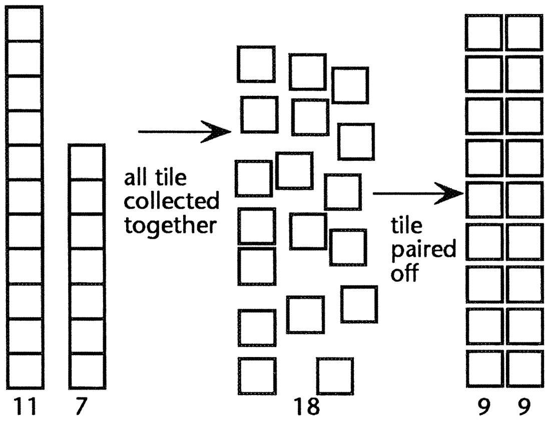
TEACHER *Please show-and-tell.*

CLARK *Sure! I just moved 2 from Abby's line to Rob's line. Now they both have 9.*



TEACHER *Thank you, Clark. Would someone else like to show a different way to make the pennies even?*

TERESE *Yes! (she comes to the overhead) I put the tile together and divided them into 2 even columns. I did this by pairing them 2 at a time. That made 9 in each.*



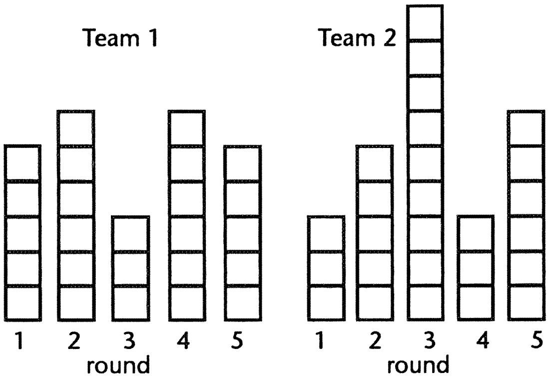
TEACHER *That's nice, Terese. If the pennies are shared equally, then each child would have 9. There is a special name for the 9 pennies in this story. We call it the average of 7 and 11.*

The lesson continues with the children representing other pairs of numbers by columns of tile and then finding the average of each pair by evening off the columns. This way of modeling the process of averaging can be extended to sets of more than two numbers as shown in Lesson 3.

LESSON 3

On the average, how many times would your children "hit the bull's eye" if they threw paper wads at the waste basket? Conduct the following experiment to find out.

Divide your class into two teams and conduct 5 rounds of shots. In each round, every child is given an opportunity to make a basket. At the end of each round, ask the children to represent their successes with a column of tile and to discuss the average number of successes at that point. Then, after 5 rounds have been completed, have them determine a final average by evening off the 5 columns of tile for each team. The illustration below shows the results of one experiment.



Discussion after each round:

Team 1

Round 1: *Five is the average so far.*

Round 2: *Our average is up a bit.*

Round 3: *Whoops! It looks like the average will be below 5.*

Round 4: *Yeah! Our average went back up. It might even be above 5!*

Round 5: *Looks like the average will be close to 5.*

Team 2

Round 1: *Three is average so far. We need more to catch the other team.*

Round 2: *Our average came up. Still need more to catch up.*

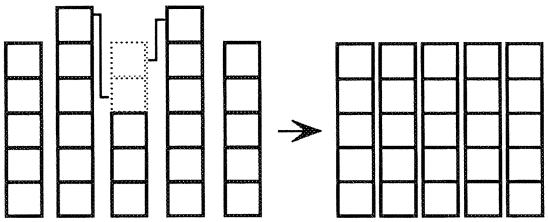
Round 3: *Wow! Our average looks like it will be more than 5! Looking good!*

Round 4: *Oops! What a bummer! Average is back down.*

Round 5: *Nice job. We think the average is close to 5.*

The next two illustrations show how the average number of successes per round for each team can be determined visually.

Team 1

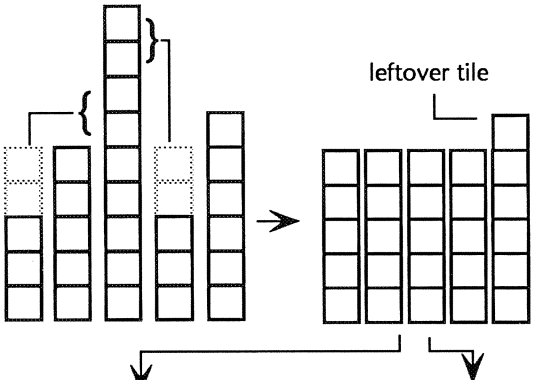


"Average is exactly 5."

"It would have been higher if the third round hadn't been so low!"

"The second and fourth rounds helped."

Team 2

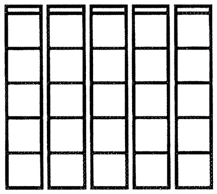


The leftover tile is one out of the 5 cubes needed to make 6 in each column. The average then is $5\frac{1}{5}$.

The leftover tile can be divided into 5 equal parts. The parts can then be distributed among the five columns.

The average then is $5\frac{1}{5}$.

"Our average is higher! If we hadn't done so well on the third round, though, we'd have been hurting!"

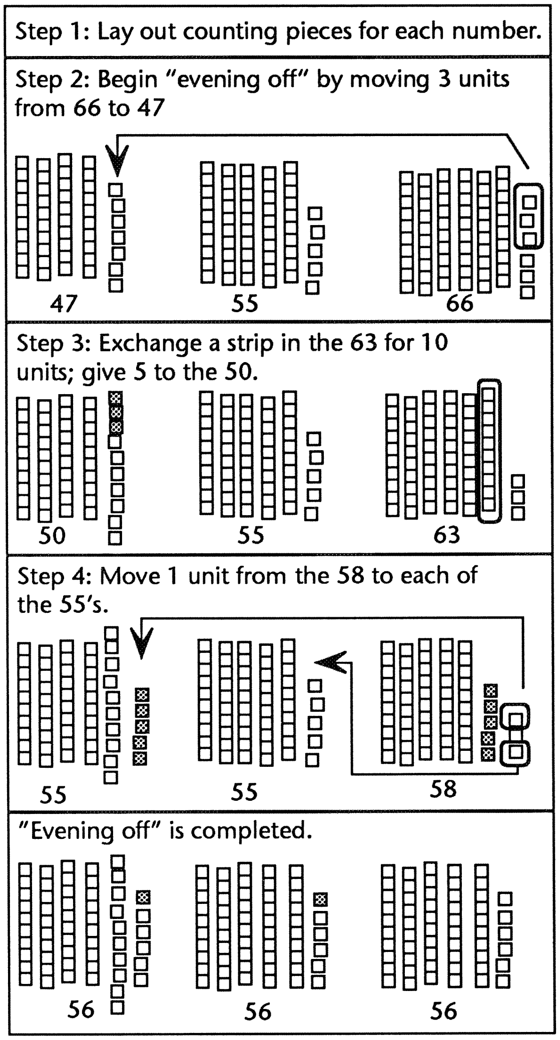


Notice the discussion that takes place after each round in this lesson. This helps children develop their sense of how large an average is and how it can vary. Notice, too, the two different ways Team 2 thought about the extra tile it had when determining its final average.

EXAMPLE 4

The average of larger numbers can be determined by evening off collections of base ten pieces. Example 4 illustrates one way of doing this.

Find the average of 47, 55 and 66



Step 1: Lay out counting pieces for each number.

Step 2: Begin "evening off" by moving 3 units from 66 to 47

Step 3: Exchange a strip in the 63 for 10 units; give 5 to the 50.

Step 4: Move 1 unit from the 58 to each of the 55's.

"Evening off" is completed.

The averaging lessons in this book focus on the "evening off" model described above. We believe that formulas or algorithms, such as the one discussed in Example 1, are best left for study in later grades.

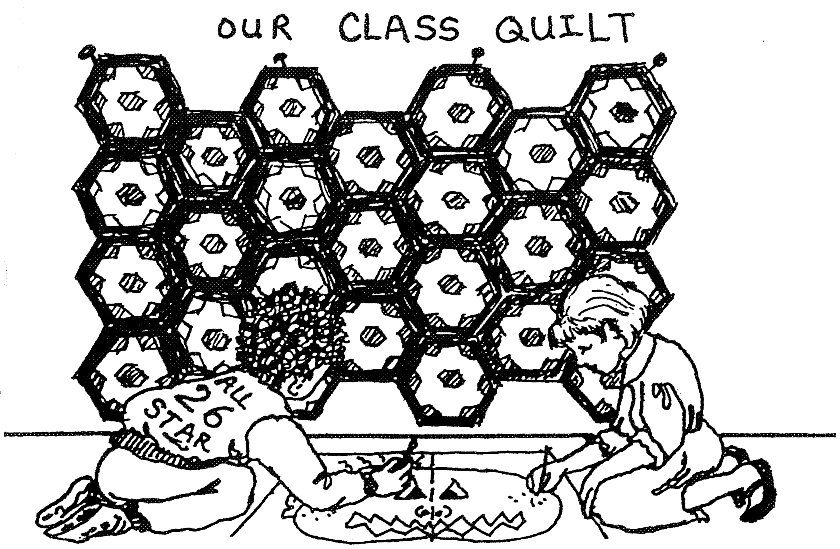
Conclusion

This chapter has discussed activities that will help your children strengthen their ability to construct and interpret graphs and see how information can be processed and summarized. We recommend that such activities be a regular part of your mathematics lessons.

Don't limit these experiences only to mathematics, however. They can be an excellent means of integrating curriculum and demonstrating applications of mathematics. For example, you could kill two (or even three) birds with one stone by analyzing data gathered from a science experiment or a social studies survey.

11 Geometry

Geometry helps us represent and describe in an orderly manner the world in which we live. Children are naturally interested in geometry and find it intriguing and motivating... *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 48*



Geometry has been an important branch of mathematics since the time of the ancient Egyptians. At first, interest in the subject was motivated by practical concerns such as surveying land. During this time, people used very informal methods for measuring and for drawing conclusions about the geometric aspects of their environment. At a later time, the Greeks began a more formal, theoretical study of the subject. The deductive methods of reasoning that are presently used in high school geometry classes trace their way back to those developed by the Greeks.

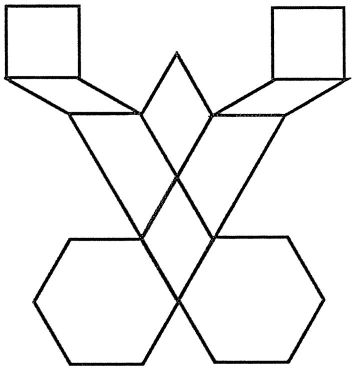
We study geometry today because it still has numerous practical applications. It also provides insight into how shapes make our world more interesting and beautiful. Geometry is a subject that helps people develop the spatial sense needed to understand their world better. In addition, there is a positive correlation between sound spatial awareness and good understanding of numerical and measurement concepts.

Children are fascinated by shapes. They have played with them since infancy and take great pleasure in building with them. Through such exploration they become aware of the properties of two- and three-dimensional shapes and of the relationships that exist among them. The geometry lessons in *Opening Eyes to Mathematics* extend this kind of activity to help your children learn more about shapes and increase their understanding of spatial relationships.

The geometry discussed in the elementary grades is often limited to the identification of shapes. While such work is very important, we feel that children should also engage in more active, dynamic experiences. In our activities, children construct shapes, move them about, and change or combine them in order to discover geometric relationships and develop spatial awareness. These sample lessons illustrate how this can be done.

LESSON 1

TEACHER *(revealing a design on the overhead) I have made a design with some pattern blocks. What things do you notice about it?*



PEARL *It looks like a person who is holding up two rocks!*

MANUEL *I think it's a bug crawling along!*

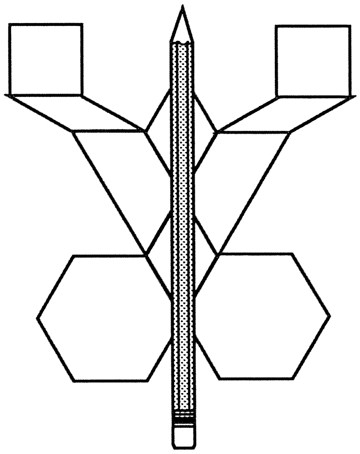
JOHN *If I look at it upside-down, it reminds me of a robot!*

CARMEN *It looks the same to me on both sides—it's like being balanced.*

MATT *I see what you mean, Carmen. The colors and the pieces match up.*

TEACHER *Can you explain further?*

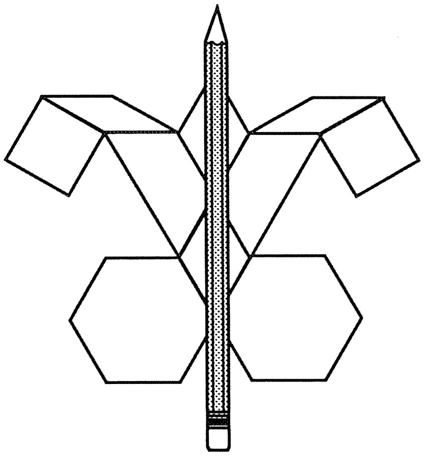
CARMEN *(at the overhead) Well, if I put a pencil right in the middle like this, the left side has the same pieces as the right. The blue diamonds are split.*



JOSE *(demonstrating) If you could somehow flip the left side over Carmen's pencil it would fall right on top of the right side.*

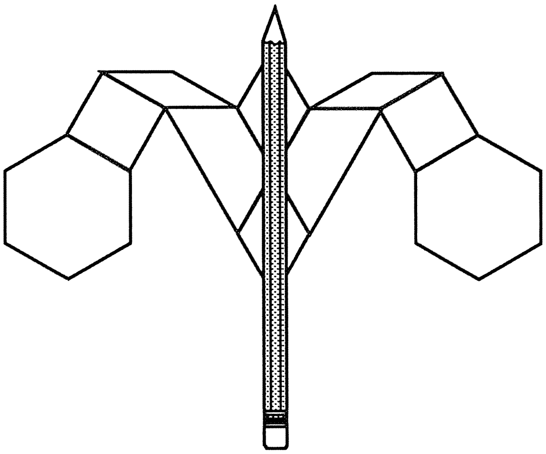
TEACHER *Yes, this design is symmetrical. It has a line of symmetry and Carmen has placed her pencil on it.*

BILL *(at the overhead) Look what happens if the orange squares are moved down like this. It looks like a person who is lifting two pails! There is still the same line of symmetry!*

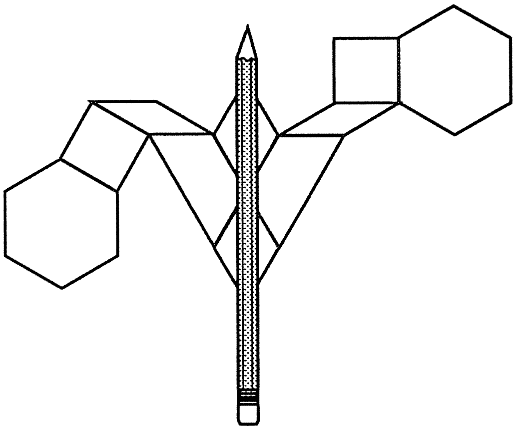


TEACHER *Can you move other pieces and still have a line of symmetry?*

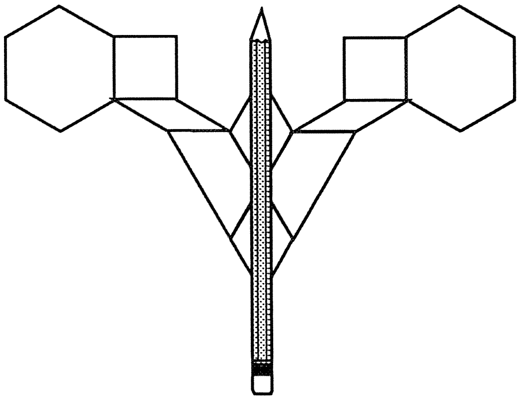
MARCY *I can! Move the yellow hexagons like this. It looks like a giant bird to me!*



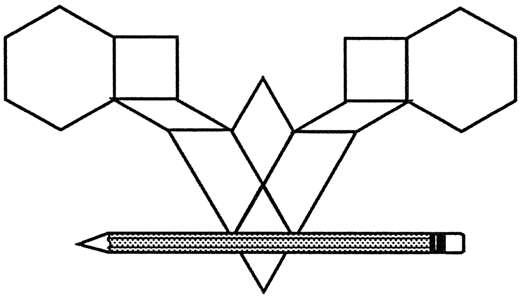
TEACHER *Could I move a square and hexagon like this?*



DAISY *Well, to keep the balance you should also move the other orange and yellow pieces.*



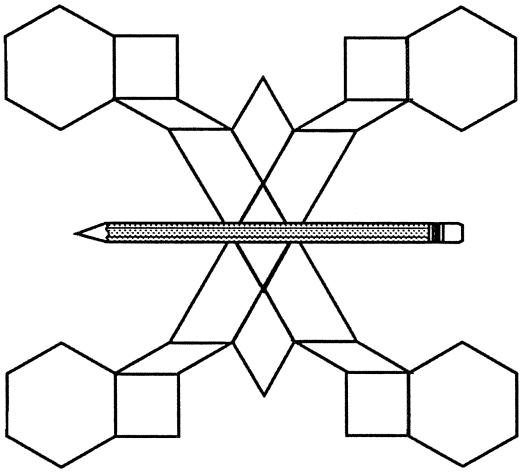
TEACHER *Suppose I put the pencil here. Is this new position a line of symmetry?*



CARMEN *No, the design isn't balanced that way. The top doesn't match the bottom; it can't be flipped over the pencil onto the bottom.*

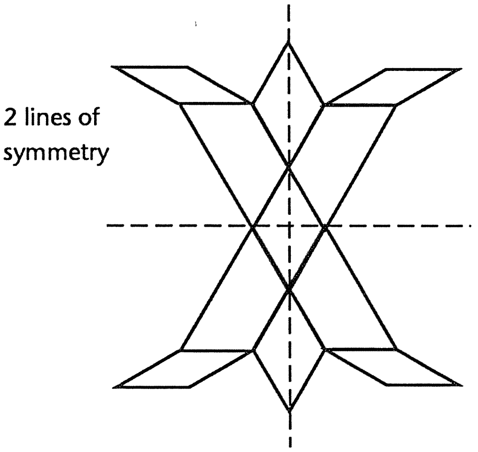
TEACHER *How can we add blocks to the design to make this new position of the pencil a line of symmetry?*

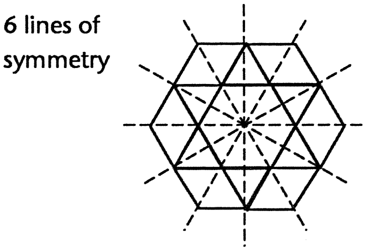
The class discusses how this might be done and children add blocks to the design



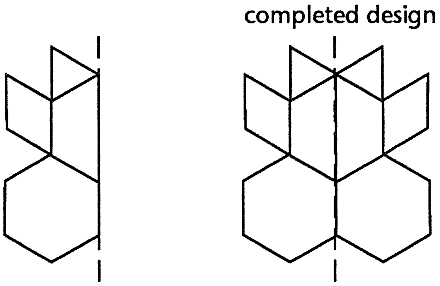
The lesson can continue in several ways, two of which are summarized here.

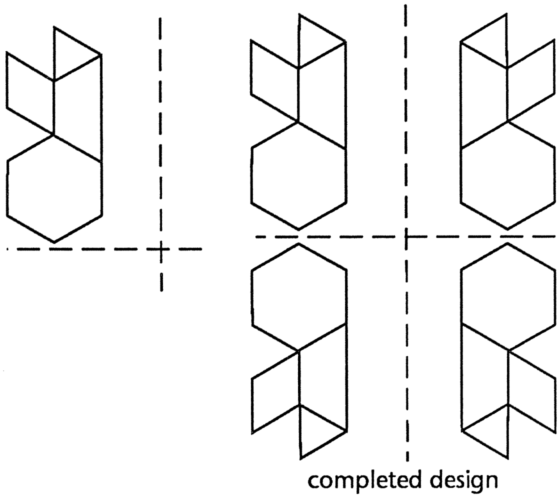
- Challenge the children to create designs of their own that have symmetry. Can they make some that have one line of symmetry? Two? Three?





- Group the children and have one member of the group make a design and identify a line that is to be a line of symmetry. The others then complete the design.

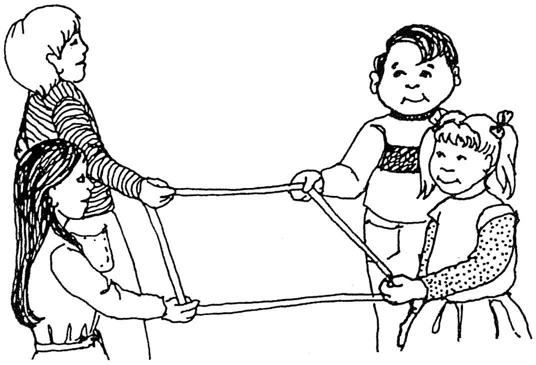




LESSON 2

(Adapted, with permission, from *Box It or Bag It Mathematics* by Burk, Snider and Symonds. See Contact Lesson 51.)

TEACHER *(after giving a large rubber band to each group of 3–4 children) Within your group, would you please stretch your rubber band to form a square—have each person hold a corner of the square.*



TEACHER *What can you say about the squares you have formed?*

CHILDREN *(offer various responses)*

There are four sides and four angles.

It looks like the units of our counting pieces.

It has the same shape as that picture on the wall.

It reminds me of a baseball diamond.

All sides should have the same length.

The angles should all be the same.

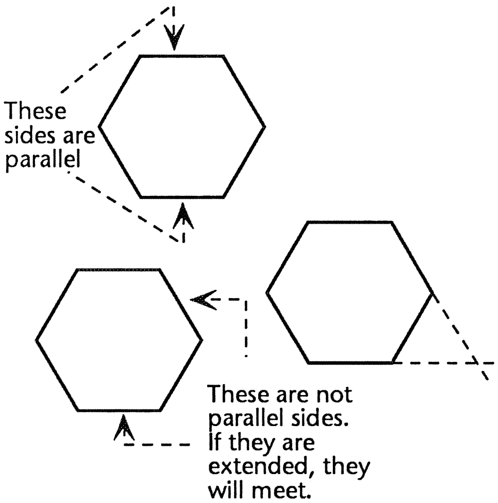
Some of the sides touch each other, but some don't.

The teacher uses these observations as a basis for discussing some related vocabulary, such as parallel lines and right angles. The children are asked to find examples that clarify the meaning of these terms.

CHILDREN *It looks like the top of the wall is parallel to the bottom.*

I see some right angles in that window frame.

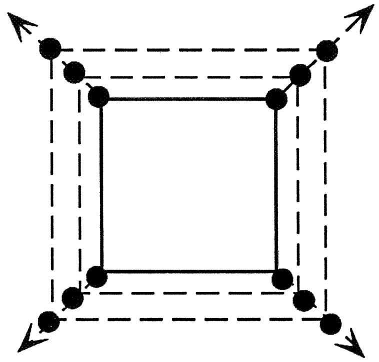
Look at this hexagon—some of its sides are parallel and some aren't.



The lesson can continue in many ways.

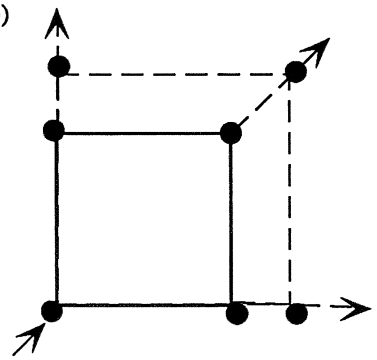
- Challenge the groups to come up with several procedures for making their squares a different size. Here are some that might be shared:

a)



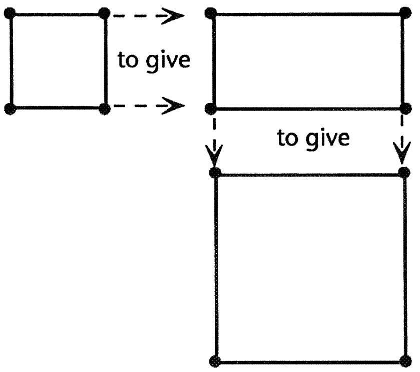
Each child pulls corners out like this

b)



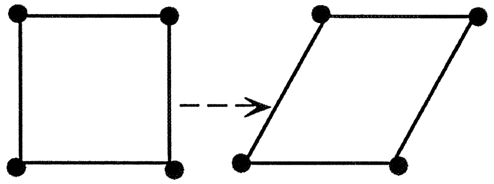
This child stays put. Others move out as shown.

c)



- Have the groups try to form a square without using their hands. As an example, some children may anchor the corners of the elastic with their necks!
- Ask the children to explore ways to change their squares into other shapes. They might notice that by releasing one corner of their square they form a triangle. Or, by holding the band in five spots, different kinds of pentagons can be created. Have them describe these shapes and search for relationships that exist among them.

LUC *Our group made this shape. It doesn't look like a square even though the sides seem to be the same.*

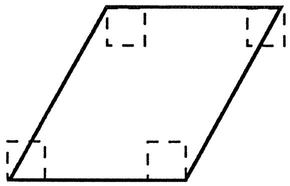


TEACHER *How is your shape different from a square?*

LUC *Some of the angles seem larger than others.*

TEACHER *How can you be sure of that?*

LUC *Well, we could take one of the mats from the counting pieces and see how its angles match with our shape. It looks like 2 of their angles are smaller than a right angle and 2 are larger.*



TEACHER *Yes. Now, how is their shape like a square?*

MAUDE *The sides are the same length.*

BRIAN *It has some parallel sides.*

AMANDA *Actually, it's like a square that has been tilted.*

TEACHER *What other shapes can you form with your rubber bands?*

The groups continue to make and describe other shapes that can be made.

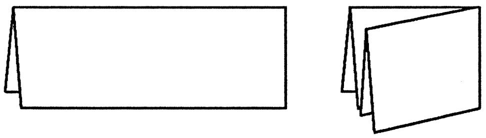
The above lessons show how it is possible to engage children in geometric activity. In the first lesson, they operate on shapes by moving them about to create symmetrical designs; in the second one, they make and classify shapes, discussing their properties and observing how they are related to one another.

In the following lesson, the children form mental pictures of a shape that is created in a paper-folding activity. They describe what they see and the concept of congruence is discussed.

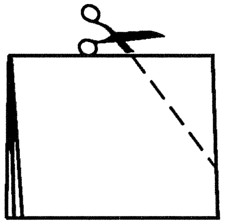
LESSON 3

The children have been given scratch paper, scissors and rulers.

TEACHER *Watch closely now, I am going to fold this paper twice.*



Now I am going to draw a straight line from one fold to the other and start a cut along this line. What do you think you will see when the part that is cut off is unfolded?



ALLEN *It looks like a triangle before it's unfolded. I think we will be able to see some triangles.*

PAULA *I think so, too! When you open the first fold, I see 2 small triangles. Then, when you unfold again, I think there will be 2 large triangles.*

TEACHER *What can you say about these triangles?*

PAULA *The small ones will be the same because one can be folded on the other. The same for the large ones.*

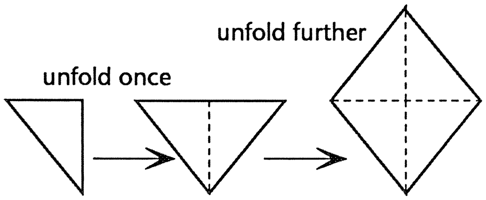
JIMMY *I think there will be 4 of the small triangles that Paula is seeing. They can all be folded onto each other.*

TEACHER *If so, we can say the triangles are congruent, meaning they have the same size and shape.*

BUCK *I'm not sure how many sides there will be, but the cut sides will all be the same length.*

ERIN *I think there will be 4 sides, 2 for each time you fold. It might be a square.*

TEACHER *(completing the cut and unfolding the shape, which will be a diamond or possibly a square):*



Can you do the same thing using your paper and scissors? Fold the paper as I did, draw a straight line from one fold to the other and make a cut along the line.

The children follow the instructions. They check out the predictions made earlier and share other observations about the shapes. For example, the children might describe the symmetry that is present or the right angles the folds create with each other. They might also determine how to make a cut that will unfold into a square.

The lesson can be extended further by folding the paper in other ways. Also, if a computer is available, the children can be asked to create a display of their shapes with a language such as Logo.

In the sample lessons, the concepts of symmetry, parallelism, congruence and angles are discussed in an informal, hands-on manner. The associated vocabulary is used in a natural context. The spirit of this approach is used in all of the geometry lessons in this program. Some other topics and concepts explored in this way are perpendicularity, segments, polygons, circles and three-dimensional shapes.

Using other geometry strategies

We help children become better observers of their world by having them look for shapes present on their bodies, in the classroom, and in the architecture of buildings. We begin by conducting a search for items and then pose various questions about them.

- line segments: Do they intersect? If extended, would they ever meet? Are they the same length? What purpose do they serve in a shape or design?
- polygons, circles and three-dimensional objects: How do they vary in size? Are there various textures within their areas? What kinds of patterns do they form? What symmetry is present? Do they have angles that are larger or smaller than a square corner? How are they alike or different? Are any exactly the same shape and size? Are any the same shape but different in size?

The activities of the Contact lessons also emphasize connections between geometry and items that are part of a child's world. These include observing the shapes that can be found on a football field and making designs by folding paper in special ways.

There are also many ways to relate geometry to other subjects in the curriculum. For example, in the dialogue below, the children think about geometric ideas as they make a wind sock.

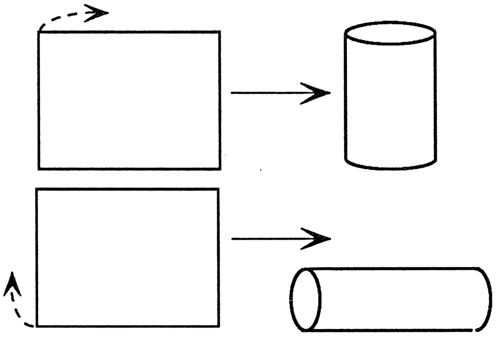
Each child has been given a 12 × 18 sheet of paper.

TEACHER *What shape is your paper?*

SANDY *It's a rectangle.*

TEACHER *Sure. Can you make a cylinder from your rectangle? Glue your paper and show the class.*

The children show different ways to form a cylinder. Some will have wider bases than others.



TEACHER *Each cylinder will be empty inside.*

MYRNA *This reminds me of a pipe my daddy was putting in. He said the pipe was hollow.*

TEACHER *Yes, our cylinders are hollow. Can you think of anything else that is hollow?*

RYAN *A drinking straw!*

JIM *Seems like I've seen hollow logs on cartoons.*

JENNIFER *I've heard my older brother talk about hollow walls. There's a space inside them.*

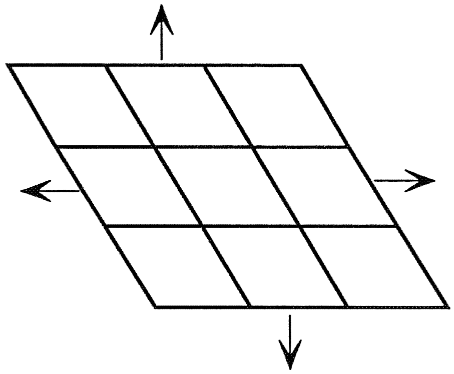
TEACHER *Wind socks are hollow, too.*

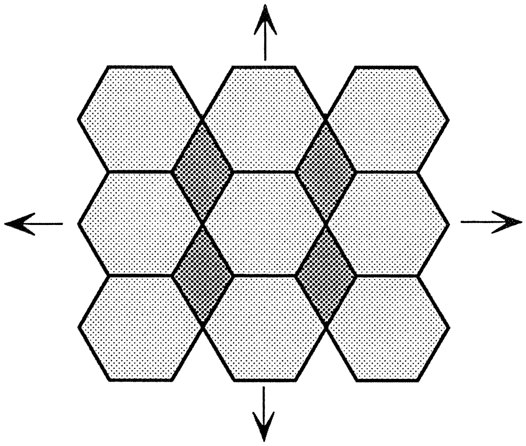
MITCH *I've seen ones that have streamers on them.*

TEACHER *You could attach some streamers to your cylinders to complete your wind socks.*

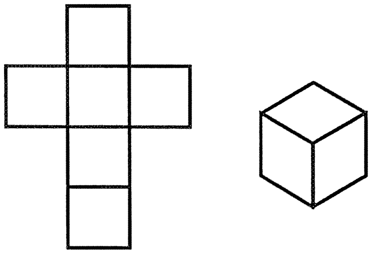
As the children finish the project, the discussion continues with questions like, "Why are wind socks hollow?" and "Do the streamers serve any purpose?"

In other lessons, your children will create colorful designs by drawing shapes on paper or by making tessellations. A tessellation is any arrangement of shapes that covers a surface without any gaps or overlaps. Shown below are two examples of tessellations formed with pattern blocks.





Spatial awareness is also developed if children are able to relate two-dimensional shapes with three-dimensional ones in a meaningful way. Most children in school have played with three-dimensional objects from early childhood. Many of the lessons build upon this experience. Students examine objects from different viewpoints and investigate ways to create objects by folding patterns of two-dimensional shapes.



This pattern of six squares will fold into a cube. Can you find other patterns of six squares that will do the same?

As you organize geometry instruction, keep in mind that language is not necessarily the main focus at this time. We believe that activities should be informally conducted, with children having plenty of opportunity to look for and describe geometric relationships in their own words. Vocabulary can grow from this as needed.

Geometric activities will help children develop a sense of measurement and of number. As children examine shapes, they will notice how these shapes vary in size and can devise ways to estimate and measure perimeters, areas and volumes. They will also form mental images of numbers by thinking about shapes in special ways. For example, the operations of multiplication and division can be modeled by rectangular arrays; fractions can be visualized by subdividing shapes into equal parts. See the chapters on measurement and numeration for additional discussion and examples of this.

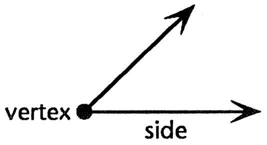
Vocabulary refresher

The following list defines and/or pictures some standard terms used in geometry. When discussing these with your children, do so informally in the context of related activities.

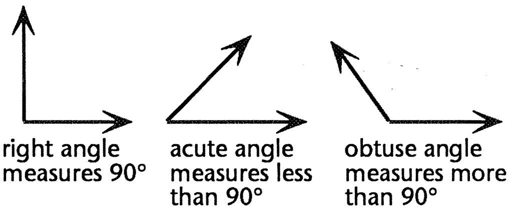
line segment: two points on a line and all the points between them.

ray:

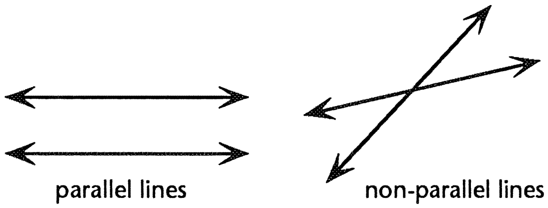
angle: two rays that have a common endpoint. The endpoint is called the vertex of the angle. The rays are the sides of the angle.



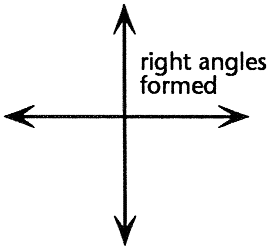
types of angles:



parallel lines: lines in a plane that will never intersect.

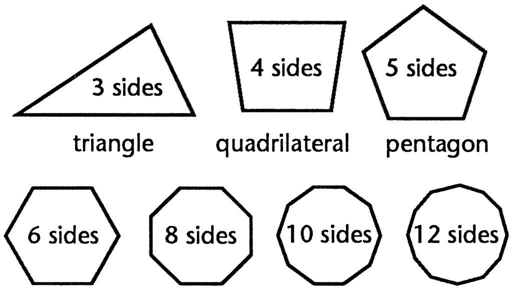


perpendicular lines: lines that form right angles.



perpendicular lines

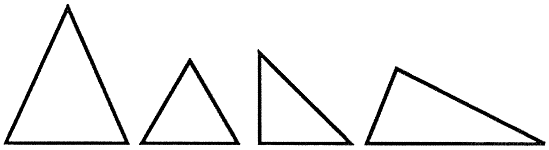
polygon: a simple closed curve whose sides are line segments.



triangle quadrilateral pentagon

hexagon octagon decagon dodecagon

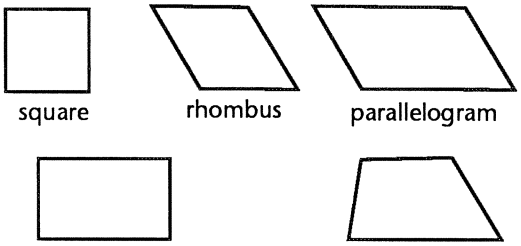
types of triangles:



isosceles equilateral right scalene

isosceles: at least two sides are same length
equilateral: all sides are same length
right: contains one right angle
scalene: all sides are of different length

types of quadrilaterals:

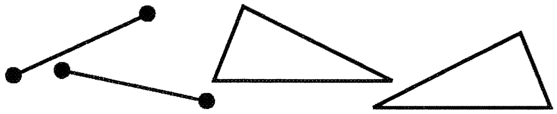


square rhombus parallelogram

rectangle trapezoid

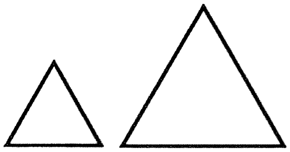
square: four sides of equal length and four right angles
rhombus: opposite sides parallel and all sides of equal length
parallelogram: two pairs of parallel sides
rectangle: four right angles; opposite sides are parallel and of equal length
trapezoid: a pair of parallel sides

congruent figures: figures that have the same size and shape. One can be placed exactly on top of the other.



congruent line segments congruent triangles

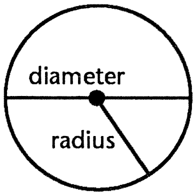
similar figures: figures that have the same shape



similar triangles

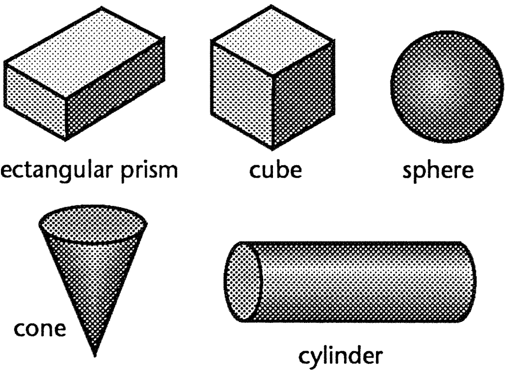
terms related to a circle:

radius: a line segment that joins the center to a point on the circle. The length of this line segment is also called the radius of the circle.
diameter: a line segment that joins two points of a circle and passes through the center. It's length is twice the radius.
circumference: the distance around the circle.



circumference

some common three-dimensional objects



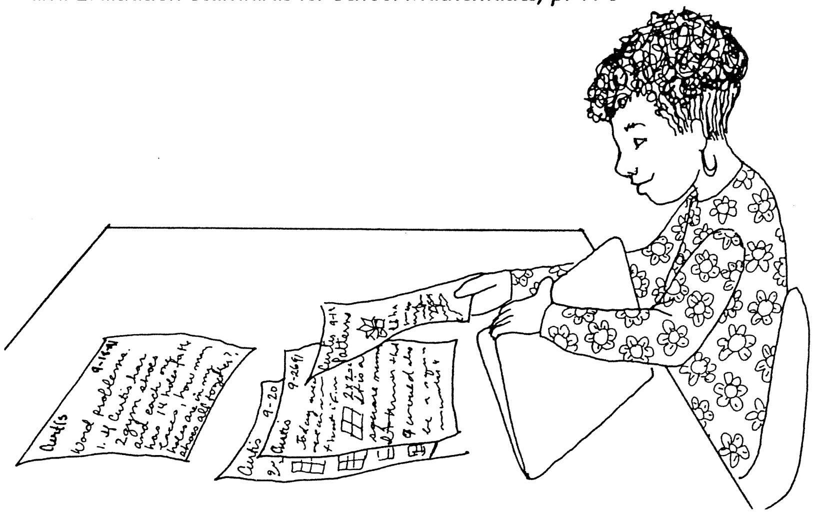
Summary

Shapes play an important role in everyday life. They come into play when arranging pencils and crayons in a school box or in developing a plan for arranging furniture in a room. Shapes are vital in designing buildings and ensuring that structures are sturdy and strong. Where would our world be without the contributions of such greats as Frank Lloyd Wright, Buckminster Fuller, Picasso or the Wright Brothers, all who have applied their genius to the creative use of shapes?

We encourage you to invite local architects, artists, engineers, carpenters and draftsmen to speak about their professions and their use of shapes. Children enjoy identifying what they are learning with what "big people do".

12 Assessment

The learning of mathematics is a cumulative process that occurs as experiences contribute to understanding. A numerical score or grades assigned at a single point in time offers only a glimpse of students' knowledge. If the goal of assessment is a valid and reliable picture of students' understanding and achievement, evidence must come from a variety of sources. *NCTM's Curriculum and Evaluation Standards for School Mathematics, p. 198*



In *Opening Eyes to Mathematics*, children are actively involved in learning mathematics. With the help of models and visual thinking, they develop mathematical understandings and explore problems from many points-of-view. In addition, they participate in mathematical discussions in which they share solutions and explain their thinking, knowing their ideas will be valued.

The learning that takes place in this program is reflected daily in this atmosphere of mathematical exploration and communication. Assessing this learning, therefore, can grow from ongoing observation of the children as they work individually or in small groups, and as they participate in whole group discussions or presentations. We feel that, to be effective, assessment needs to come from many sources and be positive. Here are some of its major purposes:

- Provide information about the mathematical growth and thinking of each child.
- Reveal the children's thoughts and feelings about mathematics.
- Encourage children and provide further opportunities that motivate them to grow.
- Serve as a basis for planning instruction and, if necessary, assigning grades.

"... the act of teaching should be founded on dialogues between teachers and students, each responding to the other on the basis of what has been said or done. Assessment refers to the process of trying to understand what meanings students assign to the ideas being covered in these dialogues; as such, it is an integral element of effective teaching. Periodic assessment provides the teacher with a basis for deciding what questions should be asked and what examples and illustrations should be used; ultimately, it offers a foundation for any meaningful dialogue between teacher and student." NCTM Curriculum and Evaluation Standards for School Mathematics, page 203.

This chapter describes several assessment techniques that will help you assess your children's learning. Much of the discussion summarizes, with permission, the recommendations for assessment presented in *Visual Mathematics* course guides. The philosophy and instructional approach of that course guide are compatible with those of this book. *Visual Mathematics* is an excellent resource and you may wish to consult it for further information. (These are available from MLC Materials.)

Assessment techniques

The assessment of learning in any classroom should relate directly to the instructional goals of each lesson. In *Opening Eyes to Mathematics,* those goals are to help each child:

- Construct understandings about mathematical concepts.
- Develop their ability to use visual thinking.
- Develop communication, reasoning and problem-solving skills.
- Learn to work cooperatively with others.
- Respect the thinking of others.
- Look at mathematics from many views.
- Strengthen their self-esteem and self-confidence.

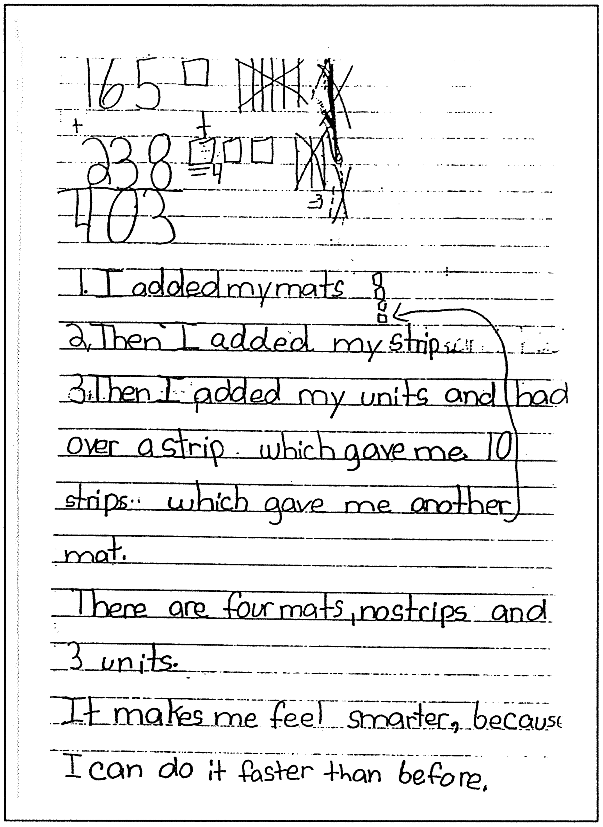
With these goals in mind, we have found the following activities to be useful assessment instruments:

- Writing activities
- Checklists
- Informal observations and questioning
- Individual conversations
- Self-evaluation
- Tests and quizzes.

We usually compile the information gathered from these sources into student folders that are helpful when planning instruction or conferencing with parents. Samples of children's work are also entered periodically into this folder.

Writing activities

Writing is an integral part of many lessons in this program. Encourage your children to prepare reports of individual and small group work and to regularly record journal entries of their thoughts and feelings. When appropriate, invite them to share their written responses with others. The accompanying discussion helps them to clarify their thinking and learn from one another. These activities reveal useful information about children's understanding and are well worth the time spent on them. They also will help you judge the nature and amount of practice your children need. Excessive practice is unnecessary for those who clearly understand a concept and unproductive for those who don't.



165
+238
403

=4
=3

1. I added my mats 8

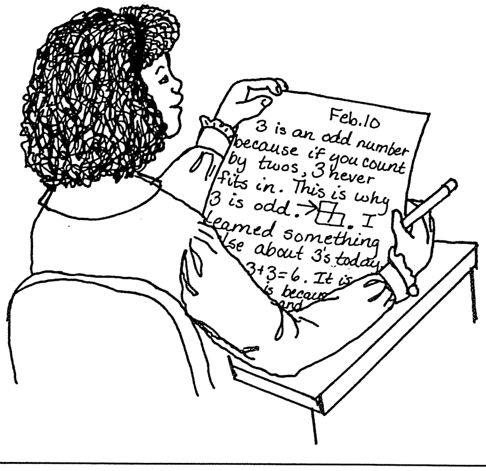
2. Then I added my strips

3. Then I added my units and had over a strip which gave me 10 strips which gave me another mat.

There are four mats, no strips and 3 units.

It makes me feel smarter, because I can do it faster than before.

As is true in all aspects of this program, we urge you to respond to your children's writing in positive ways and to indicate that everyone's contributions and feelings are valued. It helps, for example, to include at least one positive comment on each paper. This promotes a non-threatening climate for learning and the development of positive attitudes among your children. We also encourage you to model the importance of journal writing by keeping one of your own, perhaps even sharing entries from time to time with your class.



Checklists

A checklist can be a very helpful and convenient assessment instrument. It can be used to focus on those qualities of achievement or attitude that you consider important. See sample checklists below.

Checklists can be used in the classroom in several ways:

- Use over a period of days to gather information about the work of either small groups of children or of the entire class.
- Use periodically to monitor the attitudes, participation and communication abilities of your children.
- Occasionally ask children to evaluate themselves in one or more checklist categories.
- Use to observe children with a particular set of categories in mind. Since children may respond differently in a small group than in a large group, observe them in both settings.
- If possible, videotape small groups of children over a period of several days and use a checklist while viewing the tapes.

														name of child
														works well with others
														chooses appropriate unit(s) of measure
														compares unit(s) to object
														calculates correct number of units
														understands standard and non-standard units
														estimates before measuring
														records and compares data

Measuring Checklist

date: student's name	demonstrates communication	demonstrates application	demonstrates connections	uses visual models	works well with others
comments:					

Informal observations and questioning

As you implement the lessons of this program, there will be many opportunities to observe and question your children's work and thinking. Much of this can take place when they are engaged in small group activities or when they show-and-tell at the overhead. Sometimes you observe silently; other times you may want to ask a helpful question.

Questions should be designed to elicit your children's thinking and problem-solving strategies. Some questions helpful for this purpose are:

- Can you explain your thinking?
- How do you see that?
- Can you predict what will happen if ________?
- Can you describe this problem in your own words?
- Could you solve this problem in another way?
- Can you describe your thought process when you _________?
- Can you apply this knowledge to a different situation? Can you think of a time when you might use it outside of school?
- How do you feel about ___________?
- Can you draw a picture or build a model to show that ________?
- How can you be sure that __________?
- Do you see a pattern?
- Does this remind you of other discoveries?
- What pictures come to mind when you think about ________?

Find ways to record your observations quickly. Checklists, notecards and videotapes are some of the methods you might use. A videotape is useful for recording the activities of one group of children while you are free to respond to the needs of other groups.

Individual conversations

Engaging in a conversation with a child is a very special opportunity to assess their thinking about mathematics. The conversations are not intended to be a time for teaching. Rather, their primary purposes are to:

- Give insight into how the child perceives and understands mathematics.
- Allow children to demonstrate their methods of reasoning when solving a problem.
- Reveal information about children's confidence when analyzing a mathematical situation.

Conversations of this sort can take place as other children engage in independent practice. Even though they take time, they can provide information regarding the direction of instruction and are particularly helpful for children having difficulty in class. Here are some suggestions for conducting these conversations.

- Put the child at ease by talking about something of personal interest first.
- Tell the child that you are interested in how they think.
- Use how, why and when questions to probe for deeper meaning.
- Listen intently to the child. Provide plenty of time for them to think and respond.
- Converse with the child in a non-judgmental manner; be comforting and reassuring. Avoid questions that allow for only one specific answer (that you have in mind); ask open-ended questions that encourage the child to reflect. Remember, too, to check your body-language; is it consistent with your intent?
- The main purpose of the conversation is to gain insight into how the child is thinking. Instruction about concepts can be given at another time.
- When possible, use technology to your advantage. By making an audio or video tape of the conversation, you can concentrate totally on the child. Later, you can review the recording(s) to get a clearer view of that child's thought process. The tape will also let you evaluate the effectiveness of your own part in the conversation.
- The conversation need not last longer than five to ten minutes.
- Keep brief notes of each conversation.

Self-evaluation

Self-evaluation can be one of the most valuable assessment instruments. It provides teachers with direct information as to how children perceive their own work and what they feel about it. In addition, by reflecting on their own performance, they are encouraged to take responsibility for their own learning. The ability to do this is an important part of being an independent learner. (See illustration on following page.)

Tests and quizzes

Tests and quizzes are traditional ways of assessing children's performance. The spirit behind this program suggests, however, that they should not be the main source of assessment information. Further, when they are used, they should reflect the goals of the program, focusing on such things as: conceptual understanding, communication of thinking, use of visual thinking and making con-

Shade in the line to show how much you thought about math outside of class this week.

Often
10
9
8
7
6
5
4
3
2
1
Seldom

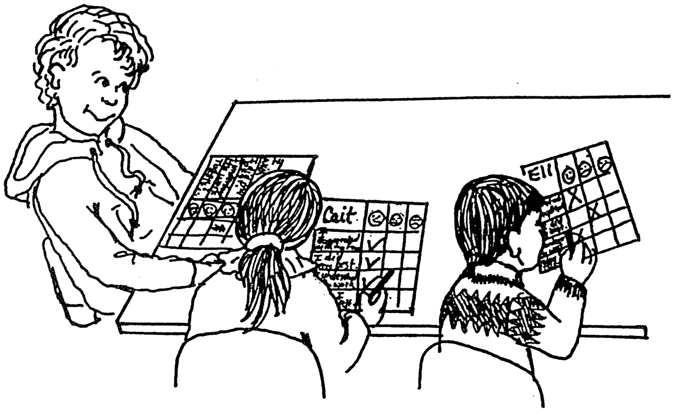
Color the line to show how well you understand __________________.

Describe any "highs" or "lows" you have experienced.

HIGH
10
9
8
7
6
5
4
3
2
1
LOW

Color the line to show how well your group worked together. If the ranking is low, tell what you could do to encourage better cooperation. If your ranking is high, tell how it was good.

HIGH
10
9
8
7
6
5
4
3
2
1
LOW



Samples of self-evaluation instruments

nections among topics.

In this program, it would be appropriate to conduct testing in different settings (individual, small groups, in class and even at home), and to make models and calculators generally available to children. Because of the writing involved, these tests will contain fewer items than traditional ones, yet require just as much time.

Consider varying the kinds of activities that you include on tests. Here are a few ideas to try:

- Ask the children to pose and solve a few problems about a selected topic. Have them write questions of varying difficulty and also explain what makes each question easier or harder.
- Present the children with a problem that has been incorrectly solved and ask them to explain the error(s).
- Have the children develop strategies for solving problems using diagrams, sketches or mental pictures.

Finally, you might anticipate changes to take place in the nature of standardized tests. Many organizations such as the National Council of Teachers of Mathematics are urging that these tests become more consistent with the curricular changes that have been recommended for school mathematics. In the meantime, we can place less emphasis on this type of test.

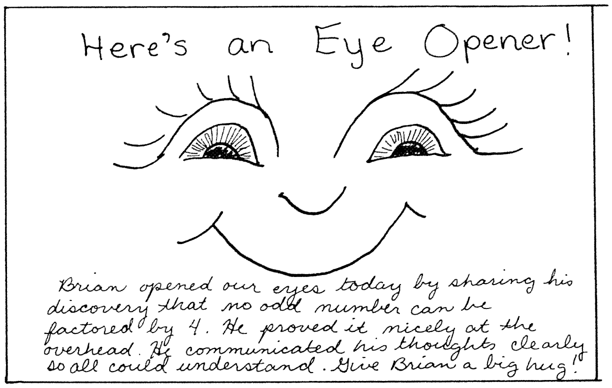
Communication with parents, administrators and colleagues

Many parents, administrators and teachers in your district may have questions about this program or even feel somewhat uncomfortable with it at first. For example, some might feel that there is too much "play" and that more paper-and-pencil drills are needed. This is not surprising since it is likely that they had quite different mathematical experiences when they were in school.

"There are, of course, barriers to the implementation of these standards, the most important being the strongly held beliefs, expectations, and attitudes of all people in education about specific aspects of the reform. A teacher who believes that speed in paper-and-pencil calculation is most important will be reluctant to let children use calculators. The administrator who has charted group scores on a standardized test for years will be reluctant to replace it. Parents who expect students to do mathematics homework on paper at a desk rather than by gathering real data to solve a problem will be surprised." NCTM Standards, page 255.

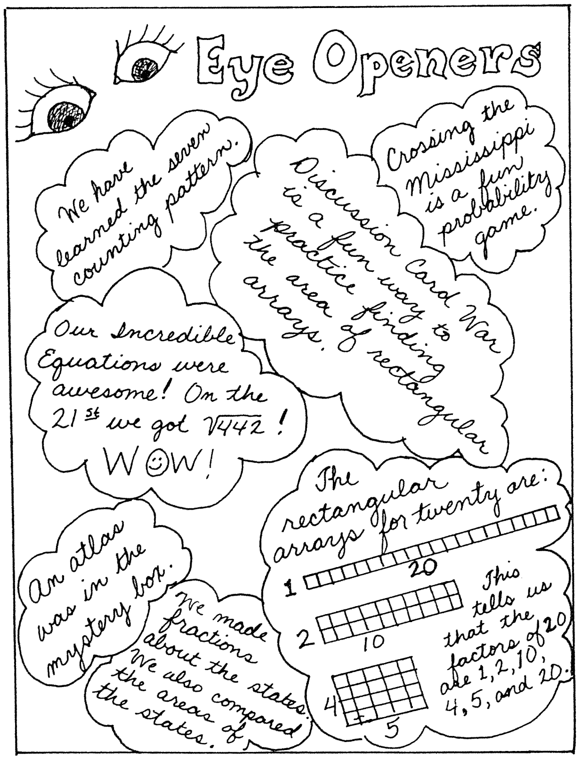
In view of this, it is very important to communicate with people about the program and seek support for it. We have found the following ideas for doing this to be helpful.

- Conduct a model lesson, or show a video of one, at an open house or PTA meeting. While it's good for the adults to see children exploring mathematics in such a lesson, it would also be helpful and fun for *them* to be the students. Activities taken from the Calendar Extravaganza or related to the Mystery Box work very nicely here. Use the lesson as a context for discussing the instructional goals of the program and the changes in mathematics education being recommended.
- Once the school year has settled down for you, invite parents, administrators, board members or colleagues to your classroom. Again, have them participate in the activities along with the children.
- Include a mathematics section in a class newspaper. This might contain articles written by children, brain-teasers for the family (or a call for some from parents), announcements of upcoming events related to math or summaries of special mathematical discoveries that children have made.
- Host a night of math enrichment for the entire family. This is wonderful opportunity for family members to enjoy mathematics together. It always turns out to be informative and fun for everyone!
- Bring in guests to speak on the role of mathematics in different careers or to discuss mathematical connections with the outside world. Be sure parents and administrators are invited to attend.
- Invite parents to assist you in the classroom or with the preparation of materials.
- Ask parents to submit brain-teasers or other items that could be used to promote mathematical discussions or explorations.
- Keep parents informed of their child's progress. Conferences are an ideal time to review the goals and expectations of the program.
- Your parents will enjoy receiving *Opening Eyes to Mathematics* post cards telling them about their child's work.

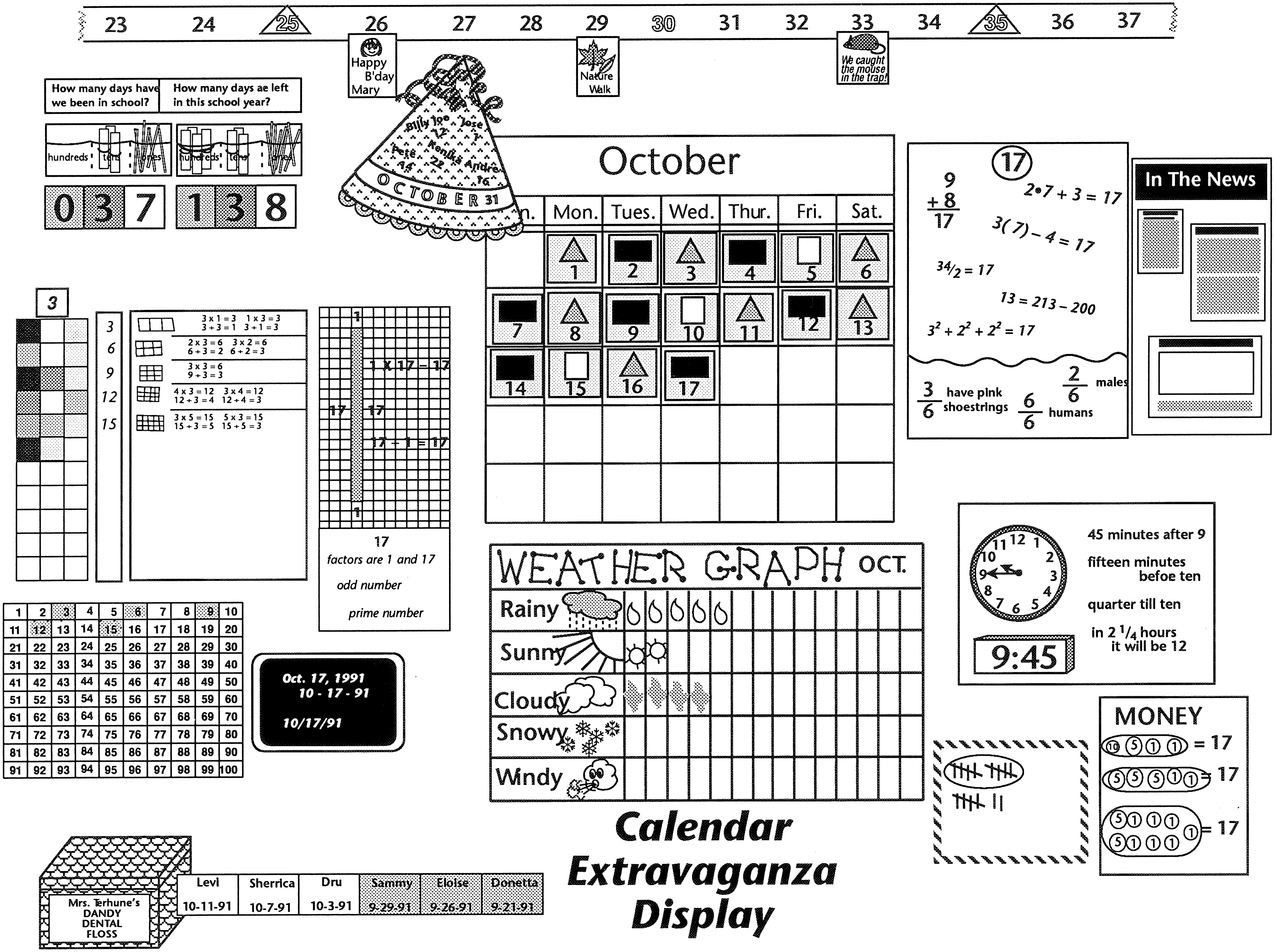


Brian opened our eyes today by sharing his discovery that no odd number can be factored by 4. He proved it nicely at the overhead. He communicated his thoughts clearly so all could understand. Give Brian a big hug!

- Keep parents informed with an Eye Openers newsletter telling them of the mathematical topics covered during the week. Encourage them to discuss topics around the dinner table. In this way, they can feel confident that, despite the absence of daily worksheets, good mathematics instruction is taking place.



Calendar Extravaganza



Calendar Extravaganza Display

Calendar Extravaganza

The calendar activities are best described as a daily extravaganza of skills and topics. Once made, it will give many, many hours of mathematical instruction for years to come and provide opportunities for daily practice of skills taught in Insight and Contact lessons. Activities are designed to be non-threatening and fun, a quick warm-up for the day ahead. They are not intended to replace those necessary hands-on conceptual experiences described elsewhere in this manual.

Skills addressed by the Calendar Extravaganza include:

- Problem Solving
- Connections
- Estimation
- Fractions
- Geometry and Spatial Relationships
- Statistics and Probability
- Children's Growth and Changes
- Odd/Even
- Patterns and Relationships
- Measurement
- Number Sense and Numeration
- Whole Number Operations

The components for the third grade calendar are designed to fit together into a twenty to thirty minute fast-popping extravaganza. While all can be done daily, most teachers vary the components for a fresher approach and to reduce the length of the lesson. Select activities from the following components that will lead to a lively discussion for your class.

- Numberline Strip
- Numberline Pockets and Records
- Numberline as a Timeline
- Weather Graphs
- Dental Floss Box
- Birthday Records
- Pattern Grid
- Chalkboard Date
- Tally Pad
- Multiplication Visual Record and Pattern Matrix
- Money Decimal Records
- Today's Array
- Incredible Equations
- People Fractions
- Clock Reading
- In The News

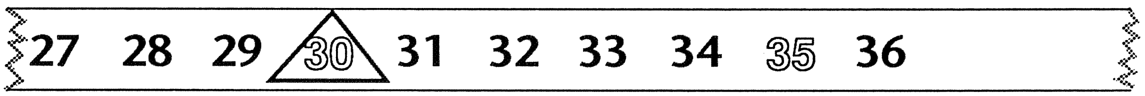
Each component highlights certain mathematical concepts, so many teachers introduce new components only after teaching the related concepts. You need not include every component every month. If your children aren't familiar with the concept highlighted in a particular component, save it for later. If you're tiring of a component, your children probably are too; remove it for a few months or permanently. Some can resurface for a quick review.

Many teachers begin their day with the calendar, but later in the school day works equally well Give your extravaganza twenty to thirty minutes each day, moving through the components at a lively pace. You may emphasize one component for two or three days, while quickly reviewing all others. Develop a child-centered, comfortable discussion of integrated math topics (as well as other topics of interest to the children).

The calendar is a "working bulletin board". Put it within easy reach so children can assist in manipulating and recording as you work through the components. Choose a large bulletin board or display space that is clearly visible to all of your children. Display the calendar elements on and around it in a decorative manner.

As each month closes, analyze some of the information gathered. This is an opportunity to expand the calendar time into Contact lesson time as the month's data gathering ends.

We find the Calendar Extravaganza to be loads of fun! We hope you will enjoy it as much as your children do.



Component
Numberline Strip

SKILLS TAUGHT

- Counting by ones, fives and tens, forwards and backwards
- Duration (How long is a week? How long is a month? How long is a school year?)
- Counting money

INSTRUCTIONAL IDEAS

Use a numberline to show the number of days school has been in session. Since this is such a valuable counting tool throughout the year, allow enough space for a lengthy display.

Record a number on the numberline *each day school is in session.* Stretch the numberline horizontally along the top of your Calendar Extravaganza. The number added each day reflects the total number of days school has been in session.

Write all numbers in *black,* except multiples of 5; write those with a bold, *red* marker. Draw a bold, *red* triangle around multiples of ten. Now both the tens and fives counting pattern are clearly seen.

Ways children may use this numberline:

- Counting forwards and backwards by creating extended number patterns for 1s, 5s and 10s
- Counting pennies, nickels and dimes
- Multiplying and dividing 5s and 10s
- Estimating and rounding off
- Predicting reasonable answers
- Experiencing many forms of addition and subtraction
- Anticipating numerals (How many more days until our 100th day? until school ends? until Martin Luther King, Jr.'s birthday?)

TEACHER *How many days have we been in school this year?*

CHILDREN *Thirty-seven.*

TEACHER *Tell me how to write 37 on our numberline strip.*

CHILDREN *Use the black marker. Put a 3 on the left for the 3 tens. Put a seven on the right for 7 ones.*

TEACHER *Let's clap on multiples of 5 as high as we can without going over and then snap the remaining ones.*

CHILDREN *(Clapping) 5, 10, 15, 20, 25, 30, 35. (Snapping) 36, 37!*

TEACHER *How many fives did we count in 37?*

MARTIN *Let's see—in order to count by fives I need to look at every number that is written in red. Seven fives!*

TEACHER *If this 37 were 37¢, how many nickels and pennies would you use if you used the minimal collection of coins?*

ZACH *That's simple! We said it took 7 fives, so that's 7 nickels. That makes 35¢. By adding 2 pennies we would make 37¢.*

TEACHER *By looking at the numberline, can you tell me the least number of dimes, nickels and pennies you could use to make 37¢?*

DONNIE *Four triangles take us to 40 and that's too far. So 4 dimes is too many. Three dimes would be 30 and a nickel (in red) makes 35. The next 2 numbers represent 2 pennies—that's a total of 37¢. That means we'd need 3 dimes, one nickel and 2 pennies for 37¢.*

TEACHER TIPS

Vary the questions from day to day. If you ask all types of questions each day it becomes boring for you as well as your students. You might ask such questions as:

Is 37 closer to 30 or 40?

What number will we be writing 20 days from now? 25 days from now?

In how many days will our school year be half completed?

Be sure to model the limitless possibilities of the numberline strip. Your children will imitate your example and soon refer to the numberline independently at other times during the school day.

MAKING INSTRUCTIONS

The numberline strip can be made in one of two ways. One way is to start with a rolled-up adding machine tape attached to the far left of your Calendar Extravaganza. Unroll a section of tape each day, keeping a continuous horizontal display. This works well, but not as effectively as the second method.

A second approach is to cut strips of colored construction paper 2" to 3" high using a different color for each month, for example, blue for August, pink for September, etc. Cut the numberline at the end of each month and begin a new color. The color pattern shows the students a colorcoded record of the days of school attended within each month. This lends itself to additional statistical analysis such as: What fraction of our total days this year were in October? How many more days did we attend in September than in August? Keep all the strips up until the school year is complete.

Component
Numberline Pockets

SKILLS TAUGHT

- Mental computation
- Place value counting
- Minimal collections

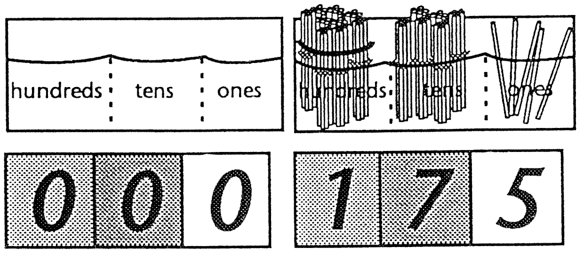
INSTRUCTIONAL IDEAS

It is fun to see the school year progress by representing all of the days in the future as well as the days past. In order to show the correspondence visually, use pockets.

Near the numberline, place three pockets representing the past and three pockets for days remaining in the future (Past on the left, Future on the right).

Above your Future pockets post the question, "How many days are left in this school year?" Above your Past pockets post the question, "How many days have we been in school?"

How many days have we been in school?	*How many days are left in this school year?*

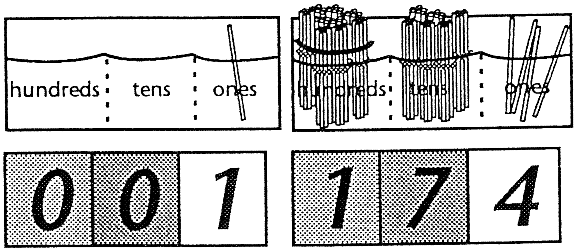


On the first day of your school year, your Past pockets are empty. In the Future pockets individual straws are grouped to show the number of days in your school year. For example, we Kentuckians would use 175 straws. Our straws are banded together in this way: 100 straws, banded as tens and again as a hundred, 7 sets of tens banded together and 5 loose straws. Make appropriate sets for your school year.

Record "000" (to represent 0 hundreds, 0 tens and 0 ones) under the Past pockets. This shows that your class has had no days together thus far in the year. Record the number of days in your school year on the numberline record under the Future pockets.

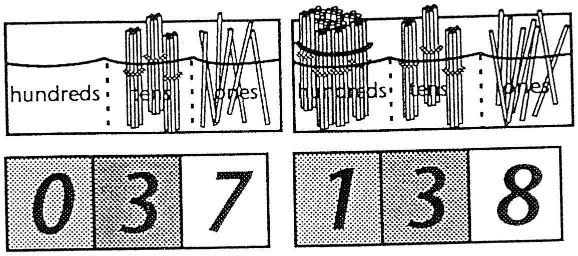
On your first day of school, remove one straw from your Future ones pocket and place it in your Past ones pocket.

How many days have we been in school?	*How many days are left in this school year?*



Each day remove one straw from your Future pockets and place it in your Past pockets. For example, on a Kentuckian's 37th day of school the Past pockets contains 37 straws and the Future pockets contains 138 straws.

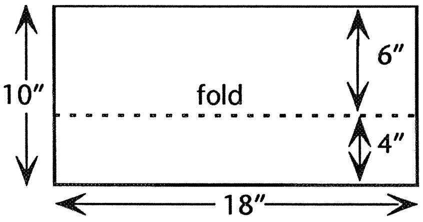
How many days have we been in school?	*How many days are left in this school year?*



At times during the year you will make new tens bundles and sometimes you will break apart tens. But you will only have *one* chance to break apart a hundred and *one* chance to create a hundred. Make all you can of each occasion! On your 100th day of school attendance, stand up and *Boogie*! (See our celebration suggestions in the Contact lessons.) Anticipation of its arrival will draw attention to the Future. Memories of fun Past activities will bring smiles of pleasure.

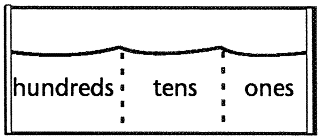
MAKING INSTRUCTIONS

Straw Pockets. Cut two pieces of clear acetate 10× 18. Fold up 4" from the bottom. Seal the edges with strapping tape



Staple, dividing each pocket into three sections.

Label the Hundreds, Tens and Ones sections with a permanent felt-tip pen.



Questions Strips. Use two sentence strips or something comparable. Label them: How many days have we been in school this year? and How many days are left in school this year?

Straws. Begin with the appropriate number for the days in your school year. Leave your ones loose. Form a 10 by bundling 10 ones together with a rubber band. Form a hundred by banding 10 tens.

The Numberline Record. Make two of these. Use three different colors of fadeless colored art paper to create a three-section card to fit under each set of pockets. Use one color for the hundreds place, a second color for the tens place and a third color for the ones place. Laminate these two records so you can write on them daily with a wipe-off pen. (Or use a permanent felt-tip pen and erase it with fingernail polish remover or duplicating fluid.)

TEACHER TIP

Rather than straws, put mats, strips and units in your bags.

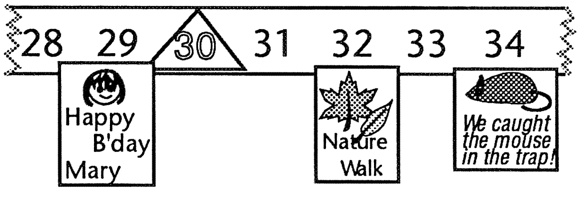
Component
Numberline as Timeline

SKILLS TAUGHT

- Recording duration
- Timelines
- Mental computations

INSTRUCTIONAL IDEAS

You can use your classroom numberline as a timeline to record special classroom happenings.



This develops an awareness of history, even in the short span of the school year. Have children create a symbol to represent those special days. Hang the symbols beneath the appropriate number.

MAKING INSTRUCTIONS

You already have your numberline made. Have a supply of 3 × 4 art paper rectangles upon which your children may illustrate each important event. To develop further ownership of the numberline, let the children decide which events need to be remembered .

TEACHING TIPS

Don't miss opportunities to do quick computations with such questions as: Today is the 38th day of school. How many school days ago did we sing Happy Birthday to Mary? How many days after Mary's birthday did we go on a nature walk? We had clues that we had a mouse among us on the third day of school. How many days did it take us to outsmart him? You don't have to ask such questions every day, however it's fun to reflect upon the past occasionally.

Component
Weather Graphs

SKILLS TAUGHT

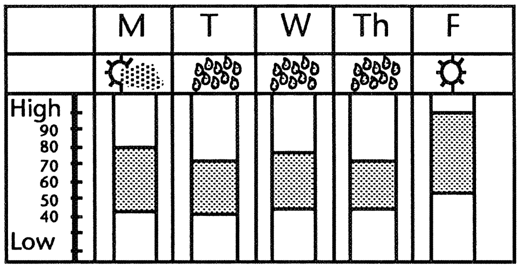
- Weather observation
- Comparing
- Weather patterns
- Graphing
- Counting
- Averaging
- Gathering information over a long period of time
- Noting newspaper, television and radio forecasts
- Fractions

Awareness of the weather and the atmospheric conditions in their world is most important to children. Weather affects so many children's activities that they have a natural curiosity about it. We have discovered that children are interested not only in the weather, but also in the predictions made by professionals. Here's a natural opportunity to gather data and display information which has been experienced by all your children.

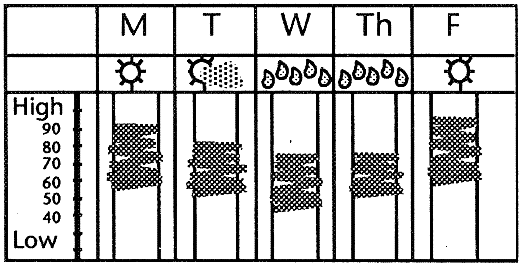
In order to maintain interest, change the type of weather graph each month. It is helpful for the children to see data organized in a variety of graphing styles.

Because changing the type of graph periodically is so vital, we have provided you with eleven different versions. You may choose to make one of each, or create your own. Just be sure to graph each version that fit your needs for space, availability of materials, and artistic ability. Have fun creating them *your* way.

Newspaper Forecast Graph



newspaper copy



classroom copy

INSTRUCTIONAL IDEAS

Begin each week this month by bringing in and posting one five-day extended forecast from your local newspaper. Keep it posted for the entire week. Make a blank copy exactly like the newspaper forecast but without any information displayed. On it record the data for that day's actual weather. Since this is posted directly beside the forecast, your children can easily compare the experts' predictions to the outcomes. This can lead to animated discussions of weather patterns and changes.

TEACHER *Let's see what the weather forecaster has predicted for today's weather.*

CHILDREN *Oh, look! The weather didn't follow the pattern that the forecaster predicted. She said it would be partly cloudy with the high in the eighties. But the sun is shining brightly and there isn't even a wisp of a cloud in the sky.*

TEACHER *Oh, really? Jody, look outside the window and check the temperature shown on the thermometer.*

ELMIRA *Wow! You won't believe this! Today looks like a scorcher. The temperature has already reached ...90...95 degrees! What a day for a swim! Here we are stuck in school. I'll surely be glad when I get home! That forecaster was fooled today!*

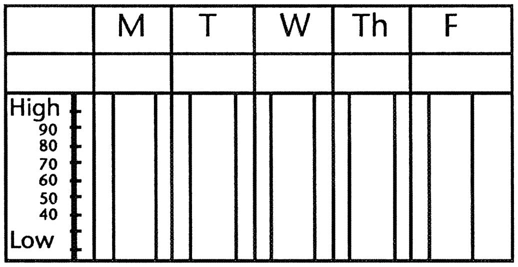
TEACHER *Yes, this would be a good day for a swim. (Draws a name from the feely box.) Serita, would you like to draw a line on our graph showing the temperature at this moment? How many of you think the temperature will go even higher than 95 degrees later today? Tomorrow morning we'll check the daily paper and find today's actual high and low temperatures.*

At the end of the week, compare the forecaster's predictions to the actual weather recorded by your class. You might want to analyze the data accumulated—what was the high temperature for the week? the low temperature? the range of temperatures? average high temperature for the week? average low temperature for the week? (See Chapter 10 for hands-on averaging strategies.)

Accumulate this data for each week in the month. At the end of the month, see if your children can determine the monthly high, monthly low, range, averages, etc.

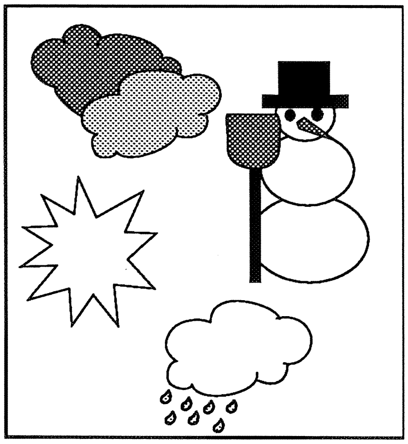
MAKING INSTRUCTIONS

Bring in a five-day extended forecast from your Sunday newspaper. For the classroom copy, match the original forecast format, but show no data.



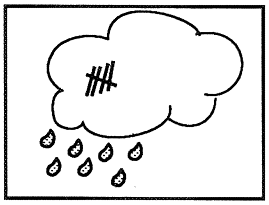
Make a feely box from a plastic 12 or 16 oz. stadium cup inserted into a man's tube sock. Place each child's name on individual slips of paper in the feely box. Don't return names to the box until all names have been drawn, so everyone has an equal opportunity to participate. However, if a child is hesitant, never insist. A child may have a friend help or do it in their place.

Weather Tally



INSTRUCTIONAL IDEAS

Tally on the cloud, snowman, sunshine or rain clouds to record your daily weather as shown.



MAKING INSTRUCTIONS

Use 11 × 18 fadeless art paper or posterboard for the background.

Clouds: Cut two shapes, one from gray and one from black. Put the gray cloud on top to record tallies.

Sunshine: Cut from yellow paper.

Snowman: Cut three white circles in different sizes. Cut hat, broom handle and coal chunks from black, carrot nose from orange, broom straws from yellow.

Rain cloud: Cut cloud and raindrops from light blue paper.

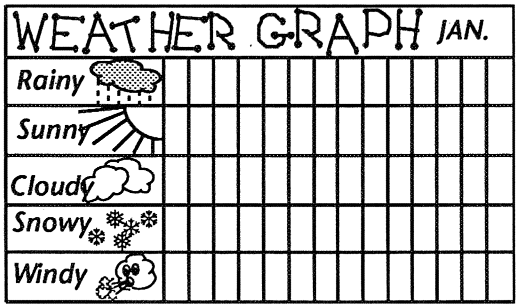
Horizontal Bar Graph

INSTRUCTIONAL IDEAS

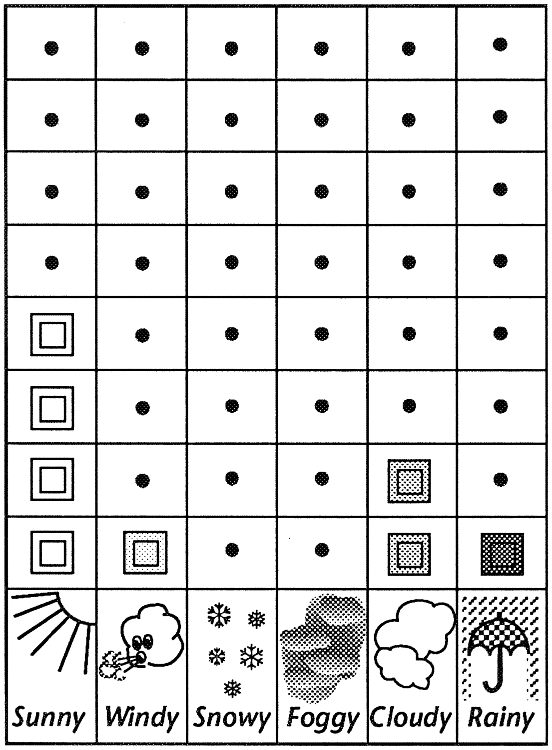
You may color in each space, draw an appropriate symbol, draw a smiley face or frown—let your class decide.

MAKING INSTRUCTIONS

Use an 11 × 18 piece of fadeless art paper or posterboard. The grid can be a different color. Allow 15–20 spaces for each row.



Vertical Bar Graph/Pie Graph



INSTRUCTIONAL IDEAS

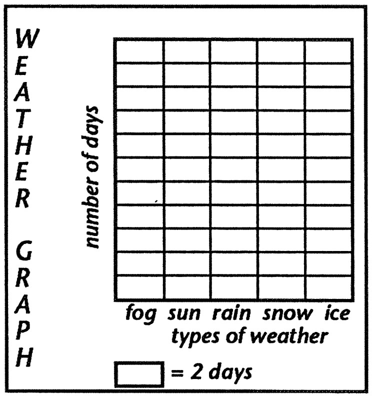
Let a volunteer slip a hex-a-link cube(s) over a pin in the appropriate column(s) to show today's weather. Choose a different color cube for each column (perhaps yellow for sunny, blue for windy, etc.).

At the end of the month, remove the hex-a-link cubes and snap them together to create a stack for each type of weather. Discuss and compare the gathered data.

MAKING INSTRUCTIONS

Use a 21 × 28 piece of white posterboard. Mark off the 21 side into six columns, each 3½" wide. Leave 4" at the bottom to illustrate each type of weather; other horizontal lines are 1" apart. White paper-punch circles make nice snow. Mount the graph on a pinning board or bulletin board.

Vertical Bar Graph: One Block = Two Days



INSTRUCTIONAL IDEAS

Start at the bottom and go up (or start from the top and go down if you feel froggy!) and fill in half a box each day. One day can be shown in a variety of ways (color half a box, draw half an X, etc.). Let the class vote to decide.

On the last day of the month, place a push pin on the center of each column. Connect the pins with yarn. Analyze the information shown on the graph.

MAKING INSTRUCTIONS

Draw the grid on a 9 × 12 piece of construction paper. Label as shown.

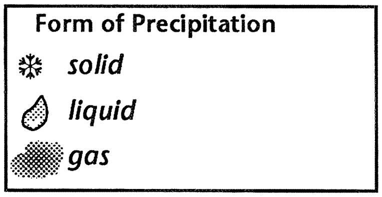
Forms of Precipitation Tally graph

INSTRUCTIONAL IDEAS

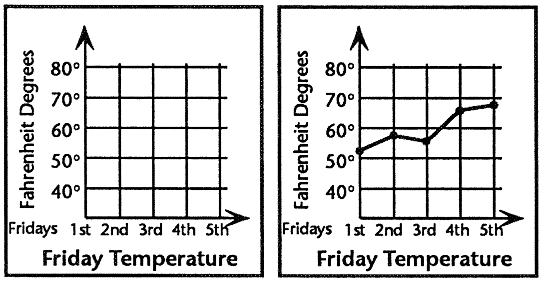
Each day make a tally beside the appropriate form of precipitation.

MAKING INSTRUCTIONS

Use a 9 × 12 piece of construction paper. Write the name of the month at the bottom after laminating.



Temperature Line Graph



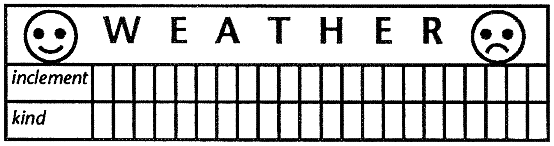
INSTRUCTIONAL IDEAS

Read the outside temperature at the same time each Friday. Decide where the day's dot will go and connect the lines. Discuss how entering or leaving a season may affect the rise and fall of the line on the graph. Mother Nature reserves the right to surprise!

MAKING INSTRUCTIONS

On a 9 × 12 piece of construction paper, draw lines 1½" apart, as shown. Leave the vertical axis unlabeled before laminating unless your temperature range varies little over the year. Use Celsius if you prefer.

Inclement/Kind Weather Horizontal Bar Graph



INSTRUCTIONAL IDEAS

Does today's weather prevent some work from being done? Is it severe for most of us or not? Draw a smiley face or a sad face to indicate the daily situation. Use during a month when your area normally has inclement weather.

MAKING INSTRUCTIONS

Cut a piece of posterboard 6 × 22. Make 1 × 2 spaces for the faces.

Variations: Shadow Days/No Shadow Days or Harmful to Local Vegetation/Beneficial for Vegetation.

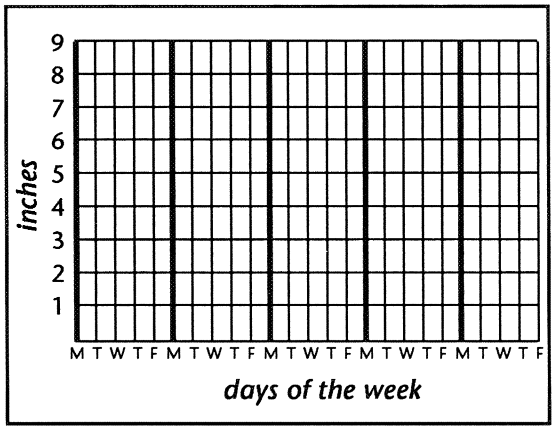
Cumulative Rainfall Line Graph

INSTRUCTIONAL IDEAS

This graph involves checking a class rain gauge daily. Make a line graph by connecting dots each day to show your monthly rain total. Your month may begin on Wednesday, so your first dot will be on the horizontal axis at the base of the third line. The dots won't rise until rain appears in the gauge.

MAKING INSTRUCTIONS

This graph takes more space, but our students love it! Use regular posterboard and make vertical lines ¾" apart. Every Monday the line is heavier to indicate that rain has collected over a longer period of time, therefore more jumps may occur on Mondays.



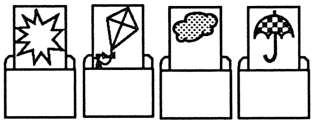
TEACHER TIPS

Check your science curriculum before making the graphs. Create them to reflect your local and state guidelines.

Easy Pocket Chart Weather Graph

INSTRUCTIONAL IDEAS

Is all this too much? Would you like a display that can be used in a variety of ways? Consider using library pockets with cards inserted to show the choices.



Just drop straws or popsicle sticks into the appropriate pocket each day.Perhaps your students can create their own graphs, individually or in groups, on the last day of the month to show the information recorded in the pockets.

Some months you may use only two pockets—in March, for example, use "lamb" and "lion" pockets.

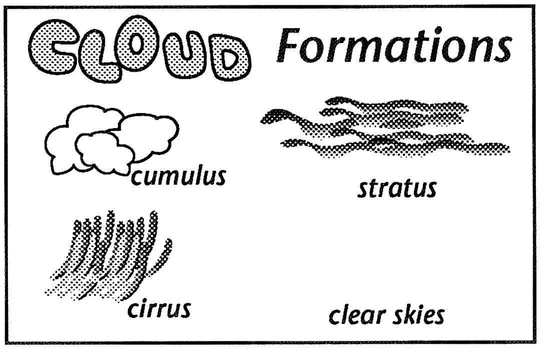
Weather Graphs: Cloud Formation Tally

INSTRUCTIONAL IDEAS

Mark tallies on the appropriate cloud for each day.

MAKING INSTRUCTIONS

Illustrate cloud formations on a sheet as large as you wish. Colored chalk works well. Use light colors so the tally marks can be easily seen over the clouds and the clear sky.



Component
Dental Floss Box

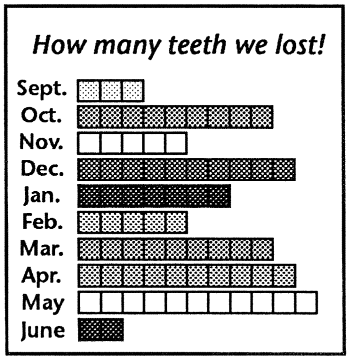
SKILLS TAUGHT

- Counting
- Honoring children's growth and change
- Keeping records for a long time
- Graphing

INSTRUCTIONAL IDEAS

Children are so proud of that new space left by a lost tooth! You wouldn't want to miss this chance to keep a permanent record of the classroom losses. Keep the dental floss box posted all year. As teeth are reported missing, pull the strip out a bit more and record the new name and date.

To make monthly variations more obvious, record each month's losses in a different color so comparisons and graphs can be made using fractions, story problems and such. At the end of the school year, cut the strip into month-long sections. Create a bar graph from the strips by gluing them to a sheet of paper.



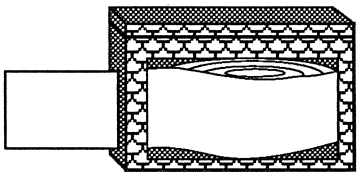
To make the graph more meaningful for the children, model the information by creating stacks of hex-a-link cubes. The numbers and colors of the cubes will correspond directly to the monthly strips.

Discuss other types of graphs you could use to show the same information. (See Chapter 10.) Divide the children into small groups and have each group use a different type of graph for showing the tooth chart information.

Statistical analysis of the lost-tooth data may surprise your children. Third graders can lose an awesome number of teeth. In fact, you may wonder how they managed to eat anything this past year!

MAKING INSTRUCTIONS

You will need a small box (see blacklines if you need a pattern). Cut an opening in the back so you can insert a rolled-up strip of paper made from 1-inch grid paper. Glue many strips together, because your class may lose as many as 150 teeth this year.



Cut a slot large enough for the strip to slide through the right side (as you look at it from the front). Cover with paper and label. Insert the paper strip and pull through the slit. You may find it necessary to add more paper to the roll if your class is extremely "snaggletoothy"!

Component
Birthday Record

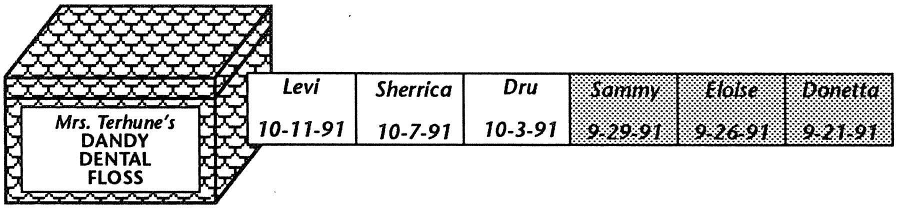
SKILLS TAUGHT

- Counting
- Comparing
- Month names
- Number of days in each month
- Fractions

INSTRUCTIONAL IDEAS

Birthdays are special to those of any age—whether 8 or 80. Keep the fun alive with a birthday hat for each month! Across the top of your calendar extravaganza, display 12 birthday hats labeled with the name and birth date of each child. These hats are a wonderful way to keep the names posted throughout the school year for all children to see.

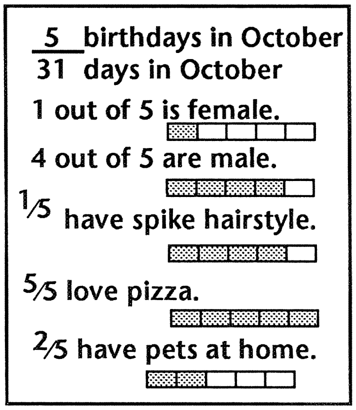
Each month, remove the current month's birthday hat from the lineup and place it near the calendar grid. On the last day of the month, record the total number of days for that month beside the month's name. The posted information can lead to a variety of mathematical analyses.



Counting. Have your children count the total number of birthdays in that month.

Comparing. After counting the number of birthdays in the month, find other months with more, less, or the same number of birthdays. How many people who have a birthday in month X also lost a tooth in month X?

Fractions. Use fractions to make comparisons involving the number of birthdays in this month. Some possibilities might be to compare the number of student birthdays during this month with the total class enrollment; male or female birthdays with the total number of birthdays in the month; birthdays of people who like pizza with the total number of birthdays; birthdays with the total number of days in the month.



MAKING INSTRUCTIONS

Locate the birthday hat pattern from the blacklines. Use colored, fadeless art paper to make 12 different hats. Label one for each month and laminate. Consider creating a color pattern such as red hat, red hat, blue hat, green hat (repeated to make 12). Decorate the hats with stickers, dots, stars or whatever suits your fancy. Add some festive zip by attaching curling ribbon at the top.

With a permanent marker, write each child's name and birth date on the appropriate hat. At the end of the year, the names can be easily erased with fingernail polish remover.

TEACHER TIPS

An alternative to the hats would be to create one large 12-month bar graph. Make two copies of the 1" grid blackline and you'll have more than enough for 12 columns. Mount them and laminate for reuse. Write each child's name in a square of their birthday month.

Component
Pattern Grid and Markers

SKILLS TAUGHT

- Counting
- Numerical recognition
- Names of the days/months
- Visual patterns
- Number patterns
- Evolving patterns
- Predictions

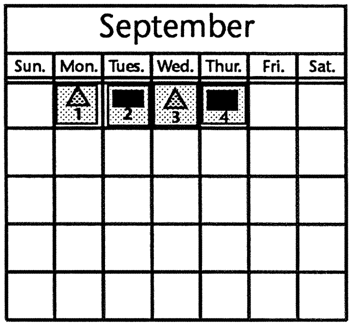
INSTRUCTIONAL IDEAS

Pattern plays such an important role in the understanding of mathematical concepts. The young child who recognizes, creates, extends and reproduces mathematical patterns will have an excellent mathematical foundation upon which to build future experiences. The Pattern Grid is a valuable tool for this purpose.

We challenge the students to discover and extend a "pattern of the month". One step of the pattern is revealed each day using markers that are hung on the calendar grid. Each marker is labeled with the day's date and reflects the corresponding step in the pattern.

It is with pride that children discover horizontal, vertical and diagonal patterns, as well as number patterns. They become skilled in guessing the pattern as well as predicting the next step in any of these directions. Number patterns help promote a connecting link with multiplication and division (see *Musical Array-ngements* booklet and cassette tape).

In the following dialogue a pattern is illustrated which uses geometric figures and repeats every five days. While it is fun to use seasonal pattern grid markers, such as pumpkins and Pilgrim hats, we have discovered interest rises if other ideas are incorporated. While this is not a time to develop concepts, it is natural to expand the use of mathematical vocabulary and to reinforce skills in need of review.



TEACHER *Let's see what number is on today's marker. What do you think it will be?*

LES *That's a pattern we can depend on—it will be a 5!*

TEACHER *Not tough enough, eh? Well, who can tell me what design will be on it?*

PEARLY MAE *Well, so far we have triangle, rectangle, triangle, rectangle. I think this one will be a triangle! I think it's a 2-step pattern.*

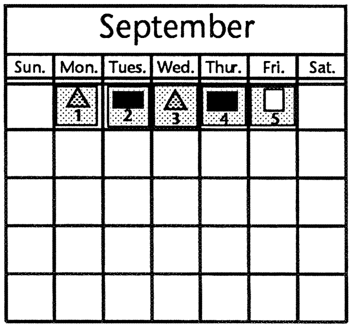
PETE *I bet you're trying to trick us. You wouldn't give us one that simple. We did ones like that in kindergarten.*

IRENE *The pattern may not be finished yet.*

PETE *Ooooh, Irene! You just might have it!*

TEACHER *Yes, Irene, the pattern is not completed. Let me show you today's marker. (Teacher holds up a marker that has a square on it.)*

PETE *A square! You really did fool us.*



TEACHER *The pattern this month uses 5 steps. Join me in clapping for triangles, snapping for rectangles and wiggling fingers for the square.*

Children join the teacher: clap, snap, clap, snap, wiggle, clap, snap, clap, snap, wiggle.

TEACHER *Could anyone suggest another way we could represent the same type of pattern?*

LARRY *I've lived in 3 different states. I could use those states. Let's say: Idaho, Mississippi, Idaho, Mississippi, Alabama, Idaho, Mississippi, Idaho, Mississippi, Alabama.*

TEACHER *Yes, that's another idea. Now, since the pattern uses 5 steps, let's count by ones and only speak aloud on the last step of the pattern?*

CHILDREN *Sure. (Whisper) 1, 2, 3, 4, (speak) 5, (whisper) 6, 7, 8, 9, (speak) 10, (whisper) 11, 12, 13, 14, (speak) 15, (and so on, as far as you like).*

TEACHER *Can anyone look at the second box in the third row and predict what we might see?*

TED *OK...mmmm...hey! That would be a square, I think!*

TEACHER *How did you figure that?*

TED *Our pattern started on a Monday, so I just counted from there—1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5—and when I landed on that box, I was on a 5. Five is the last step of the pattern and square is the last step, also. So it will be square.*

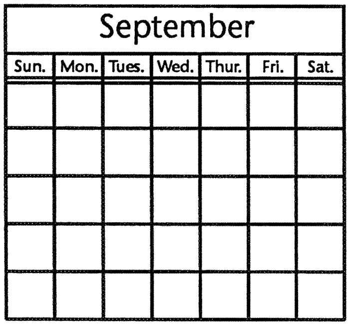
LLOYD *I did it differently. That box is on a Tuesday and we'll have to go through two full weeks and one more day to get to it—that's 15 days. The 15th step of our pattern will be square.*

You may want to have the children predict what will be seen on other days of the month. In particular, it is helpful to identify what kind of marker will be on the last day of the month. Also determine how many of those days are scheduled for school attendance. At the end of the month compare to see how many days you actually did attend school. Did snow or a flood cause school to be cancelled? Were you home from school on any holiday?

Make each day's discussion of the calendar grid brief. To ask all questions daily would kill the momentum. Remember these are *not* concept lessons, but simple review discussions. Move quickly through the calendar extravaganza—alternating your emphases. You'll be as pleased as punch with the results.

MAKING INSTRUCTIONS

The grid. You may use a commercially-prepared grid, or you may simply make your own. It is wonderful to use 5" squares, so children can see the pattern clearly. If you have bulletin board space available for this purpose, use yarn and pins to mark it off. Allow enough space at the top to place cards labeled with the abbreviations of the days of the week. Hang that month's birthday hat nearby. Or, you may choose to write that month's name on a 4 × 6 index card to display in place of the birthday hat.



The markers. Cut enough 5"-square cards to have one for each day of the month. You may have the children assist you in making the pattern markers for your grid. It is especially fun for the "creators" to know the pattern as their classmates try to guess. We are never short of volunteers to make markers. (We do, however, make the markers ourselves for the first month of school.) A great deal of planning, problem solving and critical thinking is involved in the process of making the markers, laying out the pattern, adding the numbers for the date, etc.

TEACHER TIPS

We have not suggested patterns for each month, because you know better than we what your children's strengths, weaknesses and needs are. This might be a good opportunity to review such concepts as time, fractions, open/closed curves, states East and West of the Mississippi River, or even phases of the moon.

If the month's pattern repeats every five days (as in the dialogue), use it to study multiplication and division patterns involving the number five. We like to put a chicken ring on every fifth day's marker, sing the Slicing By Fives song and do an extended number pattern lesson using the fives counting pattern.

Our children love to create a pattern by accumulating a cup of rice each day, forming pints, quarts and eventually gallons. We found through this daily accumulation, our children soon became proficient with volume. They begged us to continue it for the next month as well.

Component
Chalkboard Date

SKILLS TAUGHT

- Various ways to record dates
- Language skills

INSTRUCTIONAL IDEAS

Post a student chalkboard near the calendar grid. Ask children to volunteer ways to write the day's date. As the children make suggestions, write them on the chalkboard. Keep a quick pace by limiting the responses to 3 a day.

Leave the dates displayed all day long. As the children date their papers during the day, encourage them to write the date in a variety of ways. (Our children have even tried to stump us by suggesting methods used in other countries. What fun!)

MAKING INSTRUCTIONS

Pin up a student chalkboard near your calendar grid. (The black chalkboards available from The Math Learning Center are perfect for this.) Also keep some chalk and an eraser nearby.

Component
Tally Pad

SKILLS TAUGHT

- Counting by ones, fives and tens
- Tallying
- Addition combinations
- Math terms: vertical, diagonal, horizontal
- Multiplication and division

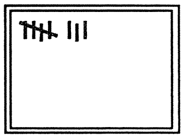
INSTRUCTIONAL IDEAS

Tallying is another way to record the number of days in each month that school is in session. It's a fun way to record information! You'll see your children using the tally method to record their own information at the most unexpected times.

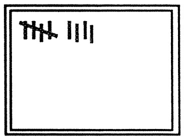
TEACHER *How many days have gone by so far in September?*

LYNETTE *Eight—that's easy! We see one 5-tally and 3 ones. That makes 8!*

TEACHER *We need to tally today. Do I need to write a vertical or diagonal line?*



THADDEUS *We need another vertical line! We only use a diagonal line when we have created a multiple of 5.*



TEACHER *How could we write this as an addition problem?*

BARRY *Five plus 4 equals 9. Or 4 plus 5 equals 9.*

TEACHER *Who thinks they can use both multiplication and addition in the same problem to tell us what the tally pad shows?*

CARLOTTA *I can! One times 5 plus 4 times 1 [(1× 5) + (4× 1)]. Or, 1 times 5 plus 1 plus 1 plus 1 plus 1 [(1 × 5) + 1 + 1 + 1 + 1].*

MAKING INSTRUCTIONS

Cut a piece of light-colored fadeless art paper into an 8" square. Laminate the paper so you can easily erase what you have written with a wipe-off pen. (Or use a permanent marker erased with fingernail polish remover.)

TEACHER TIPS

There is no need to use the tally every month. Phase out the tally altogether once students are familiar with it.

Component

Multiplication Visual Record and Pattern Matrix

SKILLS TAUGHT

- Number patterns
- Multiplication
- Division
- Extended addition
- Pattern prediction

INSTRUCTIONAL IDEAS

Your children need to become familiar with extended number counting patterns. The purpose of this component is to provide that familiarity by focusing on one particular pattern each month. For example, we explore the 2 pattern the first month of school and the 5 pattern the following month. After that the threes, fours, etc., are highlighted. By the end of the school year, each counting pattern through the nines has been explored. Likewise, opportunities have been provided for your children to compare counting patterns for likenesses and differences.

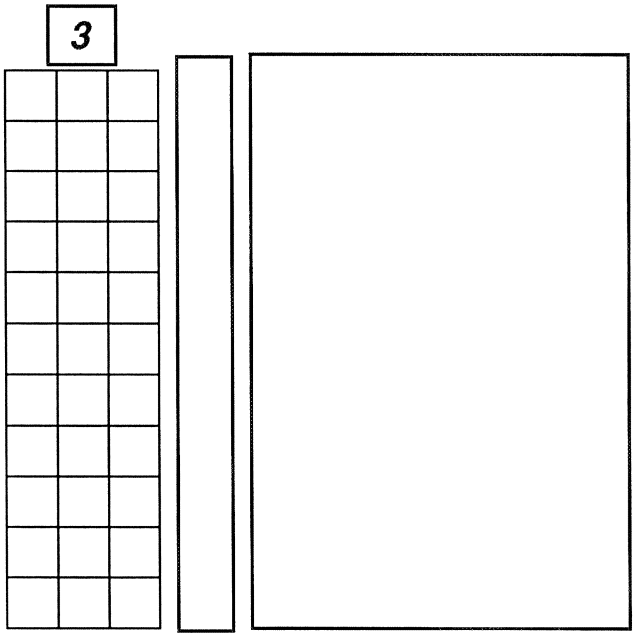
Begin by deciding which number pattern you want to explore during the upcoming month. Prepare the display to correspond (see making instructions). Below is an example of the blank display to be used with the 3 counting pattern.

When your children see this blank display for the first time, they should be able to tell that 3 is the pattern of the month to be explored. One clue is the 3 at the top of the display and the other clue is the 3 units in each row of the array.

Each day you will color in one square of the array. Use a different color in each square so each of the squares can easily be seen as an individual unit.

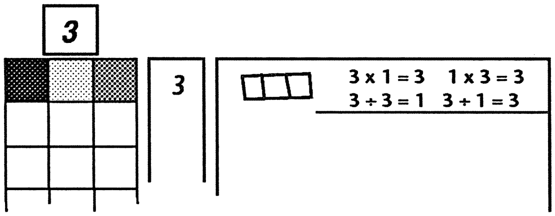
TEACHER *What will happen when we fill today's unit on the Multiplication Visual Record?*

AL *Oh, boy! When we fill in today's box we will have filled in our first row. That means we can write related multiplication and division problems on our*



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

record. We can also shade in the 3 on the hundred's matrix.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
					46	47	48	49	50
						5	58	59	60
									70

TEACHER *Would you like to snap and clap the 3 pattern today?*

LYNETTE *Yes! Let's snap 2 times and clap once—like this: snap, snap, clap. Then every time we clap, it will be on a multiple of 3.*

TEACHER *Okay, let's go for it. (The teacher and children snap and clap together.)*

TEACHER *This time let's add numbers as we snap and clap. Whisper the snapped numbers and say the clapped number aloud.*

PAOLO *Yeah. Every time we clap we should be saying a number that fits in the 3 number pattern. (The teacher and children count in unison as they snap and clap.)*

Continue filling rows by shading in one unit of the array for each day of the month. Don't forget to shade 3 blocks each Monday to make up for Saturday and Sunday. Each time a row has been filled-in, sketch the complete rectangular array to the side together with the corresponding multiplication and division facts. (Refer to the example of the 3 array on following page.)

As the blocks in the hundred's matrix are being colored, ask some prediction questions: Thirty is shaded. What will be the tenth number we color in after 30? Will the number directly under 30 be shaded? What patterns do you see horizontally? vertically? diagonally? What is the greatest number that will be shaded on the grid? What is the first number beyond 100 that would be shaded if our matrix were extended? What kind of even/odd patterns do you see from the shaded boxes? How many shaded numbers will there be in all when we finish? Will more odd or even numbers be shaded? How many numbers ending in 0 will be shaded?

At the end of the month teach your children the related number pattern song from the *Musical Array-ngements* cassette tape. Encourage your children to look at the matrix and/or the counting pattern strip to help them sing the sequence of the numbers. (Be certain that your children realize that the numbers being sung represent the area of the rectangular arrays in growing sequence.)

Move the Multiplication Visual Record and the month's matrix to a place in your room where eventually all number patterns will be displayed together. When you want to review a pattern, just direct attention to the charts and sing the song, chant the number pattern, snap-and-clap, bee-bop or do a pattern of your own.

Attach a new record to the calendar display for the next month and repeat the process with a different counting pattern. (See 4 array pattern on following page.)

By the year's end, you will have a Multiplication Visual Record posted for each counting pattern. What a wonderful reference tool. The visual learner has images upon which to build future mathematical experiences. For the tactile learner, the Visual Record recalls to mind the physical snapping and clapping. The auditory learner experiences the same number pattern through chants and songs and by merely reciting the pattern.

MAKING INSTRUCTIONS

You will make one Multiplication Visual Record display for each number pattern from 2 through 9.

For each display use two 18 × 24 sheets of construction paper to form the background. Join them to create one 36 × 24 sheet.

For each display, cut from 1" grid paper a rectangular array: the width is determined by the month's number pattern, the length is 10 squares (except the 2 pattern will need a length of 16 and the 3 pattern a length of 11). (For example, in the 3 month, you will need a 11 by 3; in the 5 month, you will need a 10 by 5; in the 7 month, you will need a 10 by 7.)

At the top left of the display, attach the blank array for that counting pattern with the multiplier displayed clearly above it.

To the right of the array, place a strip of paper, such as adding machine tape, the same length as the array. This will be used for recording the counting pattern.

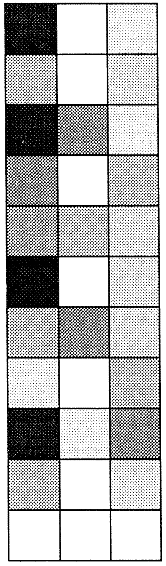
To the right of the strip, you will be recording each array in the growing sequence.

Make one copy of the large hundred's matrix for each month. Mount the matrix on a sheet of construction paper. As each row on the array is each month. Mount the matrix on a sheet of construction paper. As each row on the array is filled, the corresponding number is colored in on the matrix.

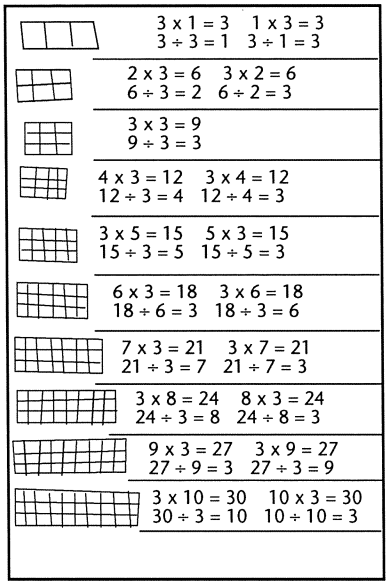
You will also use the songs from the *Musical Array-ngements* cassette tape.

Example of the 3 array

3



3
6
9
12
15
18
21
24
27
30



3 x 1 = 3   1 x 3 = 3
3 ÷ 3 = 1   3 ÷ 1 = 3

2 x 3 = 6   3 x 2 = 6
6 ÷ 3 = 2   6 ÷ 2 = 3

3 x 3 = 9
9 ÷ 3 = 3

4 x 3 = 12   3 x 4 = 12
12 ÷ 3 = 4   12 ÷ 4 = 3

3 x 5 = 15   5 x 3 = 15
15 ÷ 3 = 5   15 ÷ 5 = 3

6 x 3 = 18   3 x 6 = 18
18 ÷ 6 = 3   18 ÷ 3 = 6

7 x 3 = 21   3 x 7 = 21
21 ÷ 3 = 7   21 ÷ 7 = 3

3 x 8 = 24   8 x 3 = 24
24 ÷ 3 = 8   24 ÷ 8 = 3

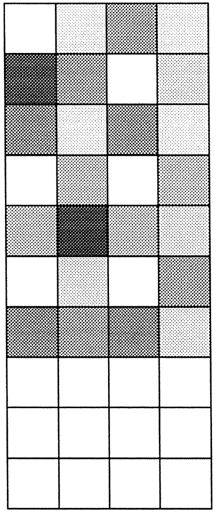
9 x 3 = 27   3 x 9 = 27
27 ÷ 9 = 3   27 ÷ 3 = 9

3 x 10 = 30   10 x 3 = 30
30 ÷ 3 = 10   10 ÷ 10 = 3

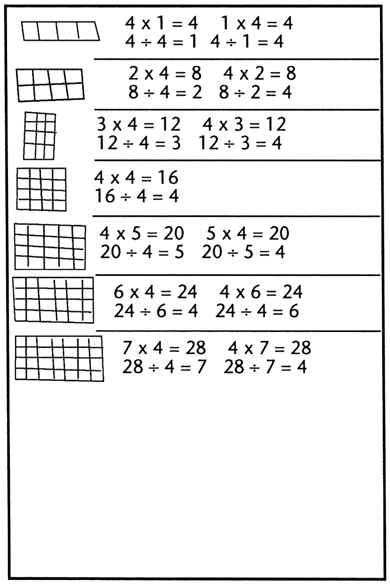
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Example of the 4 array

4



4
8
12
16
20
24
28



4 x 1 = 4   1 x 4 = 4
4 ÷ 4 = 1   4 ÷ 1 = 4

2 x 4 = 8   4 x 2 = 8
8 ÷ 4 = 2   8 ÷ 2 = 4

3 x 4 = 12   4 x 3 = 12
12 ÷ 4 = 3   12 ÷ 3 = 4

4 x 4 = 16
16 ÷ 4 = 4

4 x 5 = 20   5 x 4 = 20
20 ÷ 4 = 5   20 ÷ 5 = 4

6 x 4 = 24   4 x 6 = 24
24 ÷ 6 = 4   24 ÷ 4 = 6

7 x 4 = 28   4 x 7 = 28
28 ÷ 4 = 7   28 ÷ 7 = 4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Component
Money Decimal Records

SKILLS TAUGHT
- Counting money
- Decimals

Money Decimal Records—Version 1

Use this activity when your dates are small numbers. Using the number of the day's date, select that number of coins from each denomination of coins. (On October 3rd, 3 pennies, 3 nickels, 3 dimes, 3 quarters and 3 half dollars are selected.) Then sort the coins and determine their total value .

TEACHER *Today is the third of October. Let's place 3 of each of our translucent coins on our overhead projector and then calculate the total value of the coins.*

Teacher pulls the name of a child from the Name feely box. That child will place 3 of each of the translucent coins on the overhead projector.

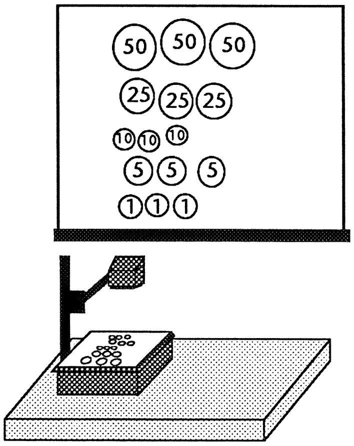
TEACHER *Thanks, Thomas. Now, we need a plan. What would be one way for us to count the total amount of money shown?*

DEREK *Well, it would really help if we sorted the coins—you know—put all the half dollars together, all the quarters together, all the dimes together, all the nickels together and all the pennies together. (Teacher or student sorts coins.)*

DEREK *Yeah, that really does help. It would help, too, if we lined them up from the largest amount to the smallest amount. That way we get the counting of those big coins out of the way first!*

TEACHER *Show me what you mean.*

Derek goes forward to the overhead projector and arranges the coins from largest value to smallest value.



SARA *Hey, that will help, Derek.*

TEACHER *Are we ready to count the coins now to see what amount of money we have?*

CHILDREN *Let's do it to it!*

CHILDREN *(Counting in unison, changing counting patterns as the coins change) $.50, $1.00, $1.50, $1.75, $2.00, $2.25, $2.35, $2.45, $2.55, $2.60, $2.65, $2.70, $2.71, $2.72, $2.73.*

TEACHER *That's right. There are 2 dollars and 73 of the 100 pennies needed to make another dollar. Now who can think of a different coin combination to make $2.73?*

MORRIE *I can! You could use 27 dimes and 3 pennies.*

TEACHER *Let's try that. Come to the overhead and show us that example. For our 27 dimes, what counting pattern should we use?*

ALICE *We would count by tens 27 times.*

Children count by tens until they reach $2.70.

TEACHER *Now we have counted all the dimes and we still have some pennies remaining. What do we do now? Any suggestions?*

DEL *Sure! Let's just switch over to counting by ones.*

TEACHER *Okay. You count by ones as I point.*

CHILDREN *$2.71, $2.72, $2.73. If we had 27 more cents, we would have another dollar.*

TEACHER *Does anyone else have another way?*

POLLY *Yes, I think we could use 5 half-dollars, 2 dimes and 3 pennies.*

TEACHER *Let's place those coins on the overhead projector, so we can see them as we count.*

Elicit a variety of answers, modeling a few. Remember, this is not your basic exposure to counting money. This is simply a quick review of concepts learned earlier. It is fun to watch the recognition of pattern evolve. At some point, a child may even notice that you are adding 91¢ to the previous day's total when you add one more of each coin.

TEACHER *If I bought food worth $2.73 at the grocery store and handed the clerk $3.00, how much change would I receive?*

CHILDREN *Twenty-seven cents.*

TEACHER *Prove it.*

The children set out the appropriate number of coins on the overhead projector in order to determine the difference between $2.73 and $3.00.

Money Decimal Records—Version 2

Use the day's date to indicate the number of coins to pull from the feely box.

TEACHER *Today is October 10th. Here, I'll pull the name of two volunteers from the Name feely box. Van and Vaunie, it looks like you two are the lucky ones. Van, will you please come to the front and pull 10 coins from our Coin feely box? And Vaunie, will you please sort the coins for Van as he pulls them from the box?*

VAUNIE *Let's see. We have 3 half-dollars, 2 quarters, 1 nickel, 1 dime and 3 pennies.*

TEACHER *So everyone can see the coins clearly, Vaunie, will you please substitute these translucent coins for the real coins and place them on the overhead projector?*

When the total number of coins has been collected and sorted, lead the class in counting the coins using the appropriate counting patterns.

Ask questions such as: I see you made $2.18 with your 10 coins. Is it possible to make $2.18 with fewer than 10 coins? What is the least number of coins you could use to make $2.18? How much would we have if Van had gotten another dime instead of the penny on the last draw? What if we kept this total for tomorrow and added an eleventh coin. How much would we have if we pulled a penny? a nickel? a dime? a quarter?

Money Decimal Records—Version 3

This version works best when the date is after the tenth of the month.

Tell a story that involves the number of the day's date as the amount of change received from some transaction.

TEACHER *(on September 14) I went to the store with $2.00 in my pocket. I bought cupcake liners so I could bake cupcakes for some special friends. My change was 14¢. How much did I spend?*

Draw from the Name feely box to select a student to lay out $2.00 in translucent coins on the overhead projector however they choose. (If the selected student seems uncomfortable, have them choose a friend to assist.)

TEACHER *I had 14¢ remaining when I left the store. How can we arrive at the amount I spent on the cupcake liners?*

YOLANDA *I know. We can trade in one of the half dollars for some smaller coins. Then we can remove coins equaling 14¢. That will leave us with the amount spent showing on the overhead.*

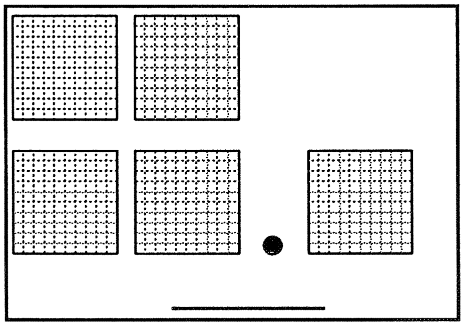
Some money trading will come into play as you use this version. For example, if the student shows the $2.00 by using 4 half dollars, one of the half dollars will need to be traded for coins of smaller value until a total of 14¢ can be removed from the screen. This would leave the amount spent ($1.86) showing on the screen.

TEACHER *(on September 15) I went shopping for new markers and only had $3.25 in my wallet. I was worried about having enough cash! I did have enough. Can you tell me without my giving any other hints how much the markers cost if my change was 15¢?*

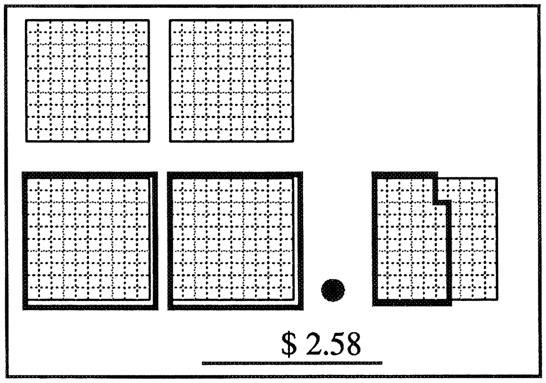
Continue making up a story problem each day. Vary the amount of money you start with, but always receive the day's date as your change. Let your children tell stories—you'll be just as entertained as they are.

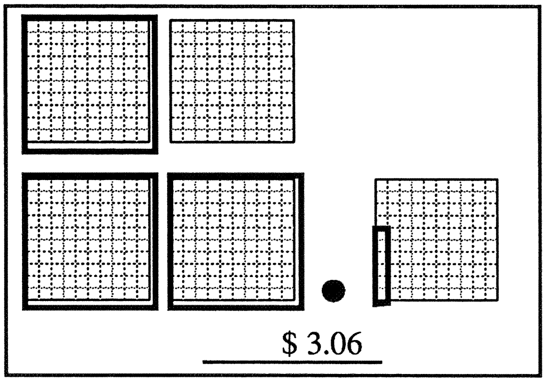
Money Decimal Records—Version 4

In this activity base ten number pieces are used to make the connection between money and decimals clear to your children. Information is recorded on a Money Record sheet (see blacklines).



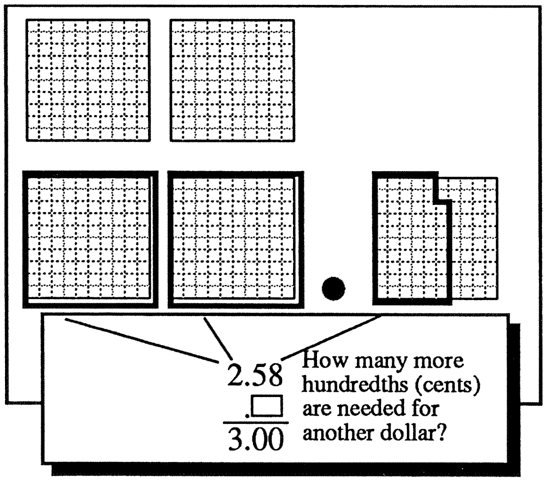
Put $4.00 in overhead coins in a container. Have a volunteer remove at least 10 coins (or enough coins to equal today's date) and count them at the overhead. Record the total amount on the Money Record Sheet, by outlining a whole square for each dollar and partitioning the square to the right of the decimal point to indicate the portion of a dollar represented by the cents. Here are some examples.





Money Decimal Records—Version 5

Follow the procedures for Version 4. Then ask your children to determine the number of cents needed to complete an additional dollar.



TEACHER TIPS

The concept of decimals is easily explored through money. Each time a collection of coins is counted, ask your children to describe the collection in terms of whole dollars and part of another dollar. Discuss the role of the decimal in separating the whole from the part of the whole.

Component
Today's Array

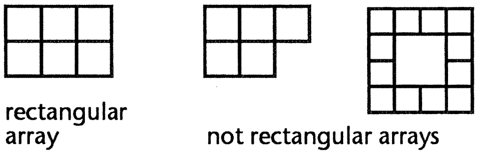
SKILLS TAUGHT

- Multiplication
- Division
- Prime numbers
- Composite numbers
- Odd and even numbers
- Factors
- Square numbers
- Square roots
- Rectangles
- Linear unit
- Dimensions
- Area
- Product

INSTRUCTIONAL IDEAS

Every day when a number is added to the calendar grid, display the corresponding number of translucent square tile on an overhead projector. The children can again visually develop the image of one more object representing one more day.

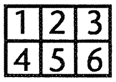
The translucent tile are a versatile tool that can be easily arranged and rearranged to form various rectangular arrays. These arrays help children learn visually about the factors of numbers.



These arrays will help your children observe the following:

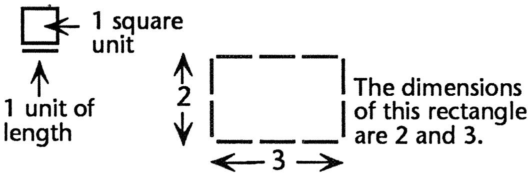
Rectangle: a 4-sided figure (polygon) that has 4 right angles.

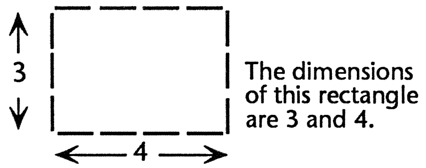
Area of a rectangular array: the number of square units used to build the array. Here each tile represents a square unit.



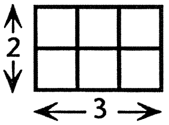
The area of this rectangle is 6 square units.

Dimensions of a rectangular array: the lengths of the adjacent sides of the array. The dimensions are measured in terms of units of length (linear or line units). In the picture below, the unit of length is the edge of a square tile.

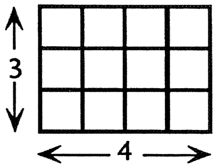




Identifying a rectangular array: an array is often referred to in terms of its dimensions.



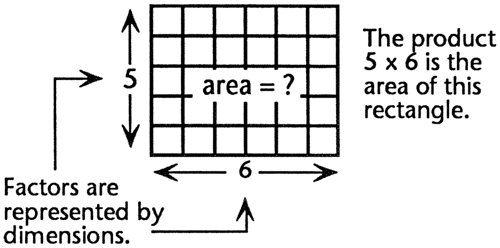
A 2 by 3 (or 3 by 2) array.



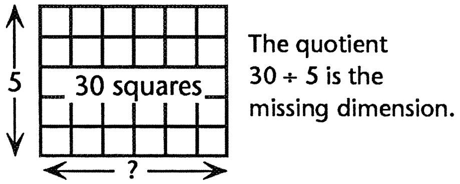
A 3 by 4 (or 4 by 3) array.

Modeling multiplication and division with arrays:

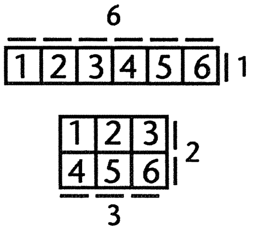
Multiplication: What is 5 × 6? or What is the area of a 5 by 6 rectangle?



Division: What is 30 divided by 5? or A rectangle has an area of 30 and one dimension 5. What is the other dimension?

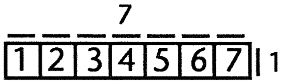


Composite numbers: Some numbers can be represented by more than one rectangular array. These are called composite numbers. Composite numbers have more than 2 factors.



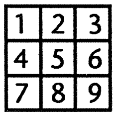
Six is a composite number. It has more than 2 factors (1, 2, 3 and 6).

Prime numbers: Some numbers have exactly 2 factors and can be represented by a rectangular array of tile in only one way (disregarding orientation). These are called prime numbers.

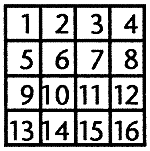


A prime number has exactly 2 factors.

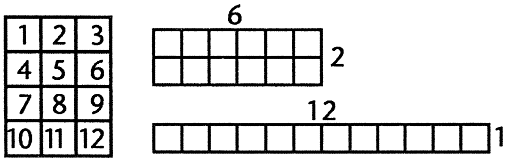
Square numbers: Numbers that can be represented by a rectangular array that is also a square are called square numbers.



9 is a square number.

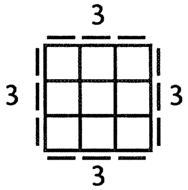


16 is a square number.



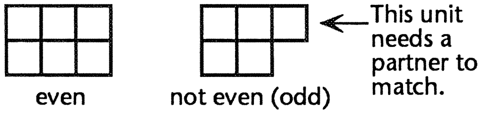
12 is a not a square number because 12 tile cannot form a square.

Each dimension of a square array represents the *square root* of the number of tile in the array.

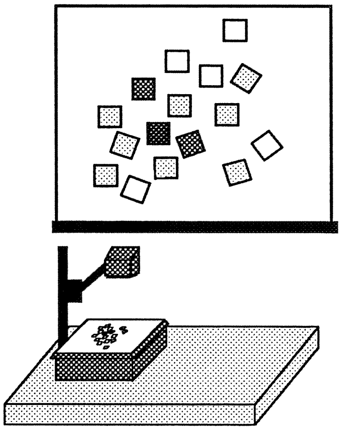


3 is the square root of 9.

Even and odd numbers: An even number of tile can always be arranged into a rectangle that has a dimension of 2. An odd number of tile will form a rectangle with a dimension of 2 that has a corner tile missing.

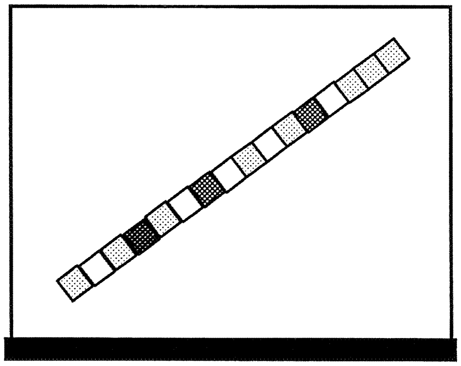


TEACHER *We have placed today's marker on the calendar grid to show that this is the sixteenth day of March. (Pulling a name from the feely box.) Laranda, would you please place 16 square units on the overhead?*

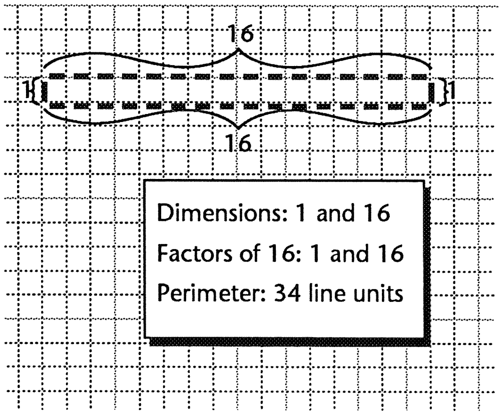


TEACHER *Who can form a rectangle from all 16 square units?*

JOSEPH *I can make a long, skinny rectangle.*

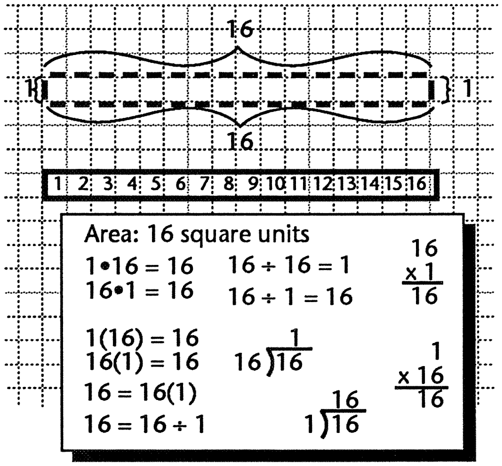


The teacher records on chart paper the observations volunteered by the children.



Dimensions: 1 and 16
Factors of 16: 1 and 16
Perimeter: 34 line units

Various ways to write expressions:



Area: 16 square units

$1 \bullet 16 = 16$
$16 \bullet 1 = 16$
$16 \div 16 = 1$
$16 \div 1 = 16$

$$\begin{array}{r} 16 \\ \times\ 1 \\ \hline 16 \end{array}$$

$1(16) = 16$
$16(1) = 16$
$16 = 16(1)$
$16 = 16 \div 1$

$$16\overline{)16}^{\,1}$$

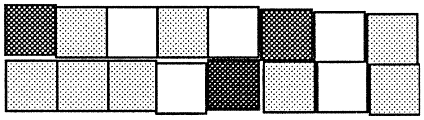
$$1\overline{)16}^{\,16}$$

$$\begin{array}{r} 1 \\ \times\ 16 \\ \hline 16 \end{array}$$

TEACHER *Can another rectangle be formed from all 16 square units?*

Another volunteer moves the tile to form a 2 by 8 array. Again, the teacher records this as the children verbalize.

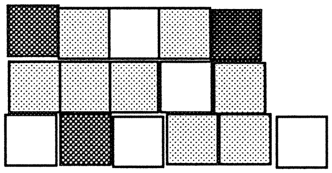
TEACHER *What information can be gathered from this array?*



TEACHER *Would anyone else like to make another rectangular array? Go for it, Amanda.*

AMANDA *I can make one with 3 in each row, I think.*

TEACHER *Rearrange them, Amanda, and see if you can make a rectangle with 3 units on one side.*



Children test the suggestion by moving the 16 tiles around to try to form a rectangle with one factor of 3.

JOSEPH *Oops. That didn't make a rectangular array. Let's try again.*

TEACHER *Could one side have 5 squares?*

JUAN *No, that won't work because every time we try it, we have a silly oddball sticking out alone messing up our rectangle.*

TEACHER *Show me what you mean.*

UNA *I don't have to make one. It's just the same as the* *one we already have. A 3×5 is the same shape as a 5×3! See?*

The child is allowed to give additional explanations if necessary.

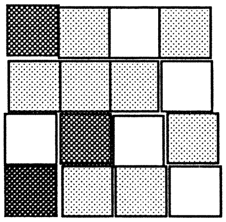
TEACHER *Well, maybe that's all we can make.*

YURI *No! Let's try 4 on one side! I think that will work!*

TEACHER *Who would like to try it?*

The volunteer moves the tile to form a 4×4 rectangular array.

DOLLY *We were right! It has 4 on one side and we used them all to make a rectangle. This rectangle is special—it's a square!*



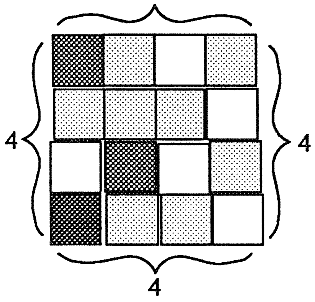
TEACHER *Is a square a rectangle?*

DOLLY *It's a special one. It has opposite sides that are parallel and 4 right angles. It is just like any other rectangle. The only difference is that a square's 4 sides are exactly the same length.*

TEACHER *Does anyone remember what we call a number that can form a square array with the colored tile?*

ZANDRA *That's not hard for us! It's just a square number! This is really neat. No matter how you turn the square, it has 4 units of length on the side. Wow, that 4 must be really important.*

TEACHER *Yes, we call 4 the square root of 16, because each side of the array is 4 linear units long.*



4 is the square root of 16

The teacher records the children's observation that 4 is an additional factor of 16. The 4×4 array is also shaded in on the grid paper. (See the complete array display on the following page.)

Component

Incredible Equations

SKILLS TAUGHT

- Problem solving
- Equations
- Sharing new mathematical ideas

INSTRUCTIONAL IDEAS

Expect to hear cheers when you move to the Incredible Equations chart, an activity most students have been eagerly awaiting. Many children will try to "gee-whiz" the entire class with a super equation.

Ask a volunteer to suggest an equation that would reflect the day's date.

Individual comfort zones will vary from child to child. Therefore, when a child suggests an equation, you may need hex-a-link cubes or other manipulatives to demonstrate how that equation is appropriate. Even if a child gives a wrong answer, encourage the class to work with it until it is fixed.

When a child gives you "2 times 4", remember that you as the teacher have several options when recording. Use a variety of ways to represent that information; for example, "2 times 4" is most often written as 2×4, but you can show that $4 + 4$, $2 + 2 + 2 + 2$, $2 \cdot 4$ or $2(4)$ could be used.

TEACHER *Who can tell me today's date by looking at the calendar grid?*

CHILDREN *Today is the 13th day of October—October 13th.*

TEACHER *Correct! I'm looking for some absolutely outstanding, incredible equations to equal 13. I hope some of you worked last night thinking of some wonderful ways to make 13!*

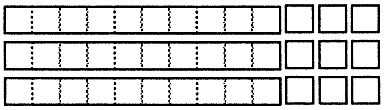
MORRIE *Oh, wow! I've got a humdinger: $5 + 3 + 5 = 13$!*

TEACHER *I'm going to record that here on the chart. Are there any more suggestions?*

VICKI *Oh, I've got one: $39 \div 3$.*

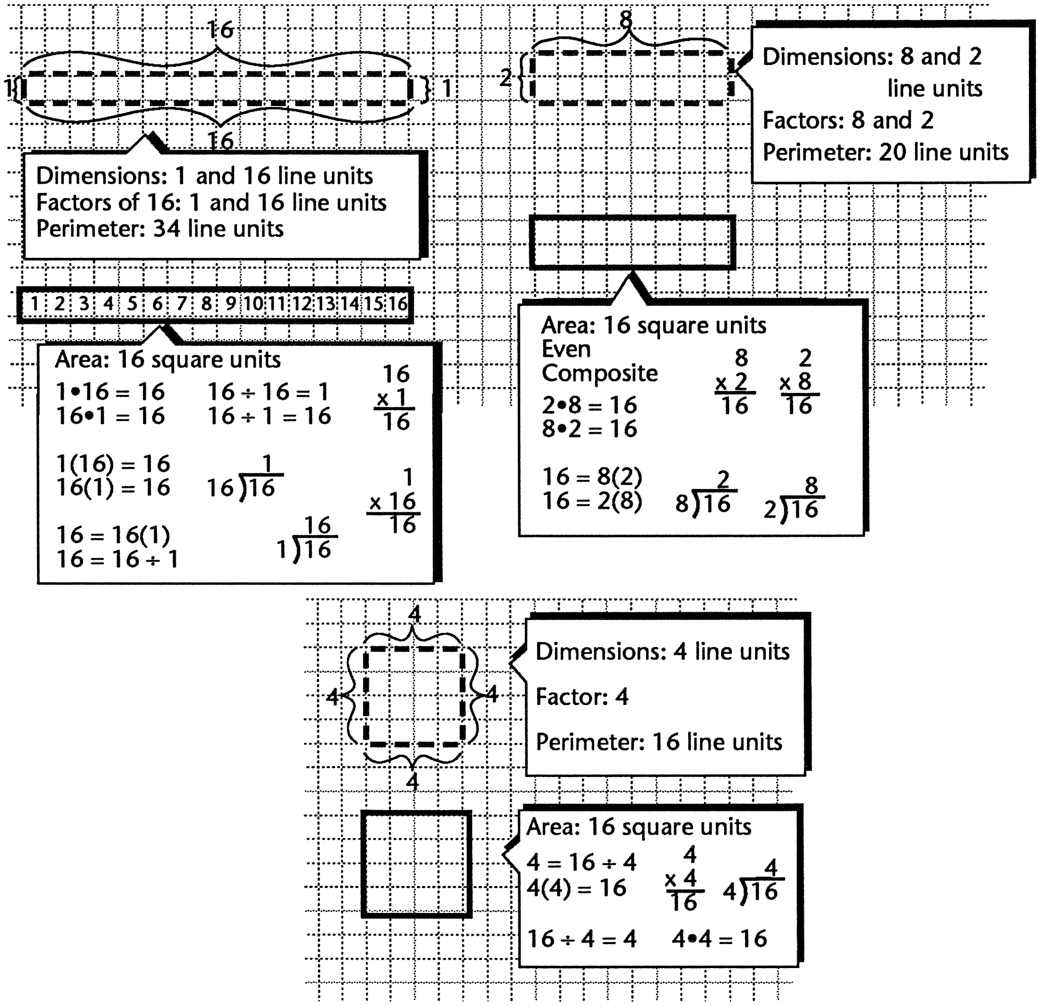
TEACHER *Let's prove that one. Can you help me?*

VICKI *(At the overhead) Sure. I'll need 3 strips and 9 units for 39. I can then make an array that has 3 rows like this:*



One dimension is 3 and the other must be 13, since each row has a strip and 3 units in it.

Complete array for 16



Dimensions: 1 and 16 line units
Factors of 16: 1 and 16 line units
Perimeter: 34 line units

Area: 16 square units

$1 \bullet 16 = 16$ $16 \div 16 = 1$ $\begin{array}{r}16\\ \times 1\\ \hline 16\end{array}$

$16 \bullet 1 = 16$ $16 \div 1 = 16$

$1(16) = 16$ $\begin{array}{r}1\\ 16\overline{)16}\end{array}$ $\begin{array}{r}1\\ \times 16\\ \hline 16\end{array}$

$16(1) = 16$

$16 = 16(1)$ $\begin{array}{r}16\\ 1\overline{)16}\end{array}$

$16 = 16 \div 1$

Dimensions: 8 and 2 line units
Factors: 8 and 2
Perimeter: 20 line units

Area: 16 square units
Even
Composite

$\begin{array}{r}8\\ \times 2\\ \hline 16\end{array}$ $\begin{array}{r}2\\ \times 8\\ \hline 16\end{array}$

$2 \bullet 8 = 16$

$8 \bullet 2 = 16$

$16 = 8(2)$ $\begin{array}{r}2\\ 8\overline{)16}\end{array}$ $\begin{array}{r}8\\ 2\overline{)16}\end{array}$

$16 = 2(8)$

Dimensions: 4 line units

Factor: 4

Perimeter: 16 line units

Area: 16 square units

$4 = 16 \div 4$ $\begin{array}{r}4\\ \times 4\\ \hline 16\end{array}$ $\begin{array}{r}4\\ 4\overline{)16}\end{array}$

$4(4) = 16$

$16 \div 4 = 4$ $4 \bullet 4 = 16$

TEACHER *That is another good equation. I'll write that one here. Are there any more volunteers.*

OTTO *I think I have one. Yesterday we said that we could make 12 by multiplying 3 by 4. So I think we could make 13 by multiplying 3 by 4 and then adding 1. Is that right?*

TEACHER *Let's check by setting up 3 fours here with the hex-a-links. Have we made 12?*

CHILDREN *Yes!*

TEACHER *Now as we add 1 more, what number have we created?*

CHILDREN *Thirteen! Hey, that was pretty good thinking.*

$$\begin{array}{r}5\\ 3\\ +5\\ \hline 13\end{array} \qquad ⑬$$

$$39 \div 3 = 13$$

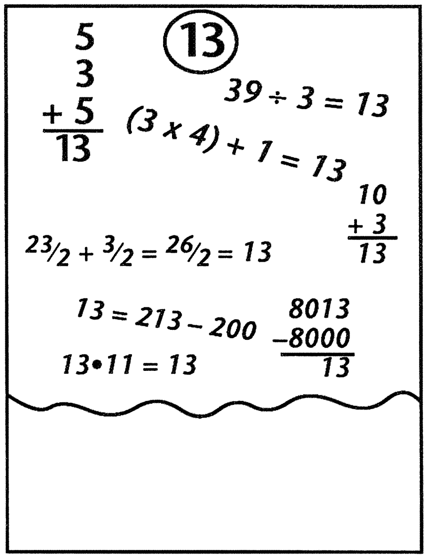
$$(3 \times 4) + 1 = 13$$

Continue accepting suggested equations for a short while, keeping the pace snappy and quick! So the magic doesn't disappear, stop while the enthusiasm is high.

This is a marvelous time to use calculators in your classroom. Choose a few children from your Name feely box to be "checkers" with their calculators. When an equation is volunteered, have it checked with the calculator. If you have an overhead calculator, it can be used to check some of the equations. We especially like the way each child can see the display.

MAKING INSTRUCTIONS

Write with markers on 24 × 36 sheets of chart paper or newsprint. After the equations have been recorded, pull a name from the Name feely box to determine who gets to take it home.



Overhead and individual calculators are available through The Math Learning Center.

Component
People Fraction Bars

SKILLS TAUGHT

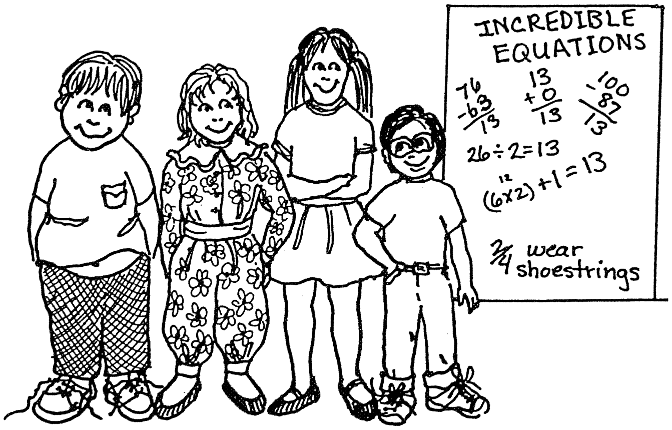
- Fraction vocabulary
- Recording fractions
- Representing fractional amounts with visual models

INSTRUCTIONAL IDEAS

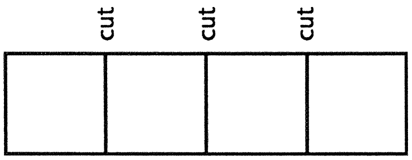
Each day invite a group of six or less children (for example, those whose middle name begins with P) to step in front of the class. These children will model people fractions.

What can be observed about these children? How can fractions be used to represent these observations?

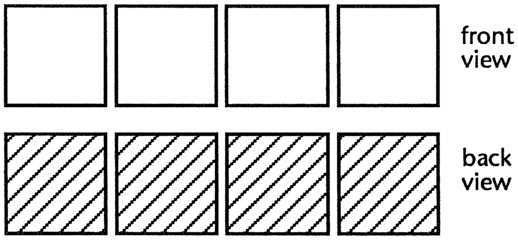
TEACHER *As I look at this fine group of four children, I notice that two of the four are wearing shoes with shoestrings. Let's record that as a fraction on our chart.*



TEACHER *Let's show that fraction with a visual model. I have cut a long, blue rectangle into four equal sections.*

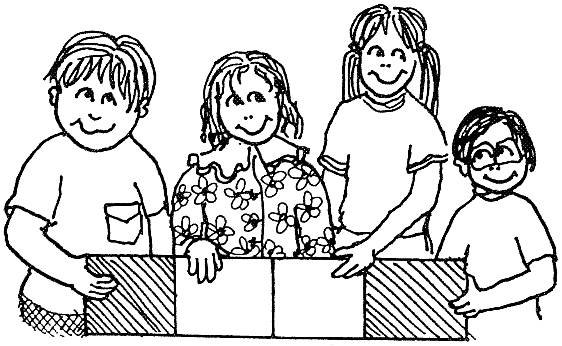


Each section is solid blue on one side and shaded on the other.



I will give one section to each of our friends. I'd like those wearing shoestrings to show the shaded sides of their cards and those without shoestrings to show the unshaded sides. By moving the cards together, we can show a long rectangle.

TEACHER *What do you notice about the rectangle?*



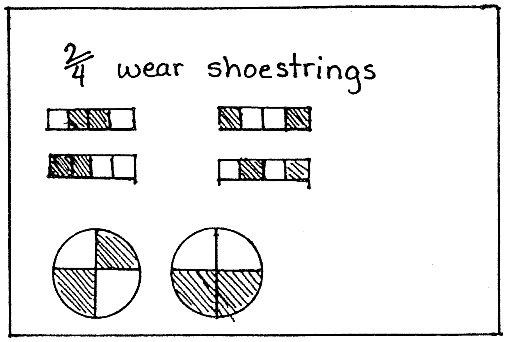
MARTHA *Two of the four sections are shaded—just like two of the four children are wearing shoestrings.*

TEACHER *(pulling a name from the feely box) Tara, will you please draw a small rectangle beside our fraction and shade it to reflect the group.*

TARA *I can draw it in several ways.*

TEACHER *(drawing a circle on the chart) How could we use this circle to show that two of the four children are wearing shoestrings?*

ARNIE *Divide the circle into four equal parts and shade two of them.*



This discussion of people fractions is only a tiny bit of what you will be doing with your children throughout the year. Spend a maximum of five minutes on this Calendar activity. The Contact Lessons of this program include similar experiences.

MAKING INSTRUCTIONS

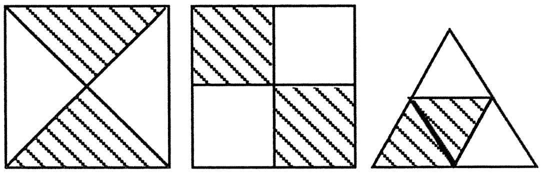
Use the bottom fourth of the Incredible Equations chart or use a separate 8 × 11 sheet of paper for this activity.

See Materials Guide for directions for making People Fractions Bars.

TEACHER TIPS

You may choose to omit People Fractions altogether on some days and use fractions to describe data gathered in other Calendar components. What fractions can be written about information shown on the Weather Graph, the Dental Floss Record, Birthday Hats, etc.

Other models can be used to depict people fractions, such as these:



Component
Clock Reading

SKILLS TAUGHT

- Counting by ones and fives
- Problem solving
- Fractions
- Before and after
- Roman numerals
- Clock awareness

INSTRUCTIONAL IDEAS

Telling time is such a difficult concept for young children because it isn't something they can reach out and touch. In all the other areas of mathematics children can be given the opportunity to work with manipulatives. But when we discuss time, regardless of how concrete we try to make it, there is still that element of abstractness. By providing the children with many, varied experiences of duration measurement, we build the foundation for telling time.

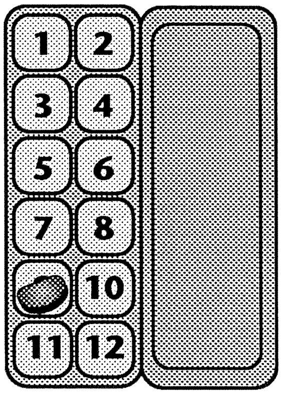
The opportunity for the children to read clocks is equally important, so incorporate this into your calendar extravaganza. Rather than worry about time itself, talk about the clock as a tool. Give your children many non-threatening opportunities to practice reading the clock in the standard round form as well as in the now-popular digital form.

An egg carton and a lima bean can become a "time machine". A time is recorded and the children discuss this time in relation to other moments in their day. Use the Name feely box to pick the time machine "shaker".

TEACHER *Ardyth, it appears that you are our lucky one today! Would you please shake our time machine?*

ARDYTH *Boy! I've been hoping you would pull my name to do this. I'm going to shake it real, real hard. Shake! Shake! Shake!*

TEACHER *Now open it. Look to see upon which the lima bean has landed to determine our hour.*



ARDYTH *It landed on 9. That means it's 9 o'clock!*

JOE *Maybe, but maybe not! We have to shake the bean again to find out if it's exactly 9 o'clock. There's only one chance that the bean will fall on 12 to make it 9:00, but how many chances that it won't fall on the 12?*

SONIA *Ummmmm...11. Right, we have 11 chances that it will land on a number other than 12. You know, it probably won't land on the 12.*

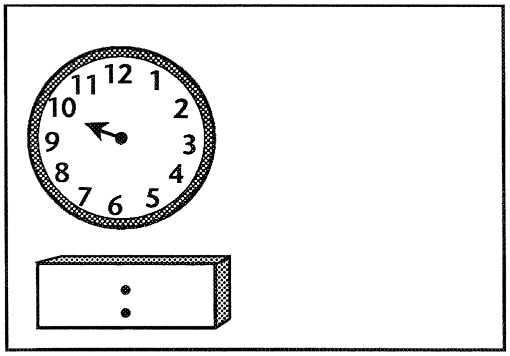
TEACHER *Now, close it and shake it again to see if it will be exactly 9:00 or not. This shake will determine the minutes.*

ARDYTH *Shake! Shake! Shake! Oh, it landed on 9 again. To show the minutes we will need to count by fives 9 times: 5, 10, 15, 20, 25, 30, 35, 40, 45—45 minutes. Man! That's almost the entire 60 minutes. That hour hand is going to need to be very close to the 10. In 15 more minutes it will become 10 o'clock.*

TEACHER *Let's record 9:45 on our round clock face. I'm going to pull a name from the feely box to do this.*

TEACHER *Shacarla, your name has appeared! Take the marker and draw the hour hand on the face of the round clock to show that 10 o'clock will be just around the corner.*

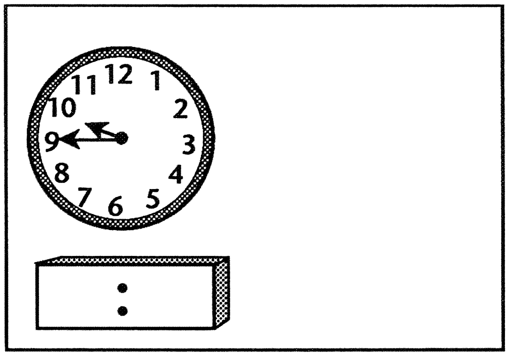
SHACARLA *I'm putting it almost on the 10 because 9:45 is very near 10 o'clock. (She draws in the hour hand between the 9 and the 10—nearer to the 10 than the 9.)*



TEACHER *Alright, now I will draw another name. This person will draw in the minute hand. Nom Pin, come on down. The time is 9:45. The minute hand needs to reflect 45 minutes past 9 o'clock.*

NOM PIN *Here, give me that marker. I know exactly how to do this! Hey guys, count with me by fives around the clock until we reach 45! That's where I'll draw the minute hand.*

Children count with the student until 45 is reached. The minute hand is drawn to the 9.



TEACHER *Now we need to put that same time on our digital clock.*

JOE *Easy smeasy! Just write 9, colon, 45. (Teacher or child records 9:45 on digital clock face.)*

TEACHER *Let's interpret the information. Exactly what is it telling us?*

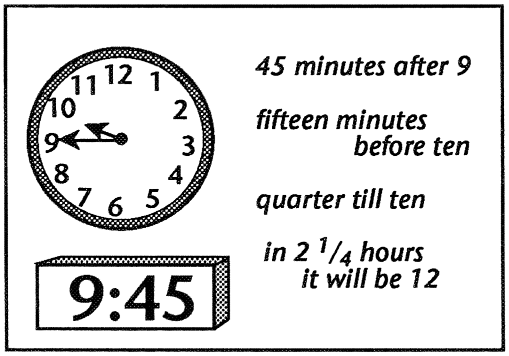
ARDYTH *Well, it is saying that the time machine has been set for 45 minutes after 9.*

LONNIE *It also says that it is a quarter till 10.*

CHET *Yeah! It also says that in 15 minutes it will be 10 o'clock.*

LYLE *Oh, Oh! I know a good one! In 2 and ¼ hours it will be 12 o'clock! I gee-whizzed you with that one, didn't I?*

Teacher records the comments from the children as they are volunteered.



TEACHER *What are we usually doing at 9:45 on a school day?*

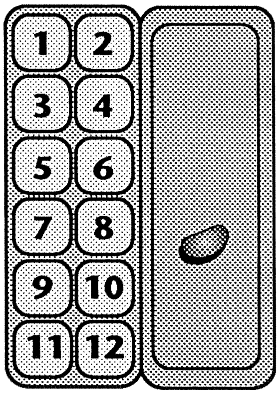
VAUNIE *That's when we have just finished creative writing and are preparing to go to P.E. Yeah; we are usually getting a little noisy at that time—you know, moving around a little more than usual because we are gettin' anxious to play.*

TEACHER *What do you expect to be doing tonight at 9:45?*

CHET *Snoozing! I go to bed at 9:00. I bet I'll be sound asleep. Or maybe I'll have my flashlight under my covers, so I can read some more in this mystery novel I checked out from the library yesterday.*

MAKING INSTRUCTIONS

Create a time machine from an egg carton and a dried lima bean (button, coin, or whatever). Use a permanent marking pen to write the numbers 1 to 12 in the sections of the egg carton.



Place the marker inside and close the lid. To determine the hour, shake the marker around vigorously in the closed egg carton. Open the lid to see where the marker landed. Close the lid again and shake vigorously. This time, see where the marker landed and then count by fives to determine the minutes to be recorded.

Duplicate the clock record sheet (see blacklines) on 9 × 12 construction paper and laminate it. Or simply create a pad of several sheets stapled together. You can tear off the top sheet each day for a child to take home.

A blackline is included showing Roman numerals on the standard clock face. Use it occasionally when children are feeling confident.

TEACHER TIPS

The repetition of this activity will slowly evolve into competent clock reading. By adding clock reading to the many duration experiences provided in other lessons, your children will slowly develop a sense of time—even though they cannot hold, measure or eat it! Be patient and allow it to develop naturally, the results are astonishing.

Component
In The News

SKILLS TAUGHT

- Current events
- World awareness
- Map skills
- Speaking and listening

INSTRUCTIONAL IDEAS

We often end the day's extravaganza by discussing current events. Children will become interested, vital citizens as they learn what is happening in their world.

Invite children to bring printed news clippings each day to share. Discuss the clippings, using this opportunity to explore geography on a large world map. Post the clippings beside the Calendar Extravaganza display. Each afternoon, return them to their owners.

Some on-going events may stimulate continued interest. It is impressive to see young children eagerly searching through the newspaper to find the latest news about the "hot topic". Ask students to leave items of special interest (or make a photostatic copy before returning them).

At times collect articles about a particular topic for several days and make a bulletin board display or create a timeline. Here you will have a record of history in the making. Perhaps fifty years from now your students will be telling their grandchildren, "I remember when...," recalling a memorable In The News discussion.

Materials Guide

The following general manipulative materials are needed to carry out the activities in *Opening Eyes to Mathematics*. It is possible that more than one teacher can share a classroom set. Conduct an inventory of the available materials in your school before ordering others.

Children will respect and carefully handle materials if the materials are well-organized and their correct use is modeled by the teacher. We have included suggestions for storing materials. In our experience, the first suggestion in each list saves the most time when distributing materials or returning lost-and-founds.

- ✂ Materials marked with a ✂ can be prepared by you or your assistants. Some of these materials can also be purchased from MLC Materials at the address given below.
- $ Materials marked with a $ are available from MLC and will need little, if any, additional preparation on your part.
- ✳ Materials marked with ✳ can be stored in a central location and easily shared by many teachers.

Order Math Learning Center materials from
MLC Materials
PO Box 3226
Salem, OR 97302
(503) 370-8130

Note: MLC Catalog symbols, preceded by a #, are included in this Guide.

Overhead Manipulatives

Each teacher will need access to an overhead projector and marking pens. If an overhead projector cannot be provided, enlarged replicas of overhead pieces can be used. Make these from poster board, laminate them and attach magnetic tape to the back. Use these pieces with a magnetic chalkboard, cookie sheet or filing cabinet.

- ✂$ Base Five Counting Pieces (#PGFO: referred to as "Base 5 Plastic Grid for Overhead Projector" in catalog)—cut apart on heavy lines
- ✂ Base Ten Counting Pieces—one set per teacher (We recommend those listed under Catalog #OH10 referred to as "Overhead Base 10 Blocks".)
- ✂ Colored Squares for Overhead Projector (#SQ)—one set per teacher
- ✂ Pattern Blocks for Overhead Projector (#PBO)—one set per teacher
- ✂ Clear Geoboard (#GBC)—one per teacher
- ✂ Overhead Calculator (#TIOH)—one per teacher
- ✂ Overhead Fraction Bars (#FBOH)—one set per teacher
- ✂ Overhead Spinner (#SPOH)—one per teacher

Storage Suggestion 1: Place each set of these manipulatives in a resealable storage bag. Label the bags and place them in a translucent storage box (available at most discount department stores). Keep the box on or near your overhead projector cart for easy access.

Storage Suggestion 2: Store the overhead collection in heavy-duty, gallon-size, resealable storage bags.

Partner Manipulative Kits
(for teams of two children)

- ✂$ Base Five Counting Pieces (#PGF)—one set per child. Purchase Base 5 Plastic Grids from MLC Materials. Or, for each child, duplicate a set on construction paper using Blackline #5 and laminate. Cut apart along the heavy lines and store in a resealable bag. (These Counting Pieces are called "Base 5 Plastic Grid" in MLC catalog.)
- ✂$ Base Ten Counting Pieces (We recommend those listed as Catalog # USM)—one set for every two children. Make individual sets of these pieces by duplicating Blackline #7 on construction paper. Laminate the page and cut the pieces apart along the heavy lines.
- ✂$ Ten-Strip Boards (#TSB)—one per child. Purchase these from MLC or create your own from Blackline #14. (Ten-Strip Board packet includes Decimal cards)
- $ Chalkboards (#MBIOS)—one per child
- ✂ Erasers—Ask your children to donate mismatched socks for this purpose. Chalk can be stored inside the sock when not in use.
- ✂$✳ Fraction Bars (#FB)—one set for every two children. Use purchased ones or prepare sets by duplicating Blacklines #90–#94 on designated colors of construction paper—halves (green), thirds (yellow), fourths (blue), fifths (purple), sixths (red), tenths (white) and twelths (orange). Laminate these Fraction Bars, cut apart, and place in a resealable storage bag.
- ✂$ Number Card Packet (#NCP)—This packet may be purchased from MLC or it can be prepared as follows: Make 1,000 cards by cutting five hundred 3 × 5 index cards in

half. Label these cards with the numbers from 1 through 1000, respectively. Randomly separate the cards into piles of 30–50 cards and place these piles in resealable storage bags. These packets are used as sources of numbers in many lessons and should be rotated among the children to provide experience with a large variety of numbers.

✂$ Linear Units (#LU)—50 to 100 for every two children. Use purchased ones or create your own by cutting coffee stirrers into 2 cm lengths.

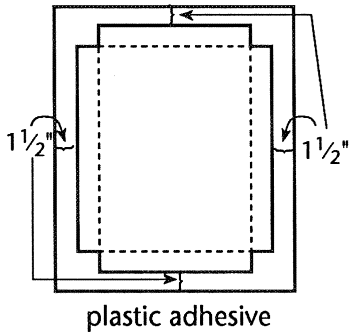
✂$ Decimal Cards—one for every two children. These may be purchased as part of the Ten-Strip Board packet or made by drawing a large dot on a 2" × 2" piece of card stock.

✂$ Individual Multiplication/Division Discussion Cards (#IDIS)—one set per child. Use purchased ones or duplicate sets from Blacklines #29–#32. Laminate, cut apart and place each set in a resealable storage bag. You may wish to have your children make an additional set that can be used at home.

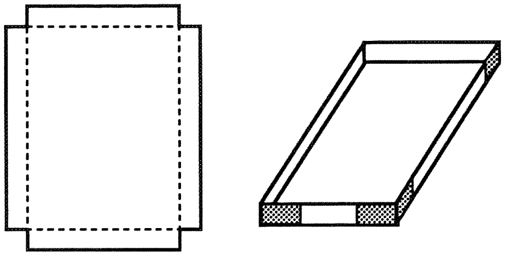
Storage Suggestion 1: Store these materials in a Basic Operations Box (BOB). The standard Box-It box from the Math Learning Center (Catalog #BL) is perfect for this purpose. Throughout the year, the contents can vary according to the concepts being explored. (For example, we begin our year with base five pieces in the BOB boxes. Later these are replaced by base ten pieces.) When a material is not being used, it can be stored in a different place. Label both ends of each box with an identification number. Likewise, except for the base ten pieces, label the contents of that box with the same number in order to make it easy to return lost-and-found items.

Follow these steps to construct your own BOB boxes:

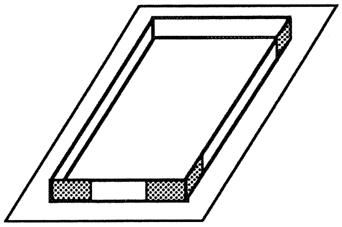
1. Cut a rectangle of adhesive plastic that is larger than the box lid.



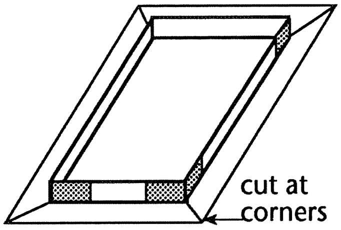
2. Assemble with strapping tape.



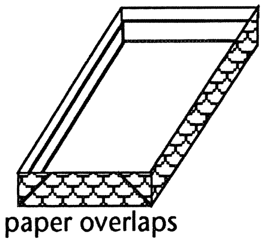
3. Remove backing from adhesive plastic covering and attach to box lid.



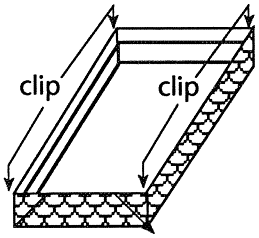
4. Smooth to remove air bubbles and clip corners.



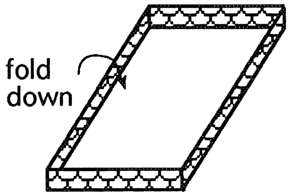
5. Fold up and lap corners.



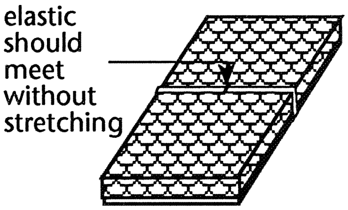
6. Clip at corners.



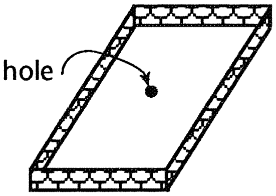
7. Fold excess to inside of lid.



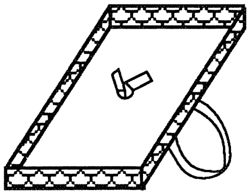
8. Place lid on box and cut one piece of $1/4$" elastic long enough to fit once around the box.



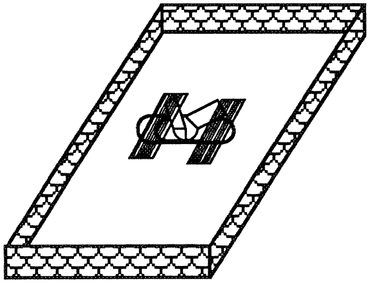
9. Remove lid and punch hole in center with scissors.



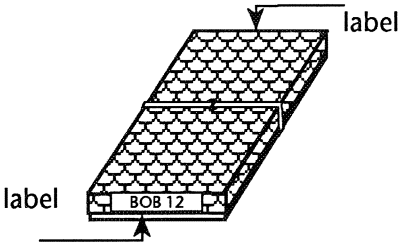
10. Insert loose ends of elastic and tie a knot.



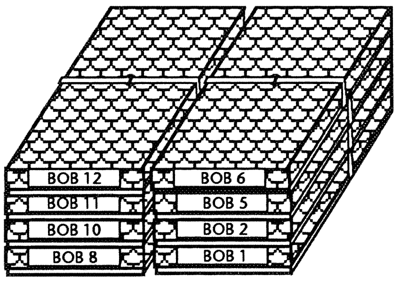
11. Slide a paper clip under knot and secure clip with strapping tape.



12. Label on both ends.



13. Boxes will easily stack for storage.



Storage Suggestion 2: An alternative storage suggestion for Partner Manipulatives is to use a labeled, heavy-duty, resealable storage bag. These may, in turn, be stored in large tubs or boxes that are clearly marked so that children may easily retrieve and replace their own materials.

Team Manipulatives Kits
(for teams of four children)

Type A Team Manipulatives

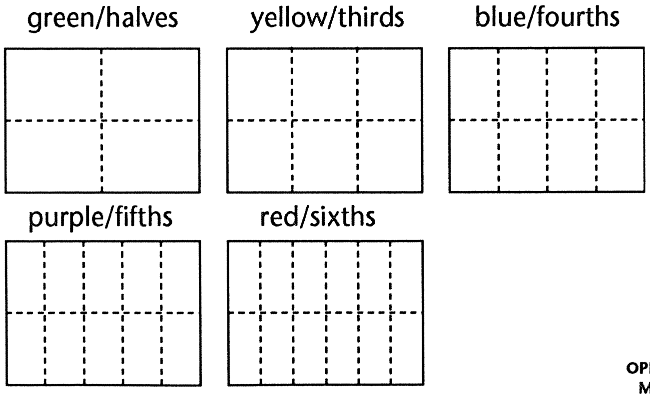
- $✻ $3/4$" Wooden Cubes (#CW75 or #CW755)—100 to 125 per team
- $✻ Hex-a-Link Cubes (#HL10 or #HL5)—100 to 125 per team
- $✻ Plastic Pattern Blocks (#PPB)—250 per team
- $✻ Popsicle Sticks (#S1000)—150 to 200 per team

Storage Suggestion 1: Store each set of manipulatives for a team in a half-size Box-It box (Catalog #BH). Assemble this box in the same way as the BOB box above. Label both ends with its contents. Color code similar manipulatives with patterns of self-adhesive plastic covering. (For example, all boxes containing wooden cubes may be covered with a shell design, while those with pattern blocks may be covered with a floral design, etc.) Lost-and-found items can then be returned easily to their rightful place.

Storage Suggestion 2: Store each set of manipulatives for a team in a translucent, plastic quart or half-gallon kitchen container with lid. Bags of objects that are alike may be stored in large tubs or boxes. Mark them clearly so children may easily retrieve and replace materials.

Type B Team Manipulatives

- ✂ Group Fraction Bars—one set for every four children. Duplicate each set on 9 × 12 construction paper using Blackline #4 to shade one of the sides. Each piece of construction paper will make two sets. Use green for halves, yellow for thirds, blue for fourths, purple for fifths and red for sixths. Laminate and then cut as shown:



✻ Toothpicks—200 for every four children

✻$ Magnifying Glasses (#MAG)—one for each child

✂✻ Elastic Loops—one for every four children. These are made by sewing together the ends of a 2-yard length of 1"-wide elastic (preferably black).

Storage Suggestion 1: For each team, store a set of these materials (with the exception of the elastic loop) in a resealable bag. When not in use, store like bags in labeled half-size boxes. Use the same pattern adhesive plastic covering for all Type B team manipulatives.

Storage Suggestion 2: Store in heavy-duty, gallon-size, resealable bag(s) or sturdy, translucent, plastic quart or half-gallon kitchen containers.

Type C Team Manipulatives

✻ Money Feely Boxes—one for every four children. Push a plastic 12 or 16 oz. soda cup into the toe of a stretchy, adult sock. Obtain a large variety of coins by asking parents to donate some pocket change. You will find it cheaper to use real money than to purchase plastic coins! The real thing is preferable and promotes a code of honor within each group.

✻ Measuring Cups—one set for every four children.

Individual Manipulatives

✂$ Large Hundreds Matrix (#LHM)—one for each child. Use purchased ones or duplicate Blacklines #38. Mount on 12 x 18 sheets of construction paper, trim, laminate and cut apart. Store on a shelf within reach of your children.

Math Journal—one for each child. Spiral-bound notebooks are useful for this purpose. Math Journals are kept in desks.

$ Calculator (#TI01 or #TI02)—one for each child. These can be purchased from MLC or perhaps children can bring some from home.

$✻ Geoboards—ideally one for each child, although they can be shared. Attach 8 to 10 rubber bands prior to distributing. (We like to use those listed under Catalog #GBC to accommodate immediate show-and-tell of creations at the overhead.)

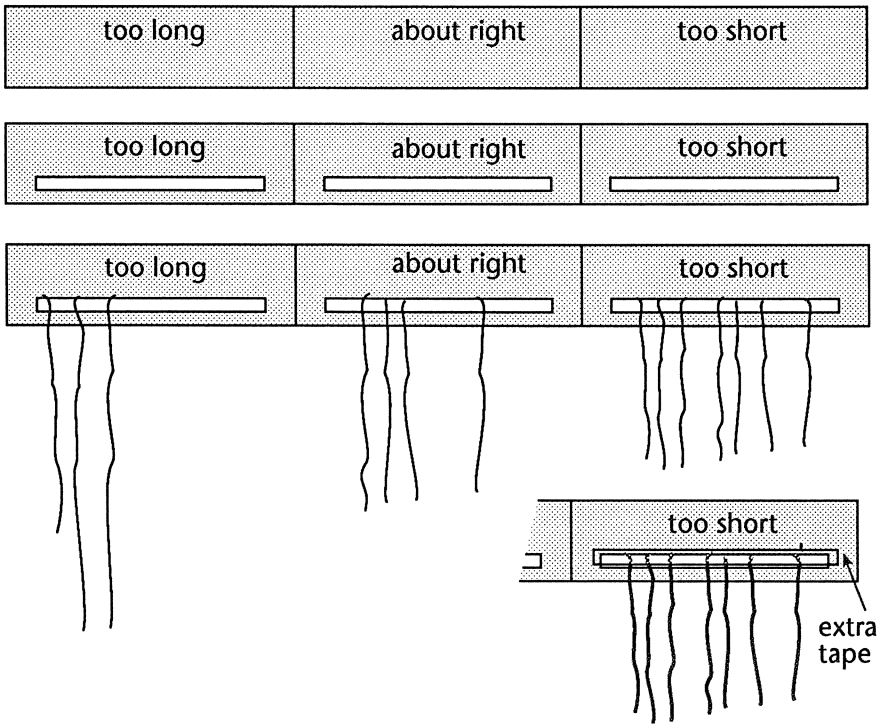
✂ Necklaces—one for each child. Form a loop by tying the ends of a 30" strand of yarn together. Use two different colors of yarn. (These are used to identify teams. For example, if you have thirty children, you will need 15 loops of one color and 15 of another. Opponents are placed side-by-side to check each other's work.)

Large Group Manipulatives

✂✻ Egg Cartons—6 with lids removed.

✻$ Colored Pom-Poms—one teacher's kit. (#SETE)

✂✻ Too Long, About Right, Too Short Graph—laminate a 24 x 10 strip of paper and label and use as shown below.



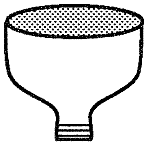
Laminate a 24 x 10 strip of paper and label.

When using these, attach loops of tape (sticky side up).

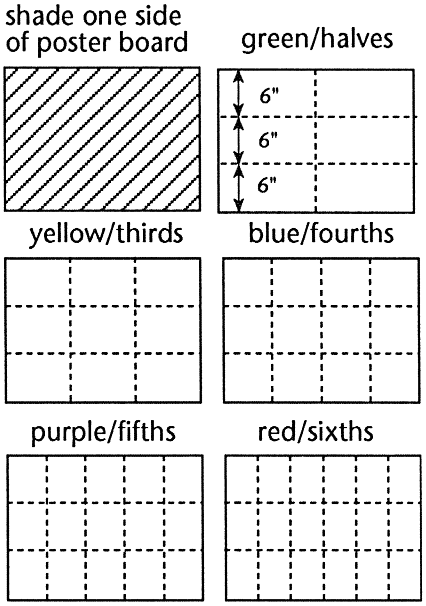
Each child attaches their string estimate of length in the appropriate section (after measuring selected object with the string).

Secure the strings with additional tape.

- ✂✳ One gallon of rice—store in an airtight container
- ✂ Several funnels. These can be made by cutting the top off two-liter soda bottles.



- ✳ Scales—one for every four children. Have several kinds available including pan, balance, kitchen and bathroom scales.
- ✳ Measuring instruments such as rulers, yardsticks, meter sticks, etc.
- ✂ People Fraction Bars—one set. You will need one sheet of each colored poster board: green, yellow, blue, purple and red. Each sheet of poster board will make three sets of bars. Create bars for halves, thirds, fourths, fifths and sixths as shown here:



Shade one side of 18 × 24 posterboard using a marker and yardstick to make slash marks. Mark off divisions as indicated for each size, then laminate and cut apart. Store each color in its own resealable bag and place in a labeled Box-It Box (or resealable bag).

- ✂$ Discussion Cards (#DIS)—one class set. Purchase from MLC and shade according to the directions. Laminate and store in resealable bags.
- ✂ Number Cubes—one of each of those described below. Assemble 10 cubes by cutting the pattern shown on Blackline #3 from poster board, laminating and folding.

Using a permanent marker, label the respective faces of nine cubes with the symbols or words listed here.

a. 0, 1, 2, 3, 4, 3
b. mats, mats, strips, strips, units, units
c. 0, 1, 2, 3, 4, 5
d. 3, 4, 5, 6, 7, 8
e. 4, 5, 6, 7, 8, 9
f. 5, 6, 7, 8, 9, 10
g. +, +, +, –, –, –
h. unit, unit, tenth, tenth, hundredth, hundredth
i. 1, 2, 6, 10, 7, 1

Keep the extra cube for occasional needs. This can be labeled with a dry, erasable marker when needed.

Construct and label a Box-It box for storing these cubes.

Metal Rings (the kind that open and snap shut). Have a variety on hand for binding books. These are used to assemble Tell It All books.

Chart Paper: Many lessons call for 24 × 36 chart paper or newsprint. You will need to have about 50 sheets on hand.

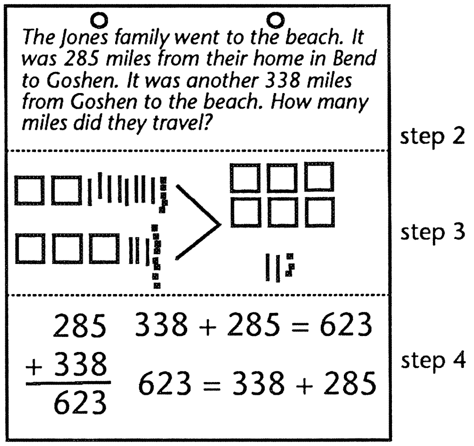
School Supplies: Your children will often use their scissors, crayons, glue, etc.

Teacher Tip: For efficient storage, punch a hole in each plastic, resealable storage bag to let the excess air escape.

Child Made Story Books

Have the children follow these directions for making Tell It All books:

Step 1: Fold a sheet of paper into thirds.



The Jones family went to the beach. It was 285 miles from their home in Bend to Goshen. It was another 338 miles from Goshen to the beach. How many miles did they travel?

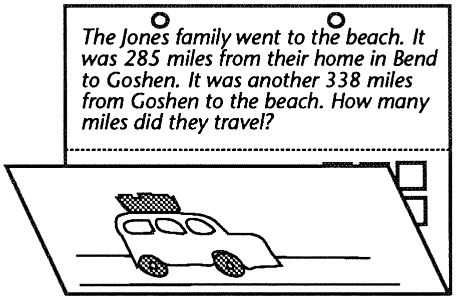
$$\begin{array}{r} 285 \\ +\,338 \\ \hline 623 \end{array} \qquad 338 + 285 = 623 \qquad 623 = 338 + 285$$

Step 2: Carefully copy the draft of their story in the top third of the paper, leaving enough space for holes to be used when binding:

Step 3: Use the middle third to model the story. This model can be a free-hand sketch or a grid paper diagram. The choice is up to the creators of the story.

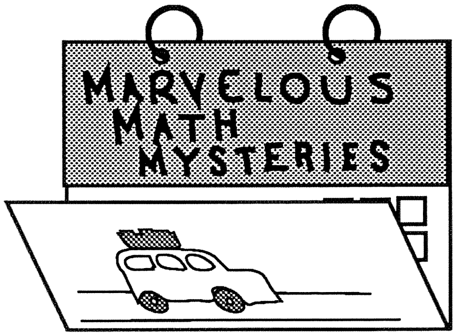
Step 4: Described the story with numbers in the bottom third of the paper.

Step 5: Draw an illustration on the folded-up bottom third of the paper.



The Jones family went to the beach. It was 285 miles from their home in Bend to Goshen. It was another 338 miles from Goshen to the beach. How many miles did they travel?

Choose volunteers to design the front and back covers of the book. Then laminate (optional) the finished stories and bind them together with metal rings.



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I

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L

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M

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N

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O

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Problem Solving: *Opening Eyes to Mathematics* provides ongoing opportunities for children to explore mathematics and develop their problem solving (and problem posing) abilities. In most lessons, mathematical investigations are used as a context for introducing concepts and strengthening skills. Other lessons, such as those listed below, focus principally on analyzing problems related to school, home life, or game situations. *See also:* TRM: Chapter 1, Vol 3: Introduction.
Vol 1: CL 40-42, 44, 64, 65, 80, 81
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Q

Quadrilateral
TRM: p. 93
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R

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Rhombus
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Right Triangle
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S

Scalene Triangle
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Spatial Visualization: *See Geometry*

Sphere
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Square
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Statistics: *See Data Analysis, Graphing*

Story Problems: *See Problem Solving*

Subtraction
TRM: pp. 34-46
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Surface Area
Vol 1: CL 71, 72
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Symmetry
TRM: pp. 86-88, 90
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Vol 3: Les 9, 37, 40, 42, 49, Appendix D
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T

Tessellations
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U

Unit
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  See also: Arrays, Addition, Division, Fractions, Measurement, Multiplication, Place Value, Subtraction

V

Venn Diagram: *See Graphing, Sorting*

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W

Weight
  TRM: pp. 62, 63, 67
  Vol 1: CL 11, 55, 56, 89
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Y

Z

Zero
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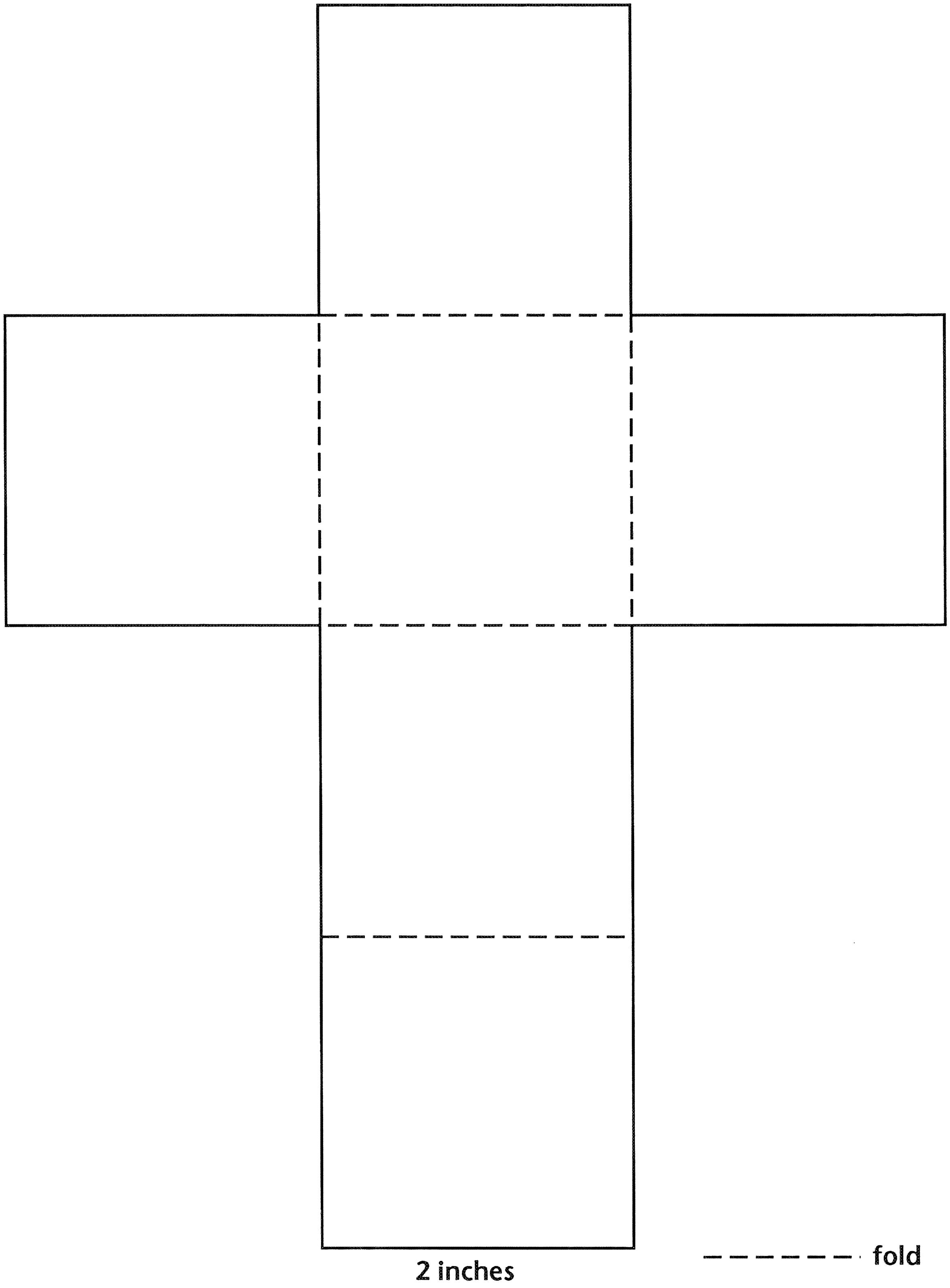
  See also: Calculating Options, Number Operations, Place Value

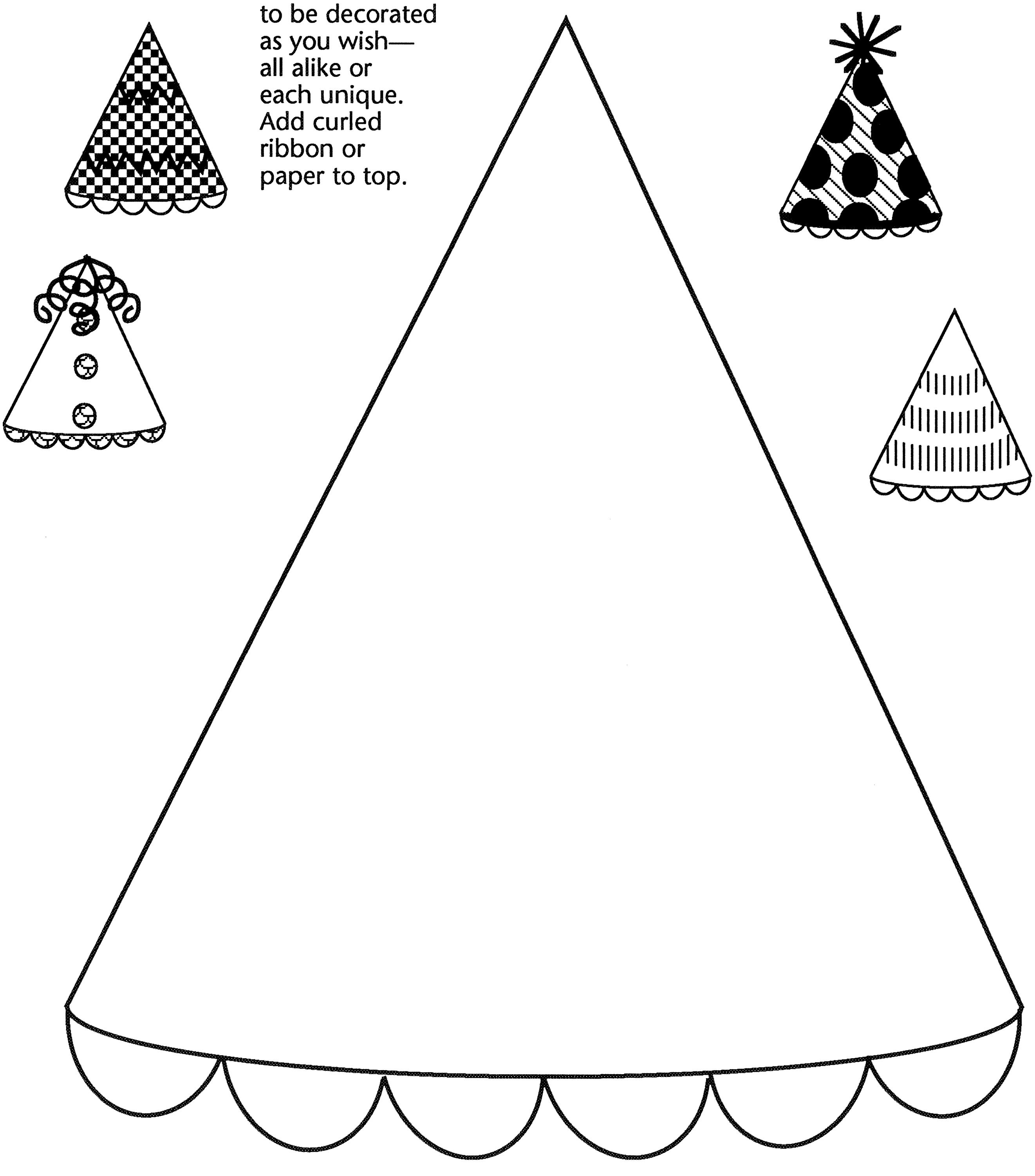
Opening Eyes to Mathematics
CALENDAR BLACKLINES—Number Cube Pattern



Opening Eyes to Mathematics
CALENDAR BLACKLINES—1" Grid

Birthday Hat

to be decorated
as you wish—
all alike or
each unique.
Add curled
ribbon or
paper to top.



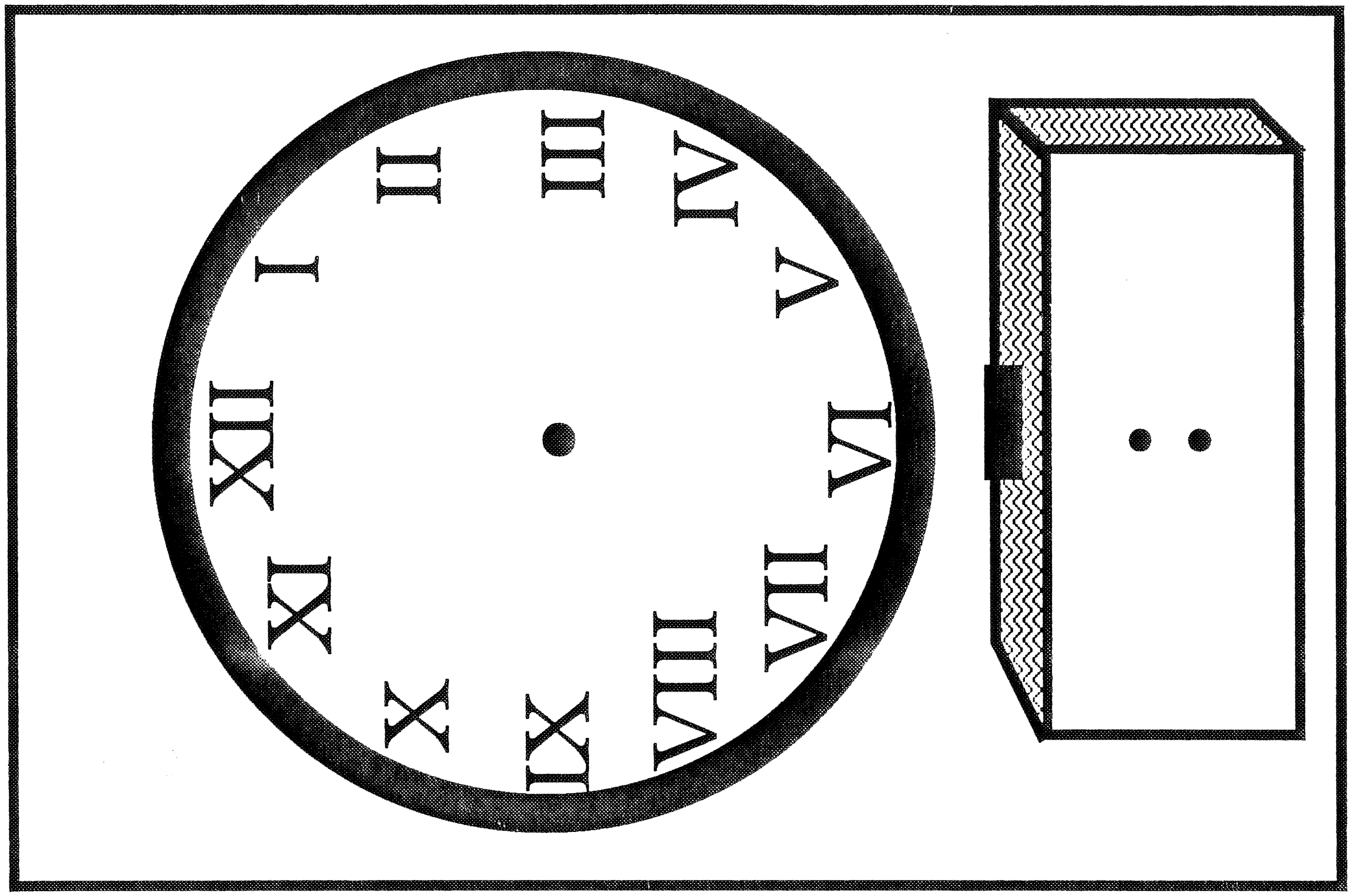
CALENDAR BLACKLINES—2 cm Grid

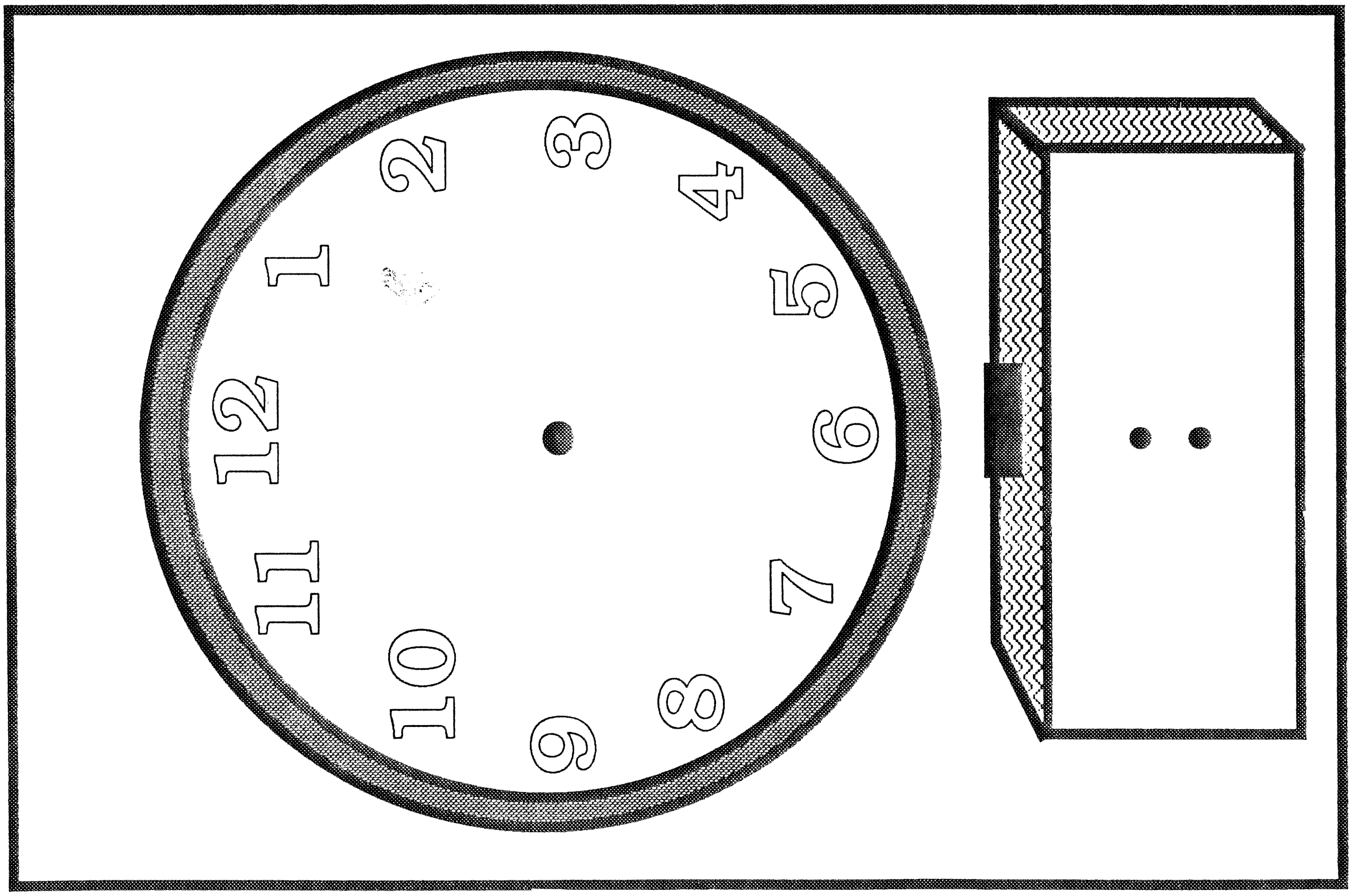
Large Hundreds Matrix

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Grid paper for Today's Array

Clockfaces





Opening Eyes to Mathematics

CALENDAR BLACKLINES—Money Record Sheet