

Unit 5 Introduction

The Excursions and Adventures in this unit both focus on two-dimensional geometry and measurement, with special emphasis on angle measures, the classification of angles and polygons, the sum of angles within polygons, the angles formed by clock hands, and reflectional symmetry. The last two Adventures explore the relationship between area and perimeter of squares and rectilinear figures.

Excursions		
Task		Targeted Concepts
A	Sneaky Scissors	Conceptual understanding of angle measurement
B	Cloud Clock	Measuring angles; angle of full rotation
C	Simultaneous Movement	Angle measure and analog clocks
D	Angle Sums 1 <i>Required before Adv. B</i>	Sum of the measures of angles in a triangle and quadrilateral
E	Two Names for Triangles	Angle classification by side length and angle measures
F	Lines of Symmetry	Classification of triangles and quadrilaterals; reflectional symmetry

Adventures		
Task		Targeted Concepts
A	Cunning Constructions	Regular polygons; factors of 360; measuring angles; angle of full rotation
B	Angle Sums 2	Patterns; sum of the measures of angles in pentagons, hexagons, etc.
C	Beyond Hexarights	Patterns; right angles; odd and even numbers
D	Passing Time	Straight angles; analog clocks; division of whole numbers
E	Maximizing Resources	Area and perimeter of rectangles and rectilinear figures
F	Square Areas	Fractional parts of a square; area; division of whole numbers; irrational numbers

Unit 5



Excursions

Task		Task Complete	Teacher Initials
A	Sneaky Scissors		
B	Cloud Clock		
C	Simultaneous Movement		
D	Angle Sums 1 <i>Required before Adv. B</i>		
E	Two Names for Triangles		
F	Lines of Symmetry		

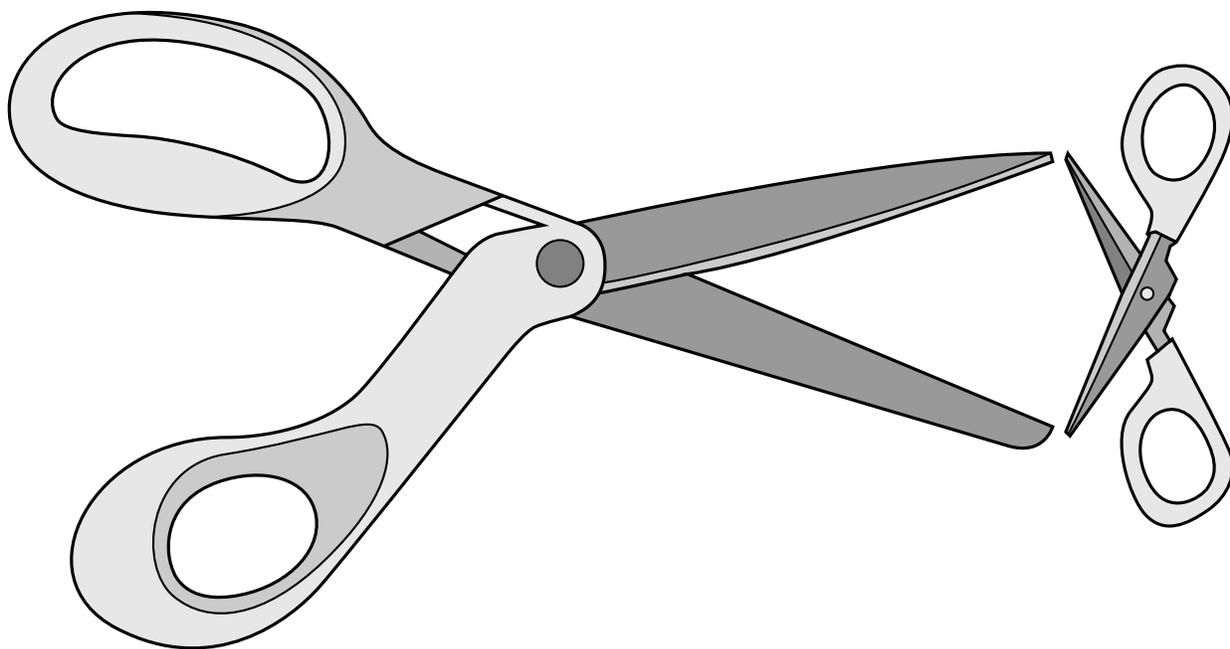


Adventures

Task		Task Complete	Teacher Initials
A	Cunning Constructions		
B	Angle Sums 2		
C	Beyond Hexarights		
D	Passing Time		
E	Maximizing Resources		
F	Square Areas		

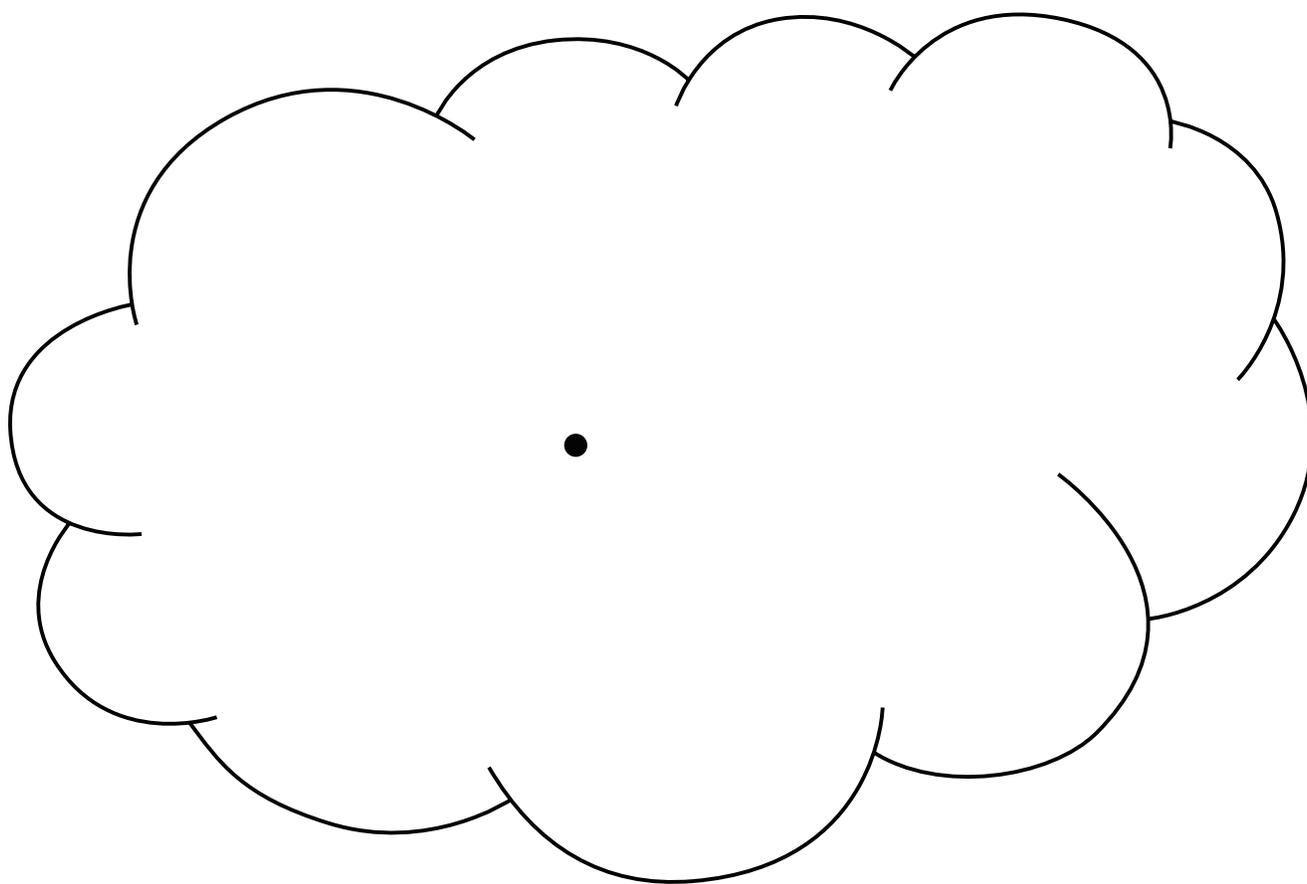
Sneaky Scissors

Which scissors are more open? Explain how you know.

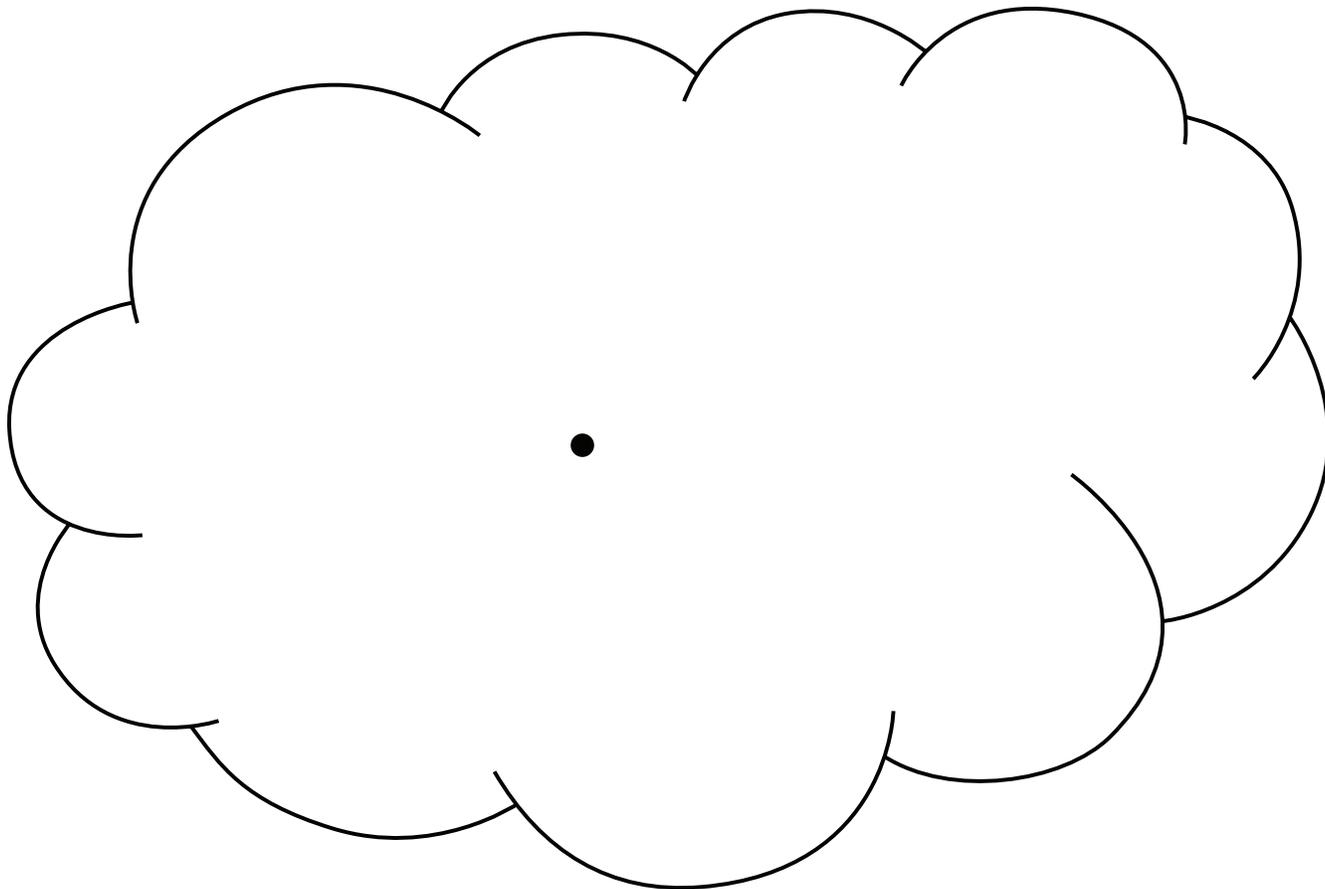


Cloud Clock

A clockmaker is making an analog clock in the shape of a cloud. The minute hand and hour hand will be centered at the dot in the cloud. Determine where the numbers 1 through 12 should be positioned inside the outline of the cloud, and mark them on the clock on the supplement page. Explain your reasoning.



Cloud Clock



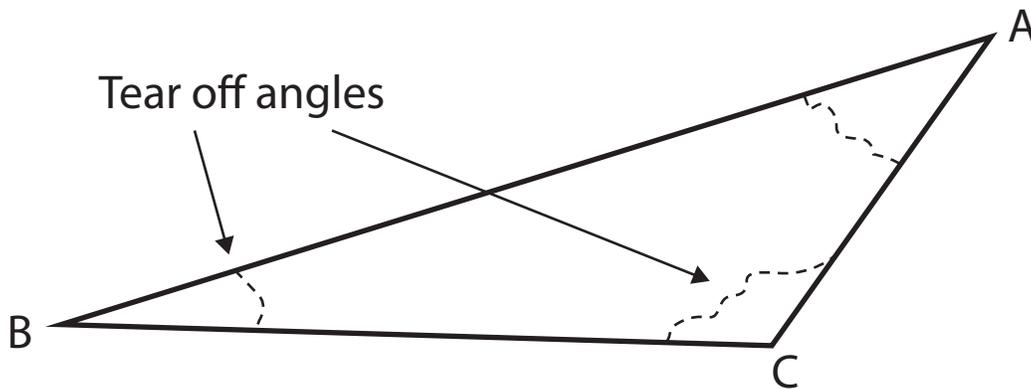
Simultaneous Movement

The hour and minute hands on an analog clock move simultaneously, although the minute hand moves significantly faster than the hour hand. Answer the following questions about this simultaneous movement.

- a. Over the course of one hour, how many degrees does the minute hand rotate?
- b. Over the course of one hour, how many degrees does the hour hand rotate?
- c. If the minute hand rotates 60° , how many minutes have passed?
- d. If the minute hand rotates 60° , how many degrees does the hour hand rotate simultaneously?
- e. If the hour hand rotates 45° , how many minutes have passed?
- f. If the hour hand rotates 45° , how many degrees does the minute hand rotate simultaneously?
- g. If the minute hand rotates 270° , how many minutes have passed?
- h. If the minute hand rotates 270° , how many degrees does the hour hand rotate simultaneously?

Angle Sums 1

What is the sum of the three angle measures of a triangle? What is the sum of the four angle measures of a quadrilateral? Follow the directions below to find out.



Cut out the **triangles** provided on the supplement page. For each triangle, tear off the three angles and place the angles together so that the vertices of the angles (points A, B, and C) are touching.

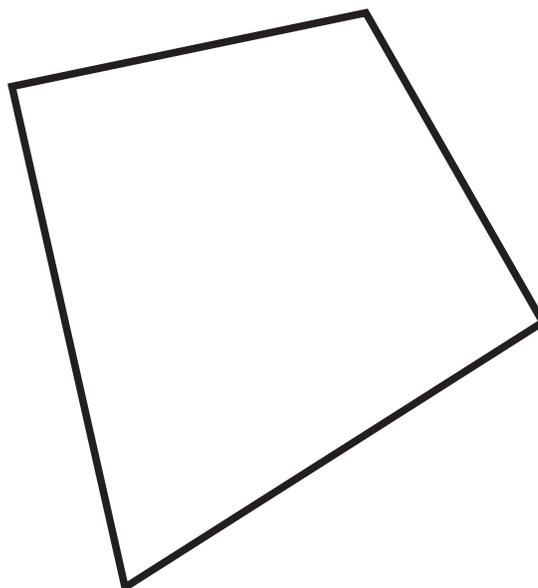
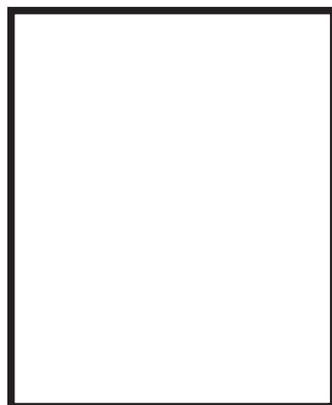
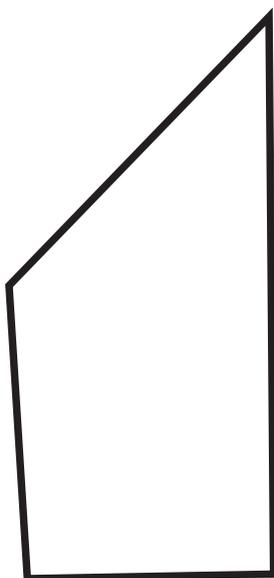
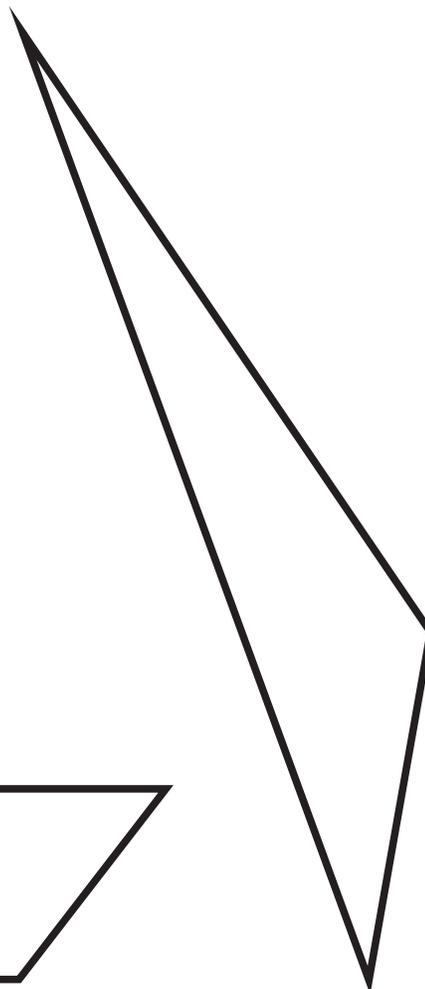
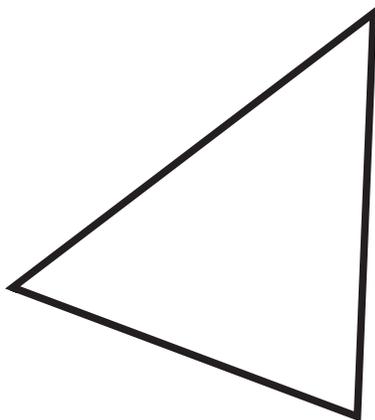
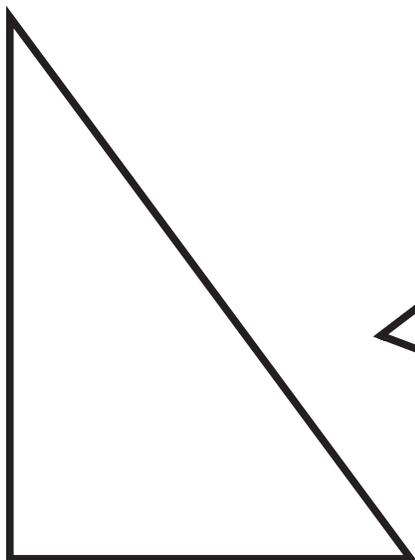
- What does this tell you about the sum of a triangle's angle measures? Are you convinced that this is true for any triangle?

Cut out the **quadrilaterals** provided on the supplement page. For each quadrilateral, tear off the four angles and place the angles together so that the vertices of the angles are touching.

- What does this tell you about the sum of a quadrilateral's angle measures? Are you convinced that this is true for any quadrilateral?

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Angle Sums 1



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Two Names for Triangles

Triangles are classified by their side lengths: scalene, isosceles, and equilateral.

- A *scalene triangle* has three sides with different lengths.
- An *isosceles triangle* has exactly two sides of equal length.
- An *equilateral triangle* has all three sides of equal length.

They are also classified by their angle measures:

- An *acute triangle* has three acute angles.
- A *right triangle* has one right angle.
- An *obtuse triangle* has one obtuse angle.

Using these terms, we can classify any triangle by its side lengths and its angle measures. In the table on the supplement page, use a straightedge to draw examples of each of the triangles. For example, in the first empty square, draw an *acute scalene* triangle. If it is impossible to draw any of the triangles, explain why it's impossible.

Two Names for Triangles

	Scalene	Isosceles	Equilateral
Acute			
Right			
Obtuse			

Lines of Symmetry

For triangles, determine:

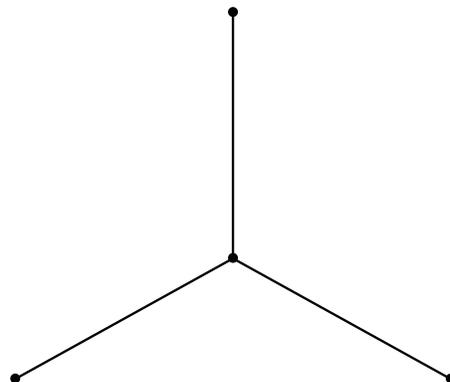
- a. What kind of triangle always has zero lines of symmetry?
- b. What kind of triangle always has exactly one line of symmetry?
- c. What kind of triangle always has exactly three lines of symmetry?

For quadrilaterals, determine:

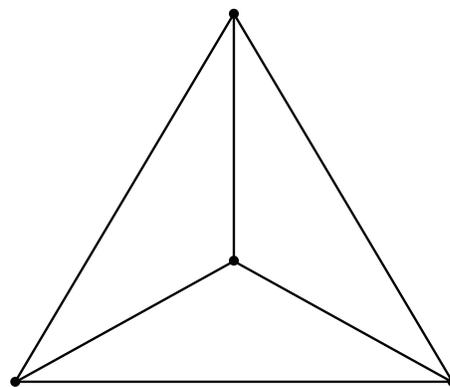
- d. What kind of quadrilateral always has exactly zero lines of symmetry?
- e. What kind of quadrilateral always has exactly one line of symmetry?
- f. What kind of quadrilateral always has exactly two lines of symmetry?
- g. What kind of quadrilateral always has exactly four lines of symmetry?

Cunning Constructions

A *regular polygon* is a polygon with equal side lengths and equal angle measures. We can make a regular triangle by drawing three equal-length lines from a point. If the lines are drawn at 120° angles ($360^\circ \div 3 = 120^\circ$), then they will be equally spaced.



Then, if we connect the ends of the lines, we will have a regular triangle (also called an equilateral or equiangular triangle).



Use this method to sketch at least four of the following shapes:

- | | |
|----------------------|---------------------|
| a. Square | b. Regular Pentagon |
| c. Regular Hexagon | d. Regular Octagon |
| e. Regular Nonagon | f. Regular Decagon |
| g. Regular Dodecagon | |

Extra Challenge: Can you use this method to construct a regular heptagon? Why does this figure provide an extra challenge?

Beyond Hexarights

You have investigated *hexarights*, six-sided shapes with all right angles. Now you will investigate what other *n-rights* are possible.

- a. Try to draw a *pentaright*. Is it possible?

- b. Try to draw a *heptaright*. Is it possible?

- c. Try to draw an *octaright*. Is it possible?

- d. Try to draw a *nonaright*. Is it possible?

- e. Try to draw a *decaright*. Is it possible?

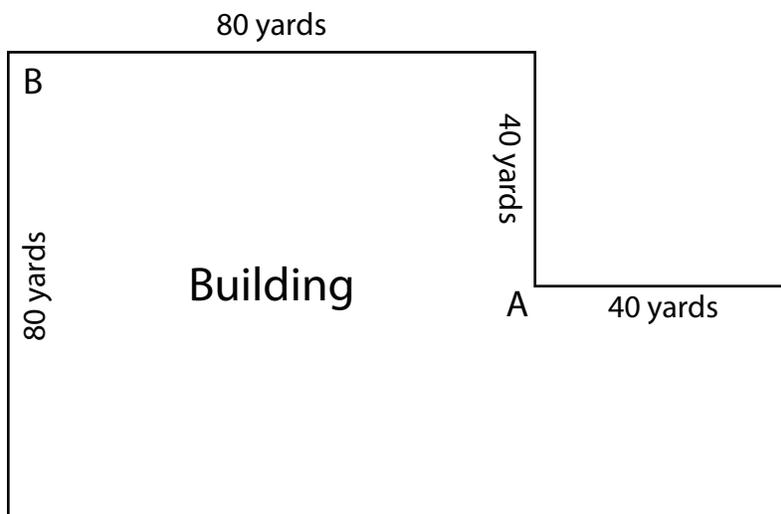
- f. Make a generalization about which *n-rights* are possible. Why are some possible but others impossible?

- g. Make a prediction about a *35-right* and a *36-right*. What do you expect, and why?

Maximizing Resources

Imagine that you have some fence that you can use to form an enclosure.

- If you had 60 one-yard sections of fencing, what would be the largest rectangular area of land you could enclose with the fence?
- If you could build your fence up against an 80-yard wall of a building, what would be the largest rectangular area of land you could enclose with the fence?
- If you needed to enclose Corner A of the building below, what would be the largest area you could enclose with the fence?
- If you needed to enclose Corner B of the building below, what would be the largest area you could enclose with the fence?

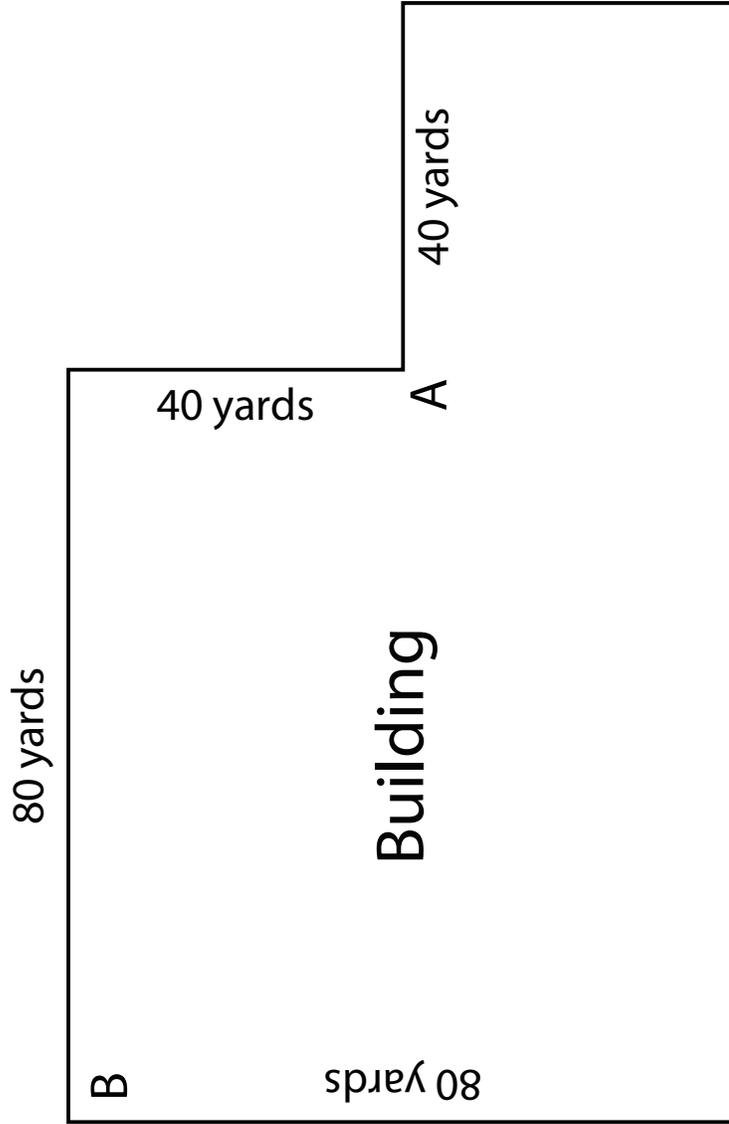


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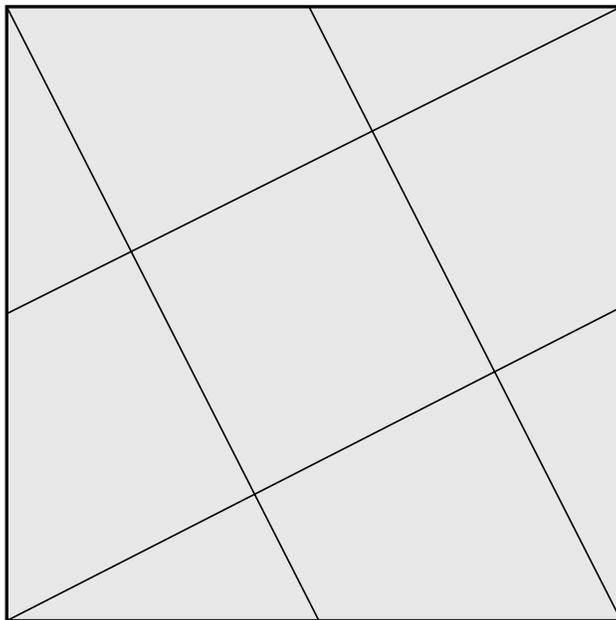
Adventure 5E Supplement

Maximizing Resources



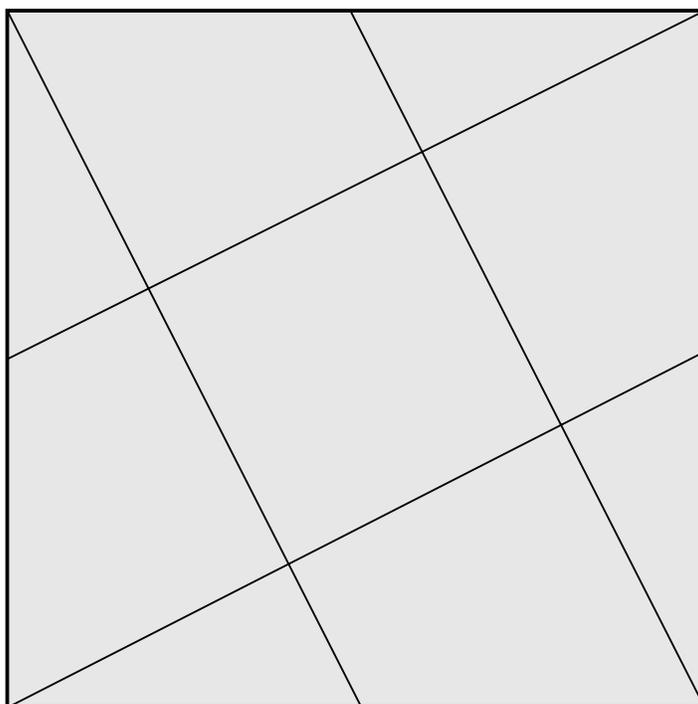
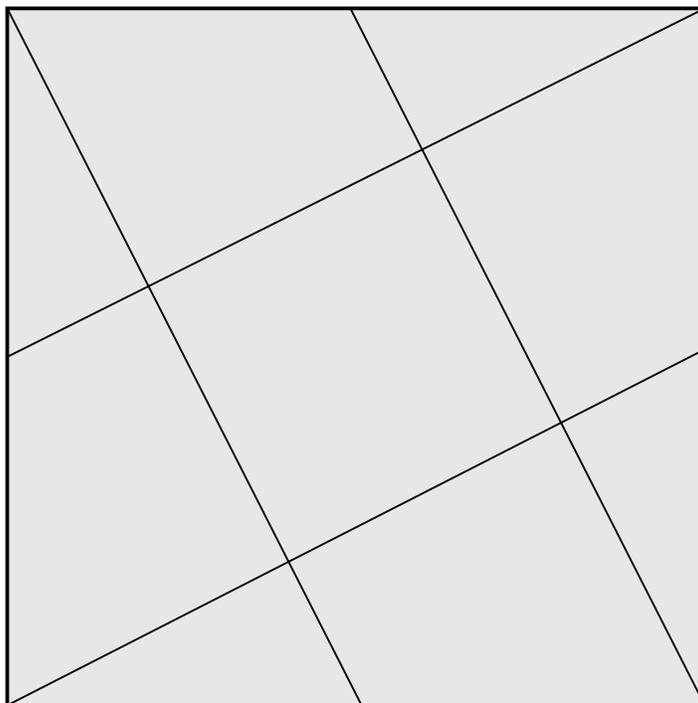
Square Areas

The outer square has sides of length 10. Use the squares on the supplement page to answer the questions below. You can cut one or both squares apart if that is helpful.



- What is the area of the smaller, inner square?
- What is the side length of the smaller, inner square, to the nearest tenth?

Square Areas



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Concept Quests

Grade 4, Unit 5 – Answer Key

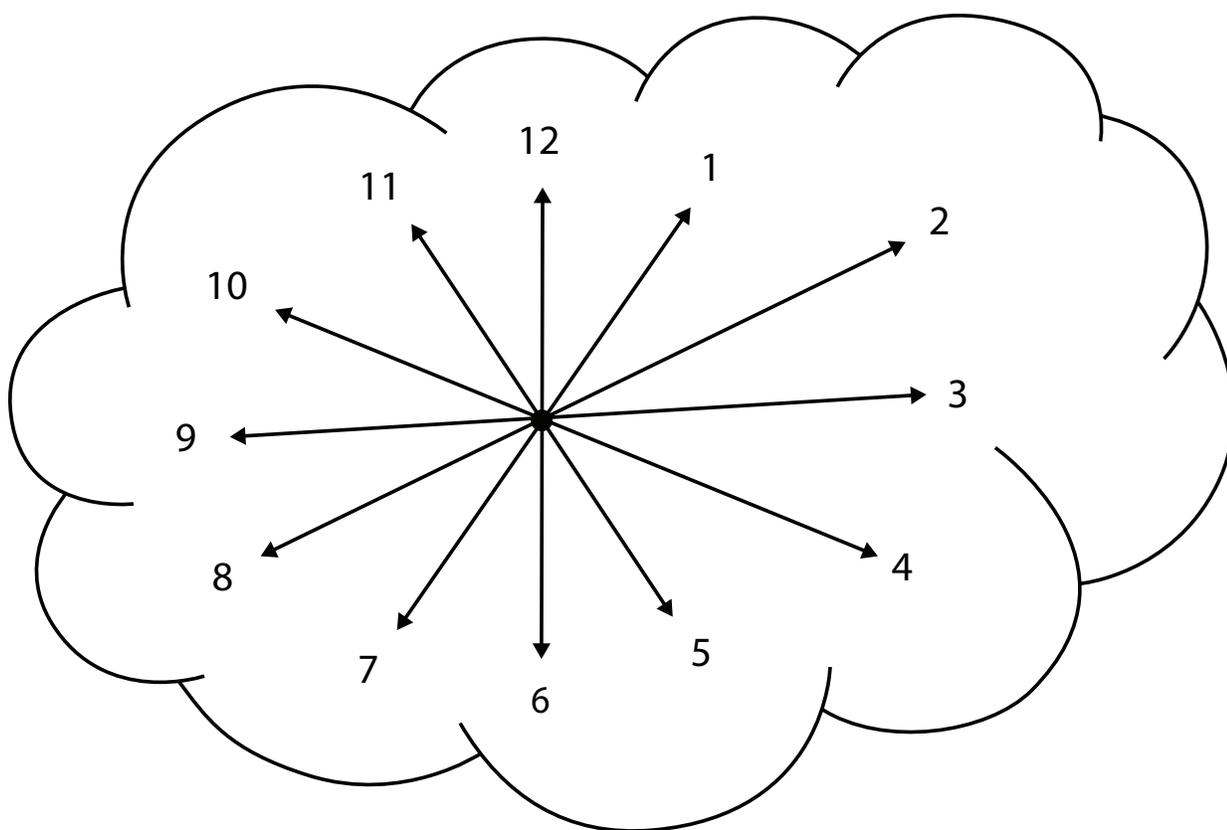
Excursion 5A: Sneaky Scissors

The small, sewing scissors are more open. The angle formed by the blades is an obtuse angle, while the angle formed by the large scissors' blades is acute.

This may surface some misconceptions if students focus incorrectly on the length of the blades, the size of the scissors, or the area revealed by the open blades.

Excursion 5B: Cloud Clock

Students should determine 30° angle increments from the anchor point. An approximation of this is provided below; students may be more accurate with their angle measures.



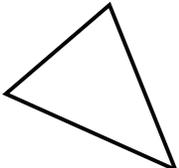
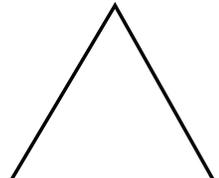
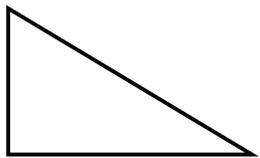
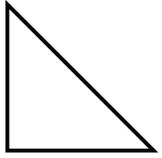
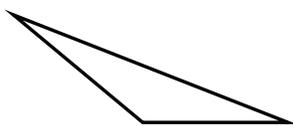
Excursion 5C: Simultaneous Movement

- a. The minute hand rotates 360° in one hour.
- b. The hour hand rotates 30° in one hour.
- c. If the minute hand rotates 60° , 10 minutes have passed.
- d. If the minute hand rotates 60° , the hour hand will move 5° .
- e. If the hour hand rotates 45° , 90 minutes have passed.
- f. If the hour hand rotates 45° , the minute hand will move 540° .
- g. If the minute hand rotates 270° , 45 minutes have passed.
- h. If the minute hand rotates 270° , the hour hand will move 22.5° .

Excursion 5D: Angle Sums 1

- a. The sum of the three angles in a triangle is 180° .
- b. The sum of the four angles in a quadrilateral is 360° .

Excursion 5E: Two Names for Triangles

	Scalene	Isosceles	Equilateral
Acute			
Right			This is impossible. An equilateral triangle has three 60° angles, so one of them cannot be 90° .
Obtuse			This is impossible. An equilateral triangle has three 60° angles, so one of them cannot be obtuse.

Excursion 5F: Lines of Symmetry

- Scalene triangles have zero lines of symmetry.
- Isosceles triangles with exactly two congruent sides always have one line of symmetry.
- Equilateral triangles have exactly three lines of symmetry.
- Parallelograms that are not rectangles or rhombi would have zero lines of symmetry. Non-isosceles trapezoids also have zero lines of symmetry.
- Kites always have exactly one line of symmetry.
- Rhombi and rectangles that are not squares have exactly two lines of symmetry.
- Squares have exactly four lines of symmetry.

Adventure 5A: Cunning Constructions

- Students will need to draw four congruent lines at 90° angles. Connecting the ends of these lines will form a square.
- Students will need to draw five congruent lines at 72° angles. Connecting the ends of these lines will form a regular pentagon.
- Students will need to draw six congruent lines at 60° angles. Connecting the ends of these lines will form a regular hexagon.
- Students will need to draw eight congruent lines at 45° angles. Connecting the ends of these lines will form a regular octagon.
- Students will need to draw nine congruent lines at 40° angles. Connecting the ends of these lines will form a regular nonagon.
- Students will need to draw 10 congruent lines at 36° angles. Connecting the ends of these lines will form a regular decagon.
- Students will need to draw 12 congruent lines at 30° angles. Connecting the ends of these lines will form a regular dodecagon. Notice the similarity to a clock face!

Extra Challenge: A regular heptagon is challenging because 360 is not a multiple of 7. Each angle would need to be $51\frac{3}{7}$ degrees.

Adventure 5B: Angle Sums 2

- The sum of the angle measures in a pentagon is 540° .
- The sum of the angle measures in a hexagon is 720° .
- For each side that is added in a polygon, 180° is added to the sum of the angle measures.

Heptagon – 900°

Octagon – 1080°

Nonagon – 1260°

Decagon – 1440°

Adventure 5C: Beyond Hexarights

- A pentaright is not possible.
- A heptaright is not possible.
- An octaright is possible.
- A nonaright is not possible.
- A decaright is possible.
- An n -right is possible, if and only if n is an even number.
- Therefore, a 35-right would be impossible, but a 36-right would be possible.

Adventure 5D: Passing Time

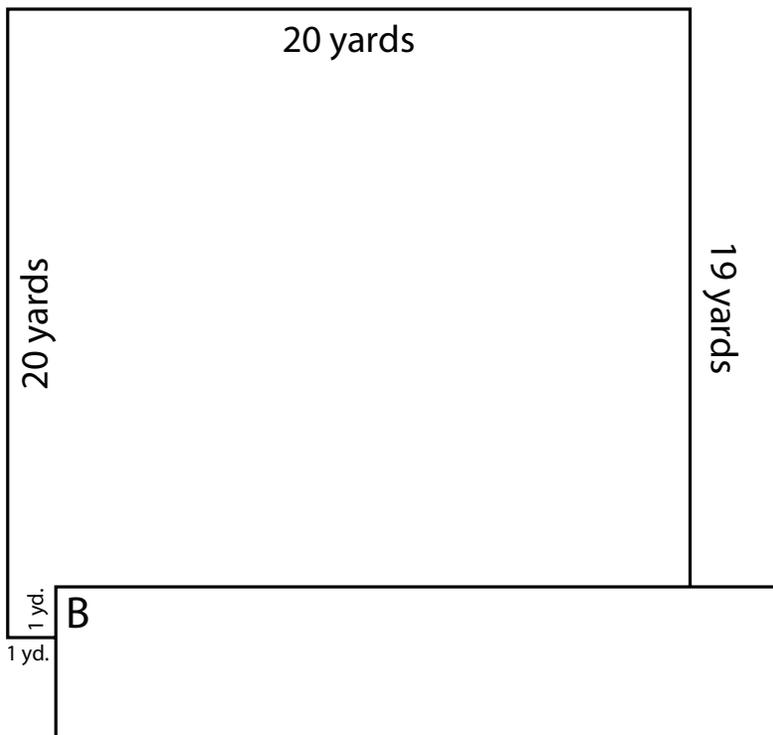
- 6:00 is the most obvious time at which the minute and hour hands form a 180° angle. There are 11 instances during any 12-hour period at which the minute and hour hands form a 180° angle. Apart from 6:00, the times below are approximate:

6:00	7:05	8:11	9:16	10:22	11:27
12:33	1:38	2:44	3:49	4:55	

- There are $12 \times 60 = 720$ minutes in 12 hours. Approximately 65 minutes ($720 \div 11$) passes between each interval.

Adventure 5E: Maximizing Resources

- The largest area that can be enclosed with 60 one-yard fence sections is 225 square yards, formed by a square with side lengths of 15 yards.
- The largest area that can be enclosed with 60 one-yard fence sections built against a wall is 450 square yards. This is formed by a 15-yard by 30-yard square, with the other 30-yard side against the wall.
- The largest area that can enclose Corner A is a 30-yard by 30-yard square, with an area of 900 square yards.
- The largest area that can enclose Corner B is 381 square yards. This can be accomplished by wrapping around corner B with a 1-yard by 1-yard square, which loses 19 square feet, as shown in the following diagram:



Adventure 5F: Square Areas

- The area of the inner square is $\frac{1}{5}$ the area of the outer square. Since the area of the outer square is 100, the area of the inner square is 20.
- For a square of area 20, the side length is approximately 4.5.

